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*PART I.*

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# THE ELEMENTS

OF

# MACHINE DESIGN

## PART I

GENERAL PRINCIPLES STRENGTH OF MATERIALS
RIVETS, BOLTS, AND OTHER FASTENINGS
JOURNALS AND SHAFTING: COUPLINGS: PEDESTALS
TRANSMISSION OF POWER BY GEARING, BELTING
ROPES AND CHAINS

BY

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# PREFACE

If originally the author had fully realised the multiplicity and complexity of the problems which arise in designing machinery, the present treatise would probably not have been written. If he had now for the first time to face the task of writing it, he would no doubt take the view that for an adequate scientific treatment of the subject a much larger treatise would be necessary. However, at the suggestion of the late Professor Merrifield the author undertook in 1877 to write a small but systematic treatise on Machine Design, and although he was shortly afterwards engaged in the onerous duties of a new Professorship the work was completed. There are now so many aids to the study of the application of scientific principles to all branches of engineering practice, and so much of engineering experience has been made accessible, that the difficulty of dealing with the subject at the time this book was written will hardly be recognised now. That it has so long continued in demand may perhaps be taken to prove that it met a want and has proved useful. Perhaps on the whole it has been not the less useful that it was not originally projected on a more ambitious scale.

In 1890 the work was revised, considerable additions were made, and the chapters relating to engine details published separately. It was again revised and enlarged in 1901. But knowledge is always advancing, and further revision and enlargement became necessary. The author felt that mere patchwork revision was no longer possible. Without any substantial alteration of plan or scope the present edition has not been merely revised, it has been almost completely rewritten. It is hoped that it has been fully brought up to date and that the modifications made will add to the completeness, accuracy, and clearness of statement of the work.

This treatise was intended to occupy a distinct field between works on Applied Mechanics and empirical books of rules and collections of examples of machine details. In common with the former, scientific rules applicable in the design of machines are given, with such an amount of explanation as is necessary to make them intelligible to readers already acquainted with the general principles of mechanics. To some extent standard proportions and examples of details are given, and empirical rules for cases which cannot be treated scientifically. But a great part of the work, and in the author's opinion the most useful part, deals with the region between applied mechanics and rule of thumb, viz. the discussion of the various considerations which should be present in the mind of the designer in applying scientific principles on the one hand, and which limit the reliance on purely practical experience on the other.

In an admirable Address to Section G of the British Association for the Advancement of Science, in 1894, Professor A. B. W. Kennedy said that 'an engineer is a man who is continually being called on to make up his mind. It may be only as to the size of a bolt, it may be as to the type of the Forth Bridge. But whatever it is, once it is settled it is decided irrevocably—it is translated into steel and iron and copper, and cannot be revoked by an act passed in another session. The matters are too complex to be dealt with mathematically or even physically. Even if they were not, there are few engineers who would have the special capacity to handle them. From this point of view the whole use of college training, of workshop practice, of practical experience is to provide the engineer later on with the means of critically examining each question as it comes up.'

That statement puts very nearly what should be the function of a treatise on Machine Design like the present, in aiding the draughtsman in his daily work. What the author aimed at was, to gather together from science and practical experience the materials for critical examination of the problems which arise in machine-designing, and, within such limitations as a short treatise permitted, to exemplify the process of criticism applied to the problems most frequently presented, whether in applying the formulæ of mechanics or in following precedents. At every

step the designer has to stop and weigh various considerations, the estimate of loads, the factor of safety, the working stress, the allowance for friction and wear, and many others. It is the multiplicity of these which render all practical problems in a sense indefinite, and from a purely scientific standpoint insoluble. But the recognition of them and the power of according to them their due relative importance in different circumstances is the distinction of the competent designer of machines.

An account is given in appropriate places of various new theoretical and experimental investigations, with references to the source of information. Some questions will be found treated in an original way leading to results which are new.

In order to avoid constant repetition, a uniform plan is adopted, as to the units employed, which is only departed from in a few cases for special reasons. Wherever there is no express statement to the contrary, the units adopted are as follows:—

Dimensions are in inches.
Loads or forces are in lbs.
Stresses are in lbs. per sq. in.
Fluid pressure is in lbs. per sq. in.
Velocities are in feet per second.
Accelerations are in ft. per sec. per sec.
Work is in foot lbs.
Speeds of rotation are in revolutions per minute, or in angular velocity per second.
Statical moments (as bending and twisting moments) are in inch lbs.

A more consistent and scientific system of units could easily be adopted, but it would involve a departure from the modes of reckoning current in the workshop.

The fault of many of the terms commonly used in the workshop and in books dealing with the subject of the strength of materials is, that they are applied to express both the forces acting on a structure and the deformations which are produced. Thus, compression means in ordinary usage either the stress acting on a bar or the strain due to its action. There is a further ambiguity arising from the use of the same words for a quantity

and an intensity. Thus elongation and compression are used either for the whole deformation or for the deformation per unit of length.

An attempt has been made to avoid some of the ambiguities arising from this double use of the same terms. The following short scheme may be useful for reference:

$$\frac{\text{Stress}}{\text{Strain}} = \frac{\text{Tension}}{\text{Extension}} \text{ or } \frac{\text{Pressure}}{\text{Compression}} \text{ or } \frac{\text{Shearing stress}}{\text{Shearing strain}} = \text{Corresponding Elasticity.}$$

$$\text{Also Extension} = \frac{\text{Elongation}}{\text{original length}}$$

$$\text{and Compression} = \frac{\text{Contraction}}{\text{original length}}.$$

*May* 1909.

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# ELEMENTS

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# MACHINE DESIGN

## CHAPTER I

### MATERIALS USED IN MACHINE CONSTRUCTION

THE parts of a machine support loads or transmit forces in accomplishing the useful work for which they are intended. They are proportioned to resist these forces without breaking or inconvenient deformation, and at the same time, for economy's sake, without unnecessary expenditure of material. The materials suitable for the construction of machines must be *strong* and *rigid*. The stronger the material, the smaller will be the bulk, and, speaking generally, the weight and cost of the machine. But the material must also be practically rigid; that is, it must not sensibly alter in size or shape under the straining actions to which it is subjected. No material is absolutely rigid, and some deformation is produced by any straining action or by change of temperature. But though such deformations must occur in machines, the engineer so chooses and proportions his materials that they are small enough to be neglected or provided for. The connecting rod of an engine, 10 feet in length, elongates and shortens about $\frac{1}{50}$th of an inch under the action of the alternate thrust and pull to which it is subjected in ordinary work, but this occasions no practical inconvenience. A long bridge girder varies in length some inches with changes of temperature, but this can be provided for by roller beds on the abutment. The changes of form which are produced in materials by the action of mechanical forces are either *temporary*,

that is, vanishing when the forces are removed; or *permanent*, that is, remaining after the straining force is removed. The former are termed elastic, the latter plastic deformations. For many reasons permanent changes of form are objectionable in machines, and hence materials for machine construction must usually be elastic materials, at least in ordinary conditions of use. Up to certain limits of loading, termed elastic limits, metals are almost perfectly elastic, and beyond those limits plastic deformations are added to the elastic deformations; this limits the range of straining action which can be permitted. Some materials which are elastic at ordinary temperatures are plastic at higher temperatures, and this is taken advantage of in processes by which materials are shaped. Some machinery parts are subjected to wear or abrasion, and to resist this they must be hard. Also some machine parts are subjected to dynamical actions, blows or impact or suddenly applied loads. The power of resisting such actions depends partly on the strength, partly on the elastic deformability of the material. A material which resists dynamical actions well is termed *tough*, one which resists them badly is termed *brittle*. Generally hardness and toughness are antagonistic qualities, and a compromise must be made securing a sufficiency of each. A material which is plastically deformed is generally hardened, but very generally this induced hardness is removable, by heating and slow cooling for some materials, or rapid for others, a process termed *annealing*. In materials which have to be subjected to rough treatment in the workshop, to rolling, bending, stamping, or hammering, toughness is of great importance, though it may be unimportant in the finished machine.

For use in construction, materials must be reduced to definite size and shape, and this is effected generally by one of three methods: (*a*) The material is reduced to a fluid state by heat and cast in a mould. (*b*) It is shaped by plastic deformation. Sometimes this is effected cold as in wiredrawing, stamping in dies, &c. More often the material is rendered plastic by heat and then shaped by rolling, forging, or hammering. (*c*) It is cut to shape by edge tools as in turning, planing, milling, sawing, filing, &c. Generally the material is roughly shaped by (*a*) or (*b*) and then more exactly by (*c*).

1. *Tests of the Quality of Materials.*—The consequences of the failure of machinery when working are so serious that a heavy

responsibility rests on the engineer to secure material for construction of known and satisfactory quality. Before accepting material it is generally subjected to tests of quality, and this is specially necessary in the case of wrought iron and steel, the properties of which vary greatly according to the care and skill exercised in their manufacture. Recently, standard specifications for material for various purposes have been prepared by the British Standards Committee,[1] and these include a statement of the tests to which materials should be subjected before reception.

Tests of material for construction are mainly directed to determine *strength* and *ductility*. Sometimes tests of chemical purity are also exacted as an indirect means of ensuring good quality, but the engineer is mainly concerned with mechanical tests. The strength and ductility are usually determined by subjecting a test bar to fracture by a gradually increased tensile load. Bend tests and temper bend tests are rough but valuable tests of ductility. Impact and fatigue tests and hardness tests are resorted to in special cases.

2. *Tensile Tests.*—A bar of ductile material such as wrought iron or mild steel, subjected to a gradually increasing tensile load, behaves in the following way:—It at first stretches uniformly along its length and proportionally to the load. At a certain load, termed the elastic limit, the proportionality of load and elongation sensibly ceases and part of the elongation is plastic or permanent set. Usually at a little greater load there is a large increase of plastic elongation without increase of load, the material is said to yield and the stress limit at which it occurs is termed the yield point. If the load is still increased the elongation also increases and more than in proportion to the increase of load. A maximum of load is reached which is conventionally termed the breaking load. But the bar does not instantly break. A short portion of the bar elongates rapidly and contracts in section, forming a local contraction or neck. During this period the load required to balance the tension in the bar is a diminishing one, in consequence of the rapid contraction of area at the neck. Finally the bar breaks. The elongation measured after fracture is not uniformly distributed along the bar. It is greatest in the short portion in which a local

[1] Published for the Standards Committee by Crosby Lockwood & Son.

contraction or neck is formed. Along the rest of the bar it diminishes a little from near the local contraction towards the ends.

If P is the greatest load carried by the bar before fracture in lbs. and $a$ the original cross section of the bar in square inches, then $P/a$ is what is termed the *breaking stress* of the material in lbs. per square inch. But this is a merely conventional quantity. The real stress on the reduced section at the moment of fracture in a ductile material is greater than the so-called breaking stress. If $P_e$ is the load at the limit of elasticity and $P_y$ the load at the yield point, then $P_e/a$ is the *elastic strength* and $P_y/a$ the *yield stress*. The ratios $P_e/P$ and $P_y/P$ are somewhat characteristic in different materials. In iron and mild steel $P_y/P$ does not fall below about 0·6. It should be noted, however, that some materials of construction, as, for instance, cast iron and stone, have strictly no definite elastic limit or yield point.

The ductility of a material in a tension test is determined by noting either the *elongation* or the *contraction of area* at the fracture. Let $l$ be the length between marks on the unstrained bar termed the *gauge length* and $l + \lambda$ the length between the same marks after fracture. Then $\lambda$ is the elongation in the gauge length $l$. The elongation per cent., which is taken as the measure of ductility, is—

$$e = 100\frac{\lambda}{l} \qquad (1)$$

If $a$ is the cross section before and $a_1$ the cross section after fracture, measured at the fracture, $a - a_1$ is the contraction of area, and the contraction per cent., which is also used as a measure of ductility, is—

$$c = 100\frac{a - a_1}{a} \qquad (2)$$

In general $e$ and $c$ have no definite ratio, but they correspond pretty closely, if the gauge length $l$ is very short. For a short enough gauge length, approximately—

$$a\,l = a_1(l + \lambda)$$

$$\frac{e}{c} = \frac{\lambda}{l} \therefore \frac{a}{a - a_1} = \frac{a}{a_1} = \frac{l + \lambda}{l}$$

The elongation can be measured more accurately than

the contraction of area, and is less affected by small local defects.

3. *Elongation Equation.*—It follows from the irregular distribution of elongation along the test bar that the per cent. of elongation for any given material is different for test bars of different proportions. The elongation is really made up of a part which is independent of the cross section and proportional to the gauge length, and a local extension near the fracture which varies with the cross section.

It was first shown by Barba that, for a given material, the per cent. of elongation is constant only when the portions of bars between the gauge points are geometrically similar, or at least when for different test bars $l/\sqrt{a}$ is constant. In tests made on test bars of different proportions, therefore, the per cent. of elongation depends both on the gauge length and the cross section. Very approximately for any given bar [1]—

$$\lambda = \alpha + \beta l$$

where $\alpha$ is a constant depending on the local and $\beta$ a constant depending on the general elongation. The per cent. of elongation is—

$$e = 100 \left(\frac{a}{l} + \beta\right),$$

the per cent. of elongation diminishing as the gauge length $l$ is greater.

It is not convenient in commercial testing to fix strictly the cross section of test bars; and, on the other hand, it is very convenient in many cases to have a constant gauge length, usually 8 inches. It is desirable, therefore, to know how the percentage of elongation varies *both with differences of gauge length and area of cross section.* The author has found that for test bars of different proportions the following relation is approximately true:—

$$e = \frac{y\sqrt{a}}{l} + \beta \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

where $y$ and $\beta$ are nearly constant for a given material. The factor 100 has been merged in the constants. The following are some average values of these constants:—

[1] Unwin, *Tensile Tests of Mild Steel and the Relation of Elongation to the Size of the Test Bar*, Proc. Inst. of Civil Engineers, clv.

CONSTANTS IN ELONGATION EQUATION

| Material | $\gamma$ | $\beta$ |
|---|---|---|
| Wrought iron | 33·0 | 20·0 |
| Steel. Ship plates. Lengthways | 67·0 | 17·6 |
| ,, ,, Across | 65·5 | 16·2 |
| ,, Boiler plates. Lengthways | 77·5 | 18·9 |
| ,, ,, Across | 56·4 | 15·7 |
| Steel axles | 39·2 | 20·6 |
| Steel tyres | 27·2 | 13·1 |
| Cast gun-metal | 8·3 | 10·6 |
| Rolled brass | 102·0 | 10·0 |
| Rolled copper | 84·0 | 0·8 |
| Annealed copper | 125·0 | 35·0 |

The Standards Committee have prescribed certain standard forms of test bars to secure as far as is practicable that the percentage of elongation in different tests should be comparable. (Report 18, *Forms of Standard and Tensile Test Pieces.*)

4. *Quality Figure.*—Strength and toughness are to a great extent antagonistic qualities. In order to embrace the two in a single quality figure various functions have been suggested. The most useful is Wöhler's. Let Q be the quality figure, $f$ the tensile strength in tons per square inch, and $e$ the elongation per cent. Then

$$Q = f + e \qquad (4)$$

Q will, of course, vary with the gauge length used in finding the elongation, and values of Q are only comparable when tests are made on similar or nearly similar bars.

QUALITY FIGURES

| | Gauge Length | For Areas of Cross Section = 0·5 | 1·0 | 2·0 |
|---|---|---|---|---|
| | Ins. | | | |
| Mild steel plates | 8 | 48 to 56 | 50 to 58 | 53 to 63 |
| Steel axles | 2 | 68 to 70 | — | — |
| ,, ,, | 3 | 64 to 66 | — | — |

These are average limits. Requirements in specifications are somewhat lower.

5. *Cold Bend and Temper Bend Tests.*—A very useful test of ductility is to bend a strip over a round bar and to note the

angle through which it can be bent without cracks appearing. The strips are usually, except in cases where this is inconvenient, $1\frac{1}{2}$ inch wide; generally the sheared edges are removed; they are bent over a bar of a diameter about three times the thickness of the strip. In temper bend tests the strip is heated to blood red, and then quenched in water not over 80° F. before bending. Copper will stand bending completely double without cracking. Mild steel plates should stand bending through 180° without cracking. Strips of steel for axles, machined to a section of $1\frac{1}{4}$ inch square, with the edges rounded to a radius of $\frac{1}{16}$ inch, bent round a bar $2\frac{1}{2}$ inches diameter, should stand bending through an angle of 90° without cracking.

## CAST IRON

6. Cast iron is a material obtained direct from the ore in a blast-furnace. It is fusible, but will not temper nor weld. According to the proportions of the charge put into the blast-furnace, the pig cast iron obtained differs in appearance, in strength, and in the purposes for which it can be used. The differences of quality are due to differences of composition of the cast iron, and especially to differences in the amount and condition of the carbon it contains. In the whitest and hardest cast irons the carbon is combined with the iron. In the greyer and softer cast irons part of the carbon is separated from the iron and mixed with it in the form of particles of graphite. The quality of the iron depends in part on the amount of other constituents present, and silicon appears to have an important influence on the form the carbon takes in the cast iron.

The cast irons of commerce are generally divided into six classes. The whiter and harder cast irons are used only for conversion into wrought iron. The greyer cast irons, classed as Nos. 1, 2, and 3, are used also for foundry purposes. The greyest iron is deficient in strength. Hence most castings are composed of mixtures of Nos. 1, 2, and 3, in proportions varied according to the judgment of the founder. The larger the casting, and the stronger it requires to be, the less is the proportion of No. 1 which is used. Generally cast iron is improved in strength by re-melting.

Cast iron usually contains 3 to $4\frac{1}{2}$ per cent. of carbon. In white iron this is entirely combined with the iron. In grey

iron from 0·6 to 1·5 per cent. is combined, and the remainder, 1·8 to 3·2 per cent., crystallises separately as graphite. The next most important constituent is silicon, which may vary from 0·15 to 5 per cent.

The following useful table, constructed by Prof. Turner, shows the relation of quality and composition in foundry irons :—

| | Combined Carbon | Graphitic Carbon | Silicon |
|---|---|---|---|
| Greatest softness | 0·15 | 3·1 | 2·5 |
| ,, hardness | – | — | under 0·8 |
| ,, general strength | 0·5 | 2·8 | 1·42 |
| ,, tensile strength | 0·47 | — | 1·8 |
| ,, crushing strength | over 1·0 | under 2·6 | about 0·8 |
| ,, transverse strength | 0·7 | — | 1·5 |

Cast-iron machine parts are formed by melting the pig iron in a cupola and pouring the melted cast iron into moulds.

A *pattern* is first made of the exact shape of the casting required. A *mould* is then formed from this in foundry sand or loam. Then the molten iron is poured into the mould. After solidification the sand is cleared away.

The patterns are commonly made of yellow pine, or, when small, of mahogany. Metal patterns are used when a great number of similar castings are required. As the cast iron contracts about $\frac{1}{8}$th of an inch per foot in each direction, the pattern is made larger than the required casting in that proportion. The amount of contraction varies with the quality of the iron and the size of the casting, and this sometimes gives rise to much difficulty and trouble.[1] Passages and apertures in castings which are so small that the sand would not resist the scouring action of the flowing metal are formed of loam, in wooden moulds termed core-boxes, and are baked before being used. Simple cylindrical parts can be moulded in loam, without the use of a core-box. Thus, the core of a pipe is formed

[1] The following small table gives the average contraction in casting of the materials most used in machines :—

| Material | Linear Contraction | Cubic Contraction |
|---|---|---|
| Cast iron | $\frac{1}{97}$ | $\frac{1}{32}$ |
| Brass | $\frac{1}{64}$ | $\frac{1}{22}$ |
| Gun-metal | $\frac{1}{130}$ | $\frac{1}{44}$ |

of loam, plastered on to a hollow metal core-bar. By rotating the core-bar and strickling off the superfluous loam with a sharp-edged board, the exact cylindrical form is obtained. The moulds for large cylinders are formed of loam, plastered over roughly built brick cylinders, strickled to the required form and dried.

Although simple forms are more easily moulded and cast than more complex forms, the skill of the moulder enables him, when necessary, to mould castings of very complicated and difficult shapes. Hence, the cast parts of machines may be more complicated in form than those which are forged. Castings, however, do not retain an altogether sharp and accurate impression of the mould. The corners of castings are usually somewhat blunt and ragged, deep hollows partially filled up, and straight lines slightly twisted. Hence, for appearance sake, castings should have broad and rounded surfaces with well-rounded edges and filleted hollows. Architectural mouldings are not suitable for castings.

Cast iron is stronger than wrought iron under pressure, and much weaker under tension. Hence, it is more suitable for compressed than for stretched machine parts. Within a limited range of stress, it is tougher than wrought iron, or undergoes a greater deformation. But its range of deformation is not great. Hence, it is not so safe as wrought iron when subjected to impact.

The tenacity or tensile strength of cast iron varies from 7½ to 15 tons per sq. in. Small cylinders of a height of 1½ to 3 times the diameter crush with 40 to 50 or more tons per sq. in. The shearing resistance is about 4 or 5 tons per sq. in. The tenacity diminishes at high temperatures, but not much for temperatures below 750° F.

When castings are contracted for, it is usual to stipulate that test bars shall be cast at the same time and of the same metal as the castings. These test bars are very commonly 3½ ft. × 2 ins. × 1 inch. They are laid on supports 3 ft. apart, with the deeper side vertical, and loaded at the centre till they break. Such bars should carry from 28 to 30 cwts. before breaking, and will deflect before fracture from 0·2 to 0·5 inch. Generally it is desirable that the iron should be ductile, and the deflection should not be less than 0·3 inch.

The special difficulty and danger in the use of cast iron is

its liability to be put into a state of internal stress, in consequence of its contraction when cooling. That contraction varies with the size and thickness of the casting and with the quality of the iron. Thus it has been found that thin locomotive cylinders contract only $\frac{1}{16}$th of an inch per foot. Heavy pipe castings and girders contract $\frac{1}{8}$th inch in 12 inches or $\frac{1}{8}$th inch in 15 inches. Small narrow wheels contract as little as $\frac{1}{25}$th inch per foot, while large and heavy wheels contract $\frac{1}{10}$th inch per foot or more. If some parts of a casting contract more than others, the thick parts, for instance, more than the thin parts, the casting is twisted and strained. If some parts of a casting solidify while others are still fluid, the former attain nearly their final dimensions, while the contraction of the latter has still to be effected. That contraction, therefore, strains the parts already set, and their resistance to deformation gives rise to stresses in the parts which are contracting. Thus a condition of initial stress is induced, sometimes great enough to fracture

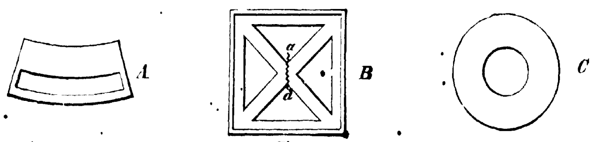


Fig. 1

the casting without the application of any external cause, and in all cases reducing the effective strength of the casting. The danger of initial stress is less when the form of the casting is simple and the thickness uniform and not excessive. It appears that the initial stress is to some extent gradually removed by molecular yielding, the alteration going on for months after the casting is made.

Suppose a casting of the form shown at *A*, fig. 1. The thin side would solidify, while the greater body of heat in the thick part still retained it in a fluid condition. When the thick part contracted, it would necessarily curve the bar and induce compression in the thin part and a corresponding extension in the thick part. In a panel of the form shown at *B*, with a thin but rigid flange, the contraction of the diagonals takes place more slowly than that of the rim surrounding them, and is very liable to cause fracture at *a a*. In a thick cylinder, such as a press cylinder, fig. 1, *C*, the outer layers solidify and

begin contracting first. The contraction of the inner layers, after that of the outer layers is completed, induces pressure in the outer layers; and the rigidity of the outer layers, causing a resistance to the contraction of the inner layers, puts them into tension. Such a cylinder will not bear so great a bursting pressure as if there were no initial strain. In fact, to obtain the greatest resistance to an internal bursting pressure, the reverse distribution of initial stress is necessary. This has sometimes been obtained by casting the cylinder with a water core, or hollow core having a water circulation through it. The interior is then cooled most rapidly. Compression of the inner layers and extension of the outer layers is the result of this mode of cooling. Castings in the form of wheels and pulleys often give much trouble. In pulleys which have a thin but rigid rim, the rim solidifies first, and the subsequent contraction of the arm breaks it by tension along the line *a a a*, fig. 2. In some cases, however, the rim breaks across near the arm, at *a b*.

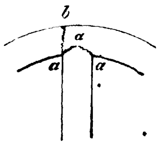



Fig. 2

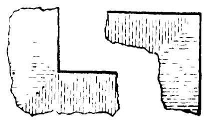


Fig 3

This appears to be due to the arms setting first. They then form a rigid abutment resisting the contraction of the rim, and bending stress is produced in the rim, causing fracture to begin outside and extend inwards.

It is because of these incalculable initial strains that cast iron is an unreliable material, where great strength is required, in structures of irregular form. The danger may be partially removed by the skill of the founder, who, by various devices, ensures as far as he can a uniform rate of cooling. But generally cast-iron structures must have excessive dimensions in order to ensure safety.

At sharp corners a plane of weakness is formed, in consequence of the way in which the crystals arrange themselves, normally to the surfaces through which heat is transmitted. This is one reason why all corners should be well rounded. Fig. 3 shows roughly the crystalline structure.

7. *Chilling.*—When castings are rapidly cooled during solidification, the separation of the graphite from the iron is prevented.

The casting has then a silvery fracture and is extremely hard. To effect this chilling, as it is termed, the mould is made of a thick block of cast iron, the surface in contact with the molten iron being protected by a wash of loam. The iron mould abstracts the heat much more rapidly than a sand mould.

8. *Malleable Cast Iron.*—This is made by surrounding a casting with oxide of iron or powdered red hæmatite, and keeping it at a high temperature, for a time varying with the size of the casting, from two or three, to thirty or forty hours, or even longer. Part of the carbon is eliminated, and the casting is converted, partially or wholly, into a tough material resembling wrought iron. Malleable castings stand blows much better than ordinary castings, but they should only be hammered when cold. The decorative parts of ironwork and pinions of wheels are often thus treated.

WROUGHT IRON

9. *Wrought iron* is a silvery nearly pure metal, fusing with difficulty, moderately hard, strong and tough. It is obtained from cast iron by eliminating the greater part of the carbon, and during the process passes into a pasty condition, so that it cannot be cast into an ingot. At a temperature of 1,500° or 1,600° Fahr. wrought iron softens, and can then be welded, a property of great importance and value. Wrought iron is used for parts of machines requiring strength and toughness, and such parts should generally have as simple a form as possible. Wrought-iron parts are first shaped by hammering or rolling at welding heat, and are then reduced to the exact form required by cutting tools. In some cases dies or swages are used to facilitate the forging of difficult forms. Large wrought-iron structures are built up of bars or plates riveted together.

The different qualities of wrought iron are commercially distinguished as merchant bar, best iron, double best, and treble best. These terms refer to the amount of working the iron has received in manufacture, and are only rough indications of quality.

Wrought iron does not vary very greatly in strength, but its value for bending, forging, &c., varies a good deal. The following is a rough classification of the ordinary qualities and their uses :—

(*a*) Iron easily worked hot, and hard and strong when cold; used for rails.

(*b*) Common iron, used for ships, bridges, and sometimes for shafting.

(*c*). Single, double, and treble best iron, from Staffordshire and other parts, where similar qualities are made. The single or double best is used for boilers. Double and treble best are used for forging.

(*d*) Yorkshire iron, from Lowmoor, Bowling, or other forges where only fine qualities are made. The best Yorkshire iron is very reliable and uniform in quality. It is used for tyres, for difficult forgings, for furnace plates exposed to great heat, for boiler plates which require flanging, &c.

(*e*) Charcoal iron. Very ductile, and of the best quality. Used for boiler tubes.

The dimensions of the ordinary rolled sections of wrought iron and mild steel, angles, tees, channels and beams have now been standardised, and a very useful set of tables has been issued by the Engineering Standards Committee giving the areas, weights, moments of inertia, and section moduli for all the standard sections.[1]

In the case of wrought iron the strength and ductility varies somewhat with the dimensions of the piece rolled, and are different in the direction of rolling and across it. The following tables give values which ought to be obtained in tests.

TESTS FOR WROUGHT IRON PLATES

| | Tensile Strength—tons per sq. inch | | Elongation per cent. in 8 inches | | Contraction per cent. | |
|---|---|---|---|---|---|---|
| | Lengthways | Across | Lengthways | Across | Lengthways | Across |
| Boiler plates— | | | | | | |
| No. 1 grade | 23–26 | 22–25 | 18 | 12 | — | — |
| No. 2 grade | 23–26 | 21–24 | 12 | 9 | — | — |
| No. 3 grade | 22–25 | 20–23 | 10 | 7 | — | — |
| Ordinary plates— | | | | | | |
| No. 1 grade: | | | | | | |
| ¼ in. thick, minimum | 23·5 | 22·5 | 16 | 12 | 25 | 20 |
| ½ in. ,, ,, | 22·5 | 21·5 | ,, | ,, | ,, | ,, |
| ⅞ in. ,, ,, | 21·0 | 20·0 | ,, | ,, | ,, | ,, |
| No. 2 grade: | | | | | | |
| ¼ in. thick, minimum | 23·0 | 20·5 | 12 | 9 | 15 | 10 |
| ½ in. ,, ,, | 22·0 | 19·5 | ,, | ,, | ,, | ,, |
| ⅞ in. ,, ,, | 20·5 | 18·0 | ,, | ,, | ,, | ,, |
| No. 3 grade: | | | | | | |
| ¼ in. thick, minimum | 22·5 | 20·0 | 10 | 5 | 12 | 8 |
| ½ in. ,, ,, | 21·5 | 19·5 | ,, | ,, | ,, | ,, |
| ⅞ in. ,, ,, | 20·0 | 18·0 | ,, | ,, | ,, | ,, |

[1] *See* Reports of Engineering Standards Committee (Crosby Lockwood). No. 1, British Standard Sections. No. 6, Properties of British Standard Sections.

TESTS FOR WROUGHT IRON BARS

| | Tensile Strength—tons per sq. in | Elongation per cent. in 8 diameters | Contraction of Area per cent. |
|---|---|---|---|
| Round and square bars: | | | |
| No. 1 grade: 1 in. diameter | $23\frac{1}{2}$ | 30 | 50 |
| ,, 5 ins. diameter | 22 | 20 | 40 |
| No. 2 grade: 1 in. diameter | 23 | 28 | 43 |
| ,, 5 ins. diameter | 21 | 20 | 35 |
| No. 3 grade: 1 in diameter | 23 | 23 | 38 |
| ,, 5 ins. diameter | 21 | 15 | 25 |
| | | In 8 inches | |
| Angles and tees | 24–21 | 24–15 | 35–25 |

10. *Case Hardening.*—The surface of wrought iron may be hardened by partially converting it into steel. This can be effected to a slight extent by making the surface bright, heating it to a red heat, then rubbing it with prussiate of potash, and quenching in water. It is far more completely effected by heating the iron in a close box, filled with bone dust and cuttings of horn and leather.

*Cold Rolled Iron.*—Wrought iron rolled cold under great pressure gets a smooth polished surface, and is found to have a greatly increased tenacity. Its ductility and toughness are, however, much diminished. Hammering iron when cold produces a similar effect. Annealing, or heating the iron to red heat and allowing it to cool slowly, restores it to its original condition.

All mechanical compression of iron when cold appears to increase its strength at the expense of its toughness and ductility. On the other hand, annealing reduces the strength, but increases the ductility and toughness. In rolling or hammering when hot, mechanical compression and annealing are going on simultaneously.

## STEEL

11. *Steel* is pure iron combined with a proportion of carbon, silicon, manganese, phosphorus and other constituents. If the pure iron were combined with carbon alone, the properties would depend directly on the percentage of carbon. But in actual steels the case is much more complex. Silicon, manganese, phosphorus, and some other constituents influence its physical properties and to some extent produce the same effect as carbon. The mildest steels contain 0·15 to 0·4 per cent. of carbon. The hardest 1·2 to 1·6 per cent.

The commercial term 'steel' includes widely different materials. Steel containing little carbon (less than $\frac{1}{2}$ per cent.) resembles wrought iron and has now superseded it for most purposes. Like wrought iron it welds easily and is not hardened when heated and suddenly cooled. This steel would be most conveniently termed 'ingot iron,' so as to mark the fact that it has been cast into a malleable ingot, while, like wrought iron, it does not temper or harden. Steel with somewhat more carbon is harder and stronger and is used for tyres and rails. Steel containing 0·5 per cent. of carbon or more has the striking property of hardening when heated and suddenly cooled and of softening again if heated, and slowly cooled.

With good steel of this character, any desired hardness can be obtained either by heating to some definite temperature and suddenly cooling it, or more commonly by first heating it strongly and cooling so as to make it excessively hard, and then tempering it or letting down the temper. The steel is reheated to a temperature indicated by the colour of the oxide which forms on its surface (straw-yellow to blue). It is then cooled in water or oil. This property of hardening makes steel suitable for cutting tools. In its hardened state it has great strength, a high elastic limit, and little ductility.

In welding steel it is important that the pieces to be united should contain the same amount of carbon. If they do not, their welding temperatures are different. Steel requires more care in smithing than wrought iron, and it is more liable to injury if worked at an improper temperature. All straining of the material when cold by hammering or punching injures steel more than wrought iron, but the damage is entirely removed by annealing.

Bauschinger found the tenacity of steel to increase with the amount of carbon nearly in the following proportion :—

$$\text{Breaking strength} = 27{\cdot}6\ (1 + C^2)\ \text{tons per sq. in.}$$

where C is the percentage of carbon. But the constituents of steel vary so much that no single rule will cover all cases.

Steel is made by acid or basic Bessemer, or by acid or basic open-hearth processes. The open-hearth steel is preferred for plates, bars and forgings.

In steel plates the strength in the direction of rolling and across it is almost exactly the same. One great advantage of

steel over wrought iron in construction is that plates of much greater area and weight and bars of much greater length can be obtained without extra cost. Steel plates of 70 sq. ft. area or 15 cwt. in weight are obtained with little difficulty, and rail bars are rolled up to 150 feet in length. The dimensions of standard rolled sections of steel are given in the Reports cited above.

The following table of average values gives the relation of composition and strength of some of the steels used in construction :—

TABLE OF STEELS

| Uses | Composition per cent. | | Tenacity | Elongation |
|---|---|---|---|---|
| | Carbon | Manganese | tons sq. in | per cent. |
| Boiler plates | 0·19 to 0·25 | 0·45 to 0·63 | 25 to 32 | 28 to 32 in 8″ |
| Forgings | 0·3 to 0·4 | 0·75 to 1·0 | 30 to 40 | 20 in 4″ |
| Gun steel | 0·3 | 0·5 | 42 | 23 in 2″ |
| Rails | 0·3 to 0·5 | 0·5 to 1·0 | 35 to 45 | 12 to 20 in 8″ |
| Laminated springs | 0·5 to 0·8 | 1·0 | 46 | 21 in 4″ |
| Volute springs | 0·8 to 1·3 | — | — | — |

The following table gives briefly the requirements usually specified for steel for different purposes. Fuller data will be found in the specifications issued by the Engineering Standards Committee

TENSILE TESTS FOR STEEL. STANDARD TEST BARS.

| | Tensile Strength—tons per sq. in. | Elongation per cent. |
|---|---|---|
| Structural steel | 28–32 | 20 in 8 ins. |
| Shipbuilding, plates | 28–32 | 20 in 8 ins.[1] |
| ,, angles, channels, &c. | 28–33 | 20 in 8 ins.[1] |
| ,, rivet bars | 25–30 | 25 in 8 diam. |
| Marine boiler plates | 28–32 | 20 in 8 ins. |
| ,, plates for flanging or furnaces | 26–30 | 23 in 8 ins. |
| Marine stay, angle, or T bars | 28–32 | 20 in 8 ins. |
| ,, rivet bars | 26–30 | 25 in 8 diam.[2] |
| Locomotive boiler plates | 26–32 | 22 in 8 ins. |
| ,, round bars | 26–32 | 22 in 8 diam.[3] |
| ,, rivet bars | 26–30 | 25 in 8 diam.[4] |
| ,, plates | 28–32 | 20 in 8 ins. |
| ,, angles | 28–32 | 20 in 8 ins. |
| ,, round bars | 28–32 | 20 in 8 diam. |
| ,, rivet bars | 26–30 | 25 in 8 diam. |

[1] 16 per cent. if under ⅜ thick.
[2] Or 27 per cent. in 4 diameters.
[3] Or 30 per cent. in 4 diameters.
[4] Or 30 per cent. in 4 diameters.

### Tensile Tests for Steel Forgings

*Standard test bars, 0·25 sq. in. area and 2 ins. gauge length or 0·5 sq. in. area and 3 ins. gauge length.*

| | Tensile Strength—tons per sq. in. | Elongation per cent. | Quality figure |
|---|---|---|---|
| Locomotives, ordinary | 25–32 | 27–20 | 52 |
| " with wearing surfaces | 40–45 | 20–15 | 60 |
| Marine, annealed | 28–40 | 29–17 | 57 |
| Locomotive crank axles | 30 | 25 | 55 |
| " straight axles | 35 | 20 | 55 |
| " straight axles, annealed or oil hardened | 35–40 | 25–20 | 60 |

The bend test-pieces for steel plates and sectional material are usually 1½ in. wide. If over ½ in. thick the arris caused by shearing is removed, and if over 1 in. thick the sheared edges are machined. They should stand bending through 180° without cracking until the internal radius is 1½ times the thickness of the test piece. The temper bend tests are made on similar bars and should satisfy the same condition. For forgings the strips are machined to 1 in. by ¾ in., and the edges rounded to a radius of $\frac{1}{16}$ in. Such strips should bend through 180° without cracking, the internal radius being ¼ in. for 32-ton steel; ⅜ for 32- to 36-ton steel; and ⅝ for 36- to 40-ton steel.

12. *Oil Tempering.*—A process of oil tempering is used to increase the strength of steel without reducing its toughness so much as it would be reduced by water tempering. The steel is heated to from 1,300° F. to 1,550° F. and suddenly cooled in rape oil. The forging is then annealed by heating to 900° F. and slow cooling. In one instance, for example, oil tempering raised the yield point from 13 to 29 tons per sq. in.; the tenacity from 31 to 43 tons per sq. in.; and the elongation in 2 ins. was only reduced from 28 to 21 per cent.

*Nickel Steel* with 3¼ to 4¼ per cent. of nickel promises to be a very valuable constructive material. Its tensile strength is 44¼ to 50½ tons per sq. in. The yield point is 27 to 32 tons per sq. in., considerably higher than in ordinary steel. Nickel rivet steel has a shearing strength of 26 tons per sq. in. and a tensile

strength of 34 tons per sq. in. The elongation of nickel steel plates in 8 ins. is from 12 to 15 per cent.

## Steel Castings

13. Steel is more fusible than wrought iron and can be cast in moulds like cast iron. At first great difficulty was found in obtaining sound castings, and the great contraction of steel in solidifying gives rise to excessive internal stresses. The difficulties in obtaining good steel castings have been for the most part overcome. By using silicon, aluminium and other constituents, the casting is rendered sound; and by prolonged annealing, the internal stresses are destroyed. Sir J. Whitworth introduced a process of casting under pressure which secures a very admirable quality of steel.

Tensile tests are made on test pieces of 0·5 sq. in. area and 3 ins. gauge length. Cold bend tests are made on test strips 1 in. × $\frac{3}{4}$ in. machined and with rounded edges, and bent to an internal radius of 1 in. The following table gives the usual requirements of strength, elongation and angle to which the cold bend strip should bend without cracking.

### Tests of Steel Castings

| | Tensile Strength—tons per sq. in. | Elongation in $3\frac{1}{4}$ diameters per cent. | Cold bend angle |
|---|---|---|---|
| Castings for marine purposes: | | | |
| Grade A | 35–40 | 15 | 60° |
| „ B | 26–35 | 20 | 120 |
| „ C | 26–35 | 15 | 90 |
| Castings for locomotives and rolling stock: | | | |
| With wearing surfaces. At least | 35 | 10 | — |
| Without „ „ „ | 26 | 15 | — |

## Copper

14. *Copper* is a reddish, rather costly metal, which can be cast, but is seldom used in that way. It is usually rolled into plates and hammered to shape. The purest copper is obtained by an electrolytic process.

Copper castings have a tenacity of 10 tons per sq. in.; copper plates for fire boxes should have a tensile strength of 14 tons

per sq. in. and an elongation of 35 per cent. in 8 ins. Copper rods a tensile strength of 14 tons and an elongation of 40 per cent. in 8 diameters. After hammering cold, copper loses ductility and requires to be annealed. In annealing it should be raised to 500° F., and then quenched in water.

### Bronze or Gun-metal

15. *Bronze* or *gun-metal* is harder and less malleable than copper. It is fusible, and makes excellent castings. It varies in quality according to the proportion of tin. Thus :—

| | | | |
|---|---|---|---|
| Soft gun-metal contains . . . . | 8 tin | to 92 | copper |
| Hard gun-metal . . . . . | 18 | ,, 82 | ,, |
| Bell metal . . . . . . . | $23\frac{1}{2}$ | ,, $76\frac{1}{2}$ | ,, |

Bearing metal sometimes consists of 82 per cent. copper, 16 per cent. tin, and 2 per cent. zinc. Ordinary bronze is not uniform in texture. Whitish spots of alloy, rich in tin, are distributed through the mass. It has been found that when it is rapidly cooled after casting the composition is more uniform, the density greater, and the strength and toughness are increased. This rapid cooling is best effected by using thick cast-iron moulds or chills, the process being analogous to the chilling of cast iron. The following minimum requirements can be obtained in bronze for machinery :

| | Tensile Strength—tons per sq. in. | Elongation per cent. |
|---|---|---|
| Bronze No. 1 . . . . . . . . | 42 | 20 |
| ,, No. 2 (malleable) . . . . | 33 | 25 |
| ,, No. 3 . . . . . . . . | 26 | 15 |

The softest bronze is used for cocks and small fittings, and a harder bronze is suitable for steps supporting rotating pieces. For bearings, an alloy containing copper, tin, and lead wears better than ordinary bronze. Sometimes phosphorus is added to facilitate casting.

*Phosphor Bronze* for bearings consists of about 79 per cent. copper, 10 per cent. tin, 10 per cent. lead, and 1 per cent. phosphorus ; but by varying the composition it can be made harder, or softer and more ductile as required. As to its strength and ductility, various tests show a tenacity of from 22 tons per

sq. in. in the softer qualities to 33 tons in the hardest. The elastic limit of the former is about 5 tons, and the latter 25. The former elongate 30 per cent. or more before fracture, and the latter 3 to 4 per cent. The contraction of area at fracture ranges from 4 to 30 per cent. Unannealed wire (16 B.W.G.) broke with from 102 tons per sq. in. to 151 tons per sq. in., and the same wire after annealing carried from 48 to 74 tons per sq. in.

*Silicium Bronze* has a much higher electrical conductivity than phosphor bronze and at the same time equal strength. It is therefore very suitable for telegraph wire in towns, where its resistance to corrosion is of great value.

*Manganese Bronze* is now made by introducing a proportion of ferro-manganese in bronze or brass. The manganese, like phosphorus, clears the alloy of oxide. Several qualities are made. No. 1, containing a good deal of zinc, can be forged or rolled hot. Simply cast it has a tenacity of 24 tons per sq. in. When rolled it has a tenacity of 28 to 32 tons and an elastic limit of 15 to 23 tons per sq. in. No. 2 is stronger and is used for castings. No. 3, which is without zinc, replaces ordinary bronze and has great strength and toughness. It has been used for large bearings and for screw propellers.

## BRASS

16. *Brass* contains from 66 per cent. copper and 34 per cent. zinc to 70 per cent. copper and 30 per cent. zinc. A little lead is often added. Common brass for cheap brasswork contains a larger proportion of zinc. Muntz metal, which can be rolled hot, contains 60 per cent. of copper and 40 per cent. of zinc, or sometimes 66 per cent. of copper, 33 of zinc, and one of lead. It is used for sheathing-plates for ships and for the tubes of locomotives. Brass is extensively used on account of its easy working and good colour. It is cheaper than gun-metal, but less strong and tough. The following requirements can be obtained in brass for machinery :—

| | Tensile Strength—tons per sq. in. | Elongation per cent. |
|---|---|---|
| Brass No. 1 | 22–26 | 30 |
| „ No. 2 | 20–24 | 25 |

*Delta Metal.*—This is a brass containing a proportion of iron. It can be worked hot or cold, or brazed. Cast in sand its tenacity is 21 tons per sq. in.; after forging 33 to 35 tons per sq. in. It is specially useful for forming special shapes by hot stamping.

17. *White Metal Alloys.*—Certain rather soft and fusible alloys have been found to support journals with less friction and wear than bronze or brass. Most of these are, however, so weak that they are used as thin linings in steps of bronze or brass. Some are so fusible that they can be melted and run into place round the journal to be supported, forming a well-fitting bearing without machining. The first white metal successfully used contained tin, antimony, and copper, and was introduced by Isaac Babbitt. Babbitt metal is a name sometimes used for all alloys of this class. True Babbitt metal was composed of 80 per cent. tin, 10 per cent. copper, and 10 per cent. antimony. It is more costly than lead-antimony alloys. The various white metal alloys contain a large percentage of either tin, lead, or zinc, with enough of other metals to give sufficient hardness. As oleic acid, which is present in some lubricants, attacks lead or zinc, the tin alloys appear to have an advantage. A common white metal alloy contains 85 to 75 per cent. of lead, and 15 to 27 per cent. of antimony. An addition of tin improves this alloy, giving greater hardness and toughness. Bismuth is added sometimes.

M. Charpy has made careful investigation of bearing alloys, and points out that they require two contradictory characteristics —hardness and plasticity. These may be combined in an alloy composed of hard grains in a plastic matrix. The alloys which have proved best for bearings in practice have in fact this structure, as is shown by microscopic examination. Thus lead-antimony alloys containing less than 13 per cent. of antimony consist of crystallites of lead embedded in a eutectic lead-antimony alloy, and those containing more than 13 per cent. of antimony consist of crystallites of antimony embedded in a eutectic lead-antimony alloy. In either case the crystallites are harder than the matrix.

TABLE OF PERCENTAGE COMPOSITION OF ALLOYS

| | Tin | Copper | Antimony | Lead | Zinc | |
|---|---|---|---|---|---|---|
| *Aluminium Alloys.* | | | | | | |
| Aluminium bronze | — | 90 | — | — | — | 10 per cent. aluminium |
| *Copper-Tin Alloys.* | | | | | | |
| Bell-metal | 23·5 | 76·5 | — | — | — | |
| Gun-metal, soft | 8 | 92 | — | — | — | |
| ,, ,, hard | 18 | 82 | — | — | — | |
| *Copper-Zinc Alloys.* | | | | | | |
| Brass | — | 70 | — | — | 30 | |
| ,, | — | 66 | — | — | 34 | |
| Muntz metal | — | 60 | — | — | 40 | |
| *Tin-Lead Alloys.* | | | | | | |
| Pewter | 80 | — | — | 20 | — | |
| Solder | 66·6 | — | — | 33·3 | — | |
| ,, | 33·3 | — | — | 66·6 | — | For plumbers |
| *Tin-Copper-Zinc Alloys.* | | | | | | |
| Bronze | 8 | 89 | — | — | 3 | Locomotive bearings |
| ,, | 16 | 82 | — | — | 2 | Car bearings |
| ,, | 16 | 77 | — | — | 7 | Heavy bearings |
| ,, | 12 | 82 | — | — | 6 | Car bearings |
| ,, | 6 | 77 | — | — | 17 | Bearings |
| *Tin-Copper-Antimony Alloys.* | | | | | | |
| White metal | 83·3 | 5·6 | 11·1 | — | — | Car bearings |
| ,, ,, | 71 | 5 | 24 | — | — | Eccentric straps |
| ,, ,, | 67 | 11 | 22 | — | — | Car bearings |
| ,, ,, | 82 | 6 | 12 | — | — | ,, ,, |
| Babbitt metal | 87 | 7·8 | 5·2 | — | — | Bearings |
| *Tin-Lead-Antimony Alloys.* | | | | | | |
| Type-metal | 10 | 2 | 18 | 70 | — | |
| Tandem and Magnolia metals (Thurston) | 5·9 | — | 16·8 | 77·7 | — | |
| Bearing metal | 37·5 | — | 25 | 37·5 | — | |
| *Copper-Lead-Antimony Alloys.* | | | | | | |
| Bearing metal | — | 10 | 25 | 65 | — | For locomotives |
| ,, ,, | — | 10 | 20 | 70 | — | |
| *Copper-Tin-Lead Alloys.* | | | | | | |
| Bronze | 8 | 76·8 | — | 15 | — | Phosphorus 0·2 |
| ,, | 13 | 82 | — | 4·5 | — | ,, 0·1 |
| Gun-metal | 10 | 78 | — | 12 | — | For bearings |
| *Other Alloys.* | | | | | | |
| Delta metal | — | 55 | — | 0·4 | 43·5 | Iron 1·0 |
| Tobin bronze | 2·2 | 59 | — | 0·3 | 38·4 | ,, 0·1 |

18. *Aluminium.*—Aluminium is the lightest of the metals used in construction, weighing per cubic foot only one-third as much as iron. It is now produced in large quantities by electro-thermal processes. The purest commercially made contains 99 to 99·7 per cent. aluminium, and small quantities of silicon and iron. It resists oxidation when nearly pure, but when it contains even 2 or 3 per cent. of silicon it oxidises in the air. Pure aluminium is too soft and weak for most purposes of construction. The weight of aluminium nearly pure is about 161 lbs. per cubic foot when cast, and 171 when rolled. The elastic limit in tension in castings is about 8,500 lbs. per sq. in., and in bars about 14,000 to 25,000. Its tenacity is about 18,000 lbs. per sq. in. in castings, and 28,000 to 40,000 in bars Its coefficient of elasticity is about 11,000,000. Alloyed with a small percentage of copper, tungsten or other metal, materials are produced of higher elastic limit and greater strength than pure aluminium. An alloy, used in plates for yacht building, had an elastic limit of 28,000 to 36,000 lbs. per sq. in. and a tenacity of 37,000 to 43,000 lbs. per sq. in. The heavier alloys have still greater strength. Aluminium brass has a tenacity of 40,000 to 90,000 lbs. per sq. in., and an elongation of 3 to 10 per cent. in 8 ins. Aluminium bronzes containing 5 to 12 per cent. of aluminium have great tenacity and toughness, some of them having 40,000 to 50,000 lbs. per sq. in. tenacity with an elongation of 30 per cent. in 8 ins., and others a tenacity of 60,000 lbs. per sq. in. and an elongation of 10 per cent. in 8 ins.

19. *Effect of High Temperatures on the Tenacity of Materials.*—As parts of machines are exposed to high temperatures—in the case of boiler shells and valves exposed to high-pressure steam, for instance—the influence of temperature on the strength cannot always be overlooked. Fortunately, in the case of iron and steel, at temperatures ordinarily reached, the strength is not seriously impaired. Most experiments show for iron or steel a small gain of tenacity between 60° and a temperature of 400° or 600° F. This gain amounts in extreme cases to ·12 to ·20 per cent. At temperatures above 600°, the strength diminishes rapidly with increase of temperature, about 50 per cent. being lost at temperatures of 1000° or 1200°.

The following tables give the relative tensile strength at

different temperatures, that at ordinary temperature being taken as 100:

*Wrought Iron*

| | | | | | |
|---|---|---|---|---|---|
| Temperature F. . . | 212° | 600° | 750° | 1000° | 1200° |
| Relative tenacity . . | 104 | 116 | 96 | 75 | 40 |

*Mild Steel*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Temperature F. . | −4° | 212° | 400° | 570° | 750° | 900° | 1100° |
| Relative tenacity . | 106 | 103 | 132 | 123 | 86 | 49 | 28 |

The coefficient of elasticity of mild steel which is 29,500,000 lbs. per sq. in. at ordinary temperatures falls to 27,700,000 at 400° F. and to 25,500,000 at 750°.

In the case of copper and the copper alloys a much more marked influence of temperature is observed. If $f$ is the tenacity in tons per sq. in. at a temperature $t°$ F., then the author has found approximately

$$f = a - b\,(t - 60)^2$$

where $a$ and $b$ are constants.

| | $a =$ | $b =$ |
|---|---|---|
| Copper . . . . . . . . | 14·8 | ·000014 |
| Rolled yellow brass . . . . | 24·1 | ·000028 |
| ,, delta metal . . . . | 31·3 | ·000041 |
| ,, Muntz metal . . . . | 14·7 | ·000029 |
| Cast gun-metal . . . . . | 12·5 | ·000021 |
| ,, brass . . . . . . | 12·5 | ·000024 |
| ,, phosphor bronze . . . | 16·1 | ·000026 |

STRENGTH OF METALS AT DIFFERENT TEMPERATURES

| | Tenacity in tons per sq. in. at temperatures F. | | | | |
|---|---|---|---|---|---|
| | 60° | 200° | 300° | 400° | 600° |
| Copper . . . . . | 14·8 | 14·5 | 14·0 | 13·2 | 10·6 |
| Rolled brass . . . . | 24·1 | 23·5 | 22·5 | 20·9 | 15·7 |
| Delta metal, rolled . . | 31·3 | 30·5 | 28·9 | 26·6 | 19·4 |
| Cast gun-metal . . . | 12·1 | 11·7 | 10·9 | 9·7 | 6·0 |
| Cast brass . . . . | 12·5 | 12·0 | 11·1 | 9·7 | 5·5 |
| Phosphor bronze . . . | 16·1 | 15·6 | 14·6 | 13·1 | 8·6 |

20. *Tenacity of Wire.*—When a metal is drawn down to a comparatively small section, its strength per unit of section increases. This is more apparent in unannealed than in annealed wire. Karmarsch has found that the total strength of a wire in lbs. is given nearly by the equation

$$P = ad + bd^2$$

where $d$ is the diameter in inches.

| | Unannealed Wire | | Annealed Wire | |
|---|---|---|---|---|
| | $a$ | $b$ | $a$ | $b$ |
| Steel | 1160 | 69000 | 165 | 62000 |
| Iron, from | 690 | 69000 | 165 | 36000 |
| ,, to | 1000 | 50000 | 275 | 31000 |
| Brass | 440 | 60000 | 300 | 31000 |
| Copper | 410 | 38000 | 0 | 25000 |

21. *Protection of Iron from Corrosion.*—One of the difficulties in the use of iron or steel is the corrosion to which these materials are liable. In many cases, as, for instance, in steam boilers, the corrosion, if allowed to proceed, may greatly weaken and endanger the structure. The corrosion is most rapid on surfaces which are alternately wet and dry, and less rapid on surfaces entirely covered by water. Cast iron obtains in the sand mould a covering of silicates which, if unbroken, is less liable to corrosion than clean surfaces of the metal. Cast iron and steel are more rapidly attacked in sea-water than wrought iron. The acids present in some woods (as, for instance, oak) cause rapid corrosion of iron in contact with them. Hence, in oak, copper bolts are generally used. The modes of protecting iron from corrosion are as follows: (1) Heating the iron to 310° F. and immersing it in a bath of pitch, maintained at a temperature of at least 210°. A little oil is generally added to the pitch. This process, known as Dr. Angus Smith's, is commonly employed for protecting water pipes. The pitch used is coal tar, from which the naphtha has been removed by distillation. (2) A tar varnish for application to surfaces which cannot be heated, consists of tar with a little tallow and resin. (3) Painting with oil paint, especially with paints which have oxide of iron as a basis. (4) Certain transparent varnishes are manufactured which protect clean

iron surfaces without altering their appearance. (5) Mr. Barff protects iron by forming on its surface a coating of magnetic or black oxide of iron. This is effected by subjecting the iron for some time to the action of superheated steam at a high temperature. (6) Temporary protection is obtained by a coating of tallow. (7) The most complete protection is obtained by immersing the iron in a bath of melted zinc, a process which is termed 'galvanising.'

Where iron is in contact with a metal electro-negative to it, and both are immersed in water, there is a voltaic action which causes rapid corrosion. If the water contains acids, as is the case sometimes with the feed water of boilers, the action is still more rapid. The irregular corrosion known as pitting and furrowing, is probably due to portions of the surface exposed being electro-negative to others, either from want of homogeneity in the material or from other causes. On the other hand, if a metal electro-positive to iron is placed in contact with it, the iron is protected from corrosion. Thus boilers are now sometimes protected by suspending inside them blocks of zinc. The zinc gradually disappears, but the iron is protected.

*Machine Drawing.*—All machines are designed as drawings from which the actual machines are constructed, the business of the workman being to carry out accurately the instructions shown by the drawing. Mechanical drawing is for the machine designer a kind of written language, and it is by means of the drawing that he makes his ideas intelligible. On the drawing the engineer studies the relative motion of the members of his machine, determines their size to ensure strength, arranges them so that they do not interfere with each other, and considers all the modifications which tend to greater efficiency of working or less cost of manufacture. The drawing also serves to indicate how the work is to be distributed to different shops and to indicate the order in which the several parts will be required. Class letters and numbers on the drawings, which can also be placed on the work if necessary, serve to prevent confusion, and facilitate the keeping of time and cost records.

*Shop Processes.*—*Pattern-making* is the making of wood models of those machine parts, of brass or cast iron, which have to be cast. The pattern-maker has to consider the allowance to be made for shrinkage in casting and for turning, boring,

or other subsequent operations of manufacture. He has also to consider very carefully how the moulding of the machine part from the pattern can be most cheaply carried out, and to arrange the pattern accordingly. Parts of the pattern must be left loose or moulding is impossible. Parts can only be moulded by the use of cores. Parts which have to be finished by cutting tools must be so placed that they are not likely to be unsound from blow holes or dirt. *Moulding* and *Founding* or *Casting* are the operations of forming the sand or loam mould of a machine part, and running into it the melted iron or brass. In green-sand moulding the mould is made from a complete wood pattern. In loam moulding the pattern is strickled to shape by boards which have the form of a generator of the surface required. Whether of sand or loam, the mould must be excessively porous to allow the escape of the steam and gases generated by the heat of the melted metal. The founder requires to know how to produce different qualities of metal, softer or stronger, by mixing different proportions of pig iron of different brands.

*Forging* is the operation of shaping wrought iron or steel, materials which become plastic without fusing. To a large extent forging consists of welding on piece by piece and shaping gradually the large pieces required. In smith work, the pieces being small, the forging is done by hand; in larger forgings machine driven or steam hammers are required. In heavy forging the highest skill is required to arrange the operation so that the material is not injured, and the mass after the operation is homogeneous and sound. *Fitting and finishing* are the operations of cutting to accurate dimensions the rough products of the foundry and forge and fitting them together. The cutting is done by steel tools which cut the metal when cold. Cutting opertaions include chipping and filing, boring, drilling, turning, planing, shaping and milling, and also in some cases abrading operations such as grinding. The surfaces most easily obtained by cutting operations are cylindrical surfaces and plane surfaces, and most ordinary cutting operations are intended to form truly cylindrical or truly plane surfaces. Conical and screw surfaces are, however, also produced, and occasionally other forms. In the best work the dimensions of fitted work are determined by standard gauges, which ensure far greater accuracy than any system of measurement.

### SECTIONAL SHADING

Fig. 4 shows the sectional shading adopted in this treatise to indicate the materials most commonly used.

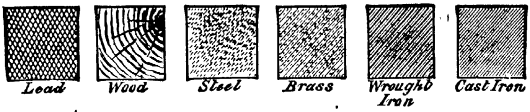



Fig. 4

Mild steel is shown by the same shading as wrought iron.

# CHAPTER II

## ON THE STRAINING ACTIONS TO WHICH MACHINES ARE SUBJECTED

22. THE forces acting on the parts of a machine comprise :—

(1) The useful load due to the effort transmitted from the driving to the working point to accomplish useful work. (2) Prejudicial resistances due to friction in the machine itself or work expended otherwise than at the working point. (3) The weight of parts of the machine. (4) Reactions of inertia due to changes of velocity of parts of the machine. (5) Centrifugal forces due to changes of direction of motion of parts of the machine. (6) Occasionally there are stresses due to constraint, preventing expansion with changes of temperature. The total action, on any member of a machine, of these forces may be called the total straining action on that member. The relative importance of the various straining actions is very different in different cases, and generally some of them are small enough to be neglected. The problems of designing are then simplified.

In each part of a machine the straining action varies with the fluctuations of the useful load and with the variations of position and velocity of the different parts of the machine. Each member must be capable of resisting the maximum straining action on that part of the machine. For each part of the machine, therefore, it is necessary to consider under what conditions the straining action is greatest. If, in consequence of changes of position or velocity, the straining action produces stresses of different kinds at different times, the member must be capable of sustaining the maximum stress of each kind. Lastly, as will be more fully explained presently, the amount or range of variation of the straining action affects the endurance of the material and therefore requires also to be considered.

23. *Steady or Dead Load, and Variable or Live Load.*— A steady load is one which is invariable during the life of the structure, and which produces a permanent and unvarying

amount of straining action. The weight of a fixed part of a machine is such a dead load. A variable or live load is a load which is alternately imposed and removed, and which produces a constantly varying amount of straining action. The weight of a railway train passing over a bridge, and the steam pressure on a piston, are live loads. A steady load can generally be very exactly estimated, and when the load is entirely of this kind, a comparatively low factor of safety affords a sufficient guarantee of security. A live load is often less easily estimated, and a load of this kind produces much more injurious effects on a structure than a dead load of the same amount. Hence, a higher factor of safety must be used for a live than for a dead load.

*Sudden loads and Shocks.*—A suddenly applied load is a load imposed without velocity, but at one instant and which continues to act during the deformation. At first such a load is not balanced by the stress, and it therefore accumulates, in deflecting or elongating the structure, a certain amount of kinetic energy which is ultimately expended in increasing the deformation, beyond the amount due to an equal steady load. If the stress does not exceed the elastic limit of the material, a suddenly applied load produces momentarily twice the stress due to the same load gradually applied or resting on the structure. Practical cases rarely approximate to these conditions.

If the load is due to a heavy body which impinges on the structure with an amount of kinetic energy previously accumulated, the stress produced may be much greater than that due to the same body resting on the structure. Such a load may be called an impulsive load and the stresses produced are said to be stresses due to shock. The problem of determining the stress is then a dynamical one, and the work of deformation is equated to the kinetic energy of the impinging load.

24. *Ultimate or Breaking Strength.*—If the straining action on a bar is gradually increased till it breaks, the load which produces fracture is termed the ultimate (statical) breaking strength of the bar. Many experiments on the breaking strength of bars of different materials, subjected to different kinds of straining action, have been made. From these it is possible to infer, by the rules of applied mechanics, the statical breaking strength of any member of a machine.

Now, obviously, the ordinary working load on a machine

part must be less than the breaking load. Practical experience has shown that it must be very considerably less. But in general it is not possible to determine by direct experiments what should be the working load, while it is easy to determine directly or from recorded experiments what is the breaking strength. Hence has arisen the custom of ascertaining the statical breaking strength of machine members and dividing it by a factor, termed a *factor of safety*, to find the proper working load. The factor of safety has been ascertained by comparing the working and breaking strength, in actual cases of machines which have proved satisfactory. Some machine members have been subjected to a given straining action for a long time and have shown no sign of failing. Others have broken down. In the former cases the factor of safety must have been sufficient, in the latter it must have been too low. By comparison of such cases values of factors of safety suitable for different materials and in different circumstances have been ascertained.

The following is a table giving general average values of such factors of safety. They are ratios of the statical breaking stress, under a load steadily and gradually increased till fracture occurs, to the ordinary working stress. They include some allowance for contingencies which may cause the actual stress to be greater than the intended working stress. But, where the straining action is imperfectly known, larger factors of safety must be adopted to provide for the unknown or neglected straining actions.

TABLE OF FACTORS OF SAFETY

| Material | Factors of Safety for | | | |
|---|---|---|---|---|
| | A dead load | A live or varying load producing | | In structures, subject to varying loads and shocks |
| | | Stress of one kind only | Equal alternate stresses of different kinds | |
| Cast iron | 4 | 6 | 10 | 15 |
| Wrought iron and steel | 3 | 5 | 8 | 12 |
| Timber | 7 | 10 | 15 | 20 |
| Brickwork and masonry | 20 | 30 | — | — |

In cast iron the factors are high to allow for unknown internal stresses due to contraction in casting. In timber they

are high because a gradual increase of deformation goes on in course of time. In brickwork and masonry, the weakness of the joints has to be allowed for, and the distribution of stress is probably very irregular.

It will be seen that the determination of the working strength, by dividing the breaking strength by a so-called factor of safety, is a purely empirical method. It remains to be seen whether a study of the properties of materials will furnish a more rational and satisfactory method.

25. *The Yield Point.*—When the load on a bar of ductile material reaches a value which may be taken roughly, in iron and steel, at $\frac{5}{8}$ to $\frac{2}{3}$ of the breaking weight, it suffers a large plastic or permanent deformation. In general such a deformation would be prejudicial or disastrous to the working of the machine. Hence generally, but not quite in all cases, the working stress due to all causes must be less than the stress at the yield point of the material. Thus, for instance, in structures of iron and steel, a factor of safety of 1·6 to 1·5 is required, merely to provide against the prejudicial deformation which would occur if the stress exceeded the yield stress. The factor of 3 in the table above, for a dead load, makes allowance for this and for contingencies and neglected straining actions.

26. *Strain and Stress.*—Every load which acts on a structure produces a change of form which is termed the *strain* due to the load. The strain may be either a vanishing or *elastic deformation*, that is, one which disappears when the load is removed; or a permanent deformation or *set*, which remains after the load is removed.

The molecular actions within the material, which are called into existence by external forces or loads, and which resist deformation, are called *stresses*.

*Elastic and Plastic Condition of Materials.*—An elastic material is one which, though it is deformed by any straining action, recovers its original condition if the straining action is removed. A plastic material is one which, when deformed by a straining action, does not completely recover its original form when the straining action is removed. Speaking broadly, any strain or deformation consists of two parts—one, an *elastic deformation* which is proportional to the straining action, and which vanishes if the stress is removed; the other, a *plastic deformation* or permanent set of indefinite amount. In all materials ordinarily

used in construction, the elastic deformation is small, and though not to be neglected, yet produces no alteration of figure or dimension which the engineer cannot provide against. The plastic deformation, however, may be very large and prejudicial. Further, for ordinary materials there is a certain range of straining action, within which the deformation is wholly, or almost wholly, elastic. But for greater straining action the deformation is largely plastic or permanent. Hence it is common to say that the working stresses must not exceed the elastic limit, or must be such as to produce no sensible permanent set.

27. *Relation of Stress and Strain. Load-Strain and Stress-Strain Diagrams.*—Suppose a bar, fig. 5, subjected to a gradually increasing axial load. If the straining action is a tension it will stretch longitudinally and contract laterally, and will ultimately break with a pronounced local contraction near the fracture. If the straining action is a pressure, it will compress longitudinally and expand laterally. Let the strains (elongations

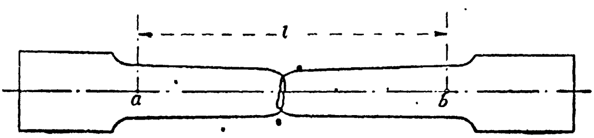



Fig. 5

or compressions) be measured on a fixed gauge length *a b*. The relation between stresses and strains may be represented by a diagram such as is shown in fig. 6.

A diagram in which the intensity of stress or stress per unit area is taken for ordinates would differ from one in which the load or total stress is taken for ordinates because the cross section of the bar alters as the load increases. Usually the diagram gives the relation of load or total stress and strain and such a diagram is considered here.

Let a pair of rectangular axes be taken and let loads or total stresses be plotted as ordinates and strains as abscissæ. Then the relation between stress and strain will be indicated by a curve. For convenience we may plot tensions upwards, pressures downwards, extensions to the right, compressions to the left. The curve for a ductile material such as wrought iron or mild steel is similar to *g f O a b c d e*. Attending at present to tension only, for a certain range of stress, the curve is almost exactly

a straight line, $O\ a$: that is, for stresses which do not exce the value $a\ a$, the strain is proportional to the stress and the stra is wholly temporary or elastic strain. At the stress indicat by the point $a$, the proportionality sensibly ceases. The strai increase faster than the stresses, and part of the strain is perm nent strain or set. The point $a$ is termed the *limit of propo tionality* or true elastic limit of the material in the given co ditions. For rolled or hammered materials there is often, but n always, a second well-marked point, $b$, where the material tak a large increase of strain, $b\ c$, without any increase of stres This point is the *yield-point*, and is often commercially b improperly called the 'elastic limit.' It is a point where t

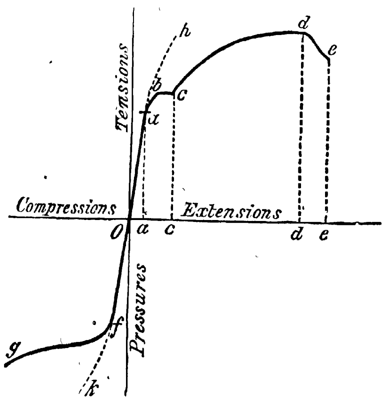



Fig. 6

defect of elasticity can be first very easily observed, even b very rough instruments. If the straining action is still graduall increased the strains increase, but faster than the stresses, and th stress-strain relation is given by the curve $c\ d$. The greate part of the strain during this period is permanent strain or se At the point $d$ the maximum stress is reached. With greate stresses, a local drawing down of the material occurs whic reduces its section so that it will no longer sustain so great load. Finally, the bar breaks with the stress $e$, $e$; the tot extension being then $O\ e$. It is well to note that the so-calle *breaking stress* of a material is the stress $d\ d$, reckoned per uni of area of the primitive section of the bar. The real intensity o

stress as the section reduces, reckoned per unit of reduced section, increases to the end of the test, and is much greater than the so-called breaking stress. For compression, the curve is like $O\ a\ f\ g$, with a less well-marked yield point than in tension. In compression the area increases as the block is compressed, and hence the straining action increases up to the end of the test. The stress, reckoned per sq. in. of the compressed area, approaches a constant value termed the pressure of fluidity.

For cast materials like cast iron and the harder qualities of steel, the stress-strain curve is like $k\ f\ O\ a\ h$, with no well-marked elastic limit or yield point.. In such materials the total strain before breaking is much smaller than in materials having a yield point, and such materials are said to be brittle or wanting in toughness. On the other hand, brass and bronze have no marked elastic limit or yield point but have great plastic deformation before breaking, and these materials are tough.

28. *Working Limits of Stress.*—There is no clear agreement at present as to the principle on which the greatest safe-working stress should be fixed. The condition that any large deformation is practically inconvenient fixes the yield-point, for such materials as have a strongly marked deformation at that point, as a limit of stress which should not be exceeded. But many materials have no such yield-point. Next, the limit of proportionality has been fixed on as marking the safe limit of working stress. It is clear that, if the working stress produces only a temporary or elastic deformation, then the material is unchanged by the stress, and may be expected to bear any number of repetitions of the stress.

Hence most of the older writers fixed on the elastic limits $a$ and $f$ on the diagram as marking out limits to the stresses which would be safe. The stresses corresponding to the points $a$ and $f$ are called the primitive elastic strengths of the material. The small defects of elasticity observable with very refined measurements with stresses less than the elastic strength, and which generally disappear after a few successive loadings, may, with a good deal of probability, be ascribed to initial defects of homogeneousness in the material.

29. *Variation in the position of the elastic limit.*—It is now understood that the elastic limit or limit of proportionality is not such a fixed point as had hitherto been supposed. Its

position can be altered by straining actions. Tensile straining may raise the limit in tension. Compressive straining may lower the limit in tension. Further, since most materials are subjected to some straining in manufacture, the primitive elastic limit found in a bar as received may be an artificially raised elastic limit. Reversals of stress frequently repeated appear generally to lower the elastic limit. If repeated equal tensions and compressions are applied, the elastic limits in tension and compression tend to values, equal for tension and compression, which Bauschinger suggested should be termed the natural elastic limits of the material.

Dr. Stanton found the following results on specimens of Swedish Bessemer steel tested statically and also after more than a million alternations of tensile and compressive stresses :

| | Tons per sq. in. | | |
|---|---|---|---|
| | No. 1 | No. 2 | No. 3 |
| Statical breaking strength | 47·6 | 43·7 | 43·7 |
| Primitive elastic limit in tension | 27·7 | 25·0 | 25·0 |
| Range of alternating stresses | 31·3 | 29·1 | 29·1 |
| Greatest tensile stress during alternations of stress | 18·3 | 16·9 | 12·1 |
| Natural elastic limit in tension after alternations of stress | 17·3 | 16·4 | 12·6 |

The elastic limit in tension was lowered by alternations of stress to about six-tenths of its primitive value.

30. *Failure of machine parts after long periods of working.*—It has long been known that machine parts, especially machine parts exposed to alternations of tension and thrust, have broken after long periods of service, though there had been no ascertainable change in the conditions or the amount of straining action to which they were subjected. In the case of railway axles this is so well understood that they are regularly removed from service, after having accomplished a fixed limit of mileage. Mr. Longridge's Reports for the Boiler and Engine Insurance Company make frequent mention of crank shafts and piston rods which, after years of service, broke unexpectedly without any assignable reason, except that metals after many alternations of stress suffer a deterioration, which may be termed *fatigue*, which render them incapable of sustaining stresses previously

safely carried. The following are cases of crank shafts quoted from the Report of 1894.

| | Calculated Stress due to steam pressure, lbs. per sq. in. | Millions of revolutions before breaking |
|---|---|---|
| Wrought Iron . . . . . | 9660 | 123 |
| " " . . . . . | 10080 | 46 |
| " " . . . . . | 9920 | 130 |
| Whitworth steel . . . . . | 14400 | 41 |

The inertia stresses are not included. They would probably a little reduce the stresses given. The breaking stresses are much below the yield stress of the materials.

*Fatigue due to the action of live loads, causing constant variation of stress.*—The researches of Wöhler (1871), since repeated by others, prove that the stress at which a material fractures, when subjected to a constantly varying amount of straining, depends on the *range of variation* of stress and on the *number of repetitions* of the change of load. Hence, the safe working stress in such cases must be lower, the greater the variation of stress and the greater the number of repetitions of loading. Now most machine parts, such as the crank shaft or piston rod of an engine, suffer a complete cycle of variation of stress in every revolution of the engine and should be capable of resisting a practically infinite number of repetitions of the cycle of changes. A locomotive may run 60,000 or 80,000 miles before being overhauled and defective parts replaced. In that time the working parts will have been subjected to probably 20,000,000 changes or reversals of stress. Wöhler's researches prove that parts, subjected to variations of stress for indefinitely long periods, must be designed for much smaller working stresses than would be safe with a steady load and smaller in a ratio depending on the range of variation of stress.

Ewing, Rosenhain, Humphrey and others have shown by microscopic investigation, that, when deformation of a metal exceeds certain limits, the crystalline particles are altered in shape by *slips* at the natural cleavage planes of the crystals. It has also been shown that when a bar is subjected to variations of stress such as would ultimately cause fracture, slips are observable at an early stage of the test. Gradually these slips increase in number and are massed together, and ultimately develop

into fissures or cracks. These cracks, as repetitions of straining proceed, extend till the bar fractures. A fracture due to fatigue of this kind is very characteristic. The fissure gradually formed is indicated by a very fine grained surface surrounding a coarser grained part corresponding to the final sudden fracture.

31. *Mathematical expression for Wohler's law.*—Let K be the statical breaking strength of a material and suppose that the stress in a bar or machine part varies from $k_{max}$ to $k_{min}$, so that the range of variation of stress is $= \Delta = k_{max} - k_{min}$. In using this expression, if tensions are reckoned positive, pressures must be reckoned negative, so that, if the two stresses are of different sign, the range of stress is equal to their sum $[k_{max.} - (-k_{min}) = k_{max} + k_{min}]$. Let the number of changes of load be indefinitely great. Then Wöhler's researches show that fracture will occur, for some value of $k_{max.}$ less than K, and so much smaller, the greater the range of stress Δ. That is, the breaking strength for the live load is less than K. Hence, in designing a structure for such a varying load. the ultimate strength is to be taken at some value $k$ less than K, which is determined with reference to Δ.

For example, Wöhler found that a bar was equally safe to resist varying bending and tensile straining actions, or sustained an equal very large number of repetitions of loading, before breaking, when the maximum and minimum stresses had the following values, for which Δ is nearly constant :—

| | Pounds per sq. in. $k$max. | min. | Δ |
|---|---|---|---|
| *For Wrought Iron* | | | |
| In tension only . . . . . . . . . | 18713 to | 31 | 18682 |
| In tension and compression alternately . . | 8317 to | −8317 | 16634 |
| *For Steel* | | | |
| In tension only . . . . . . . . . | 34307 to | 11436 | 22871 |
| In tension and compression alternately . . | 12475 to | −12475 | 24950 |

These experiments are sufficient to show that the breaking and therefore also the working stress depends very much on the range of variation of stress, being much greater when a bar is subjected to stress of one kind only than when subjected alternately to stresses of opposite kinds, and in both cases being less than for a steady load with no variation.[1]

[1] Wöhler's experiments agree with and confirm the earlier experiment of Sir W. Fairbairn, communicated to the Royal Society, on the

Let, as before, K be the breaking strength per unit of section, for the given material, and for a load once gradually applied. Let $k_{max.}$ be the breaking strength for the same material subjected to a variable load ranging between the limits $k_{max}$ and $\pm k_{min.}$, and repeated an indefinitely great number of times. $k_{min.}$ is + if the stress is of the same kind as $k_{max}$ and K, and − if the stress is of the opposite kind, and it is supposed that $k_{min.}$ is not greater than $k_{max}$ Then the range of stress is $\Delta = k_{max.} \mp k_{min}$, the upper sign being taken if the stresses are of the same kind, and the lower if they are different. Hence $\Delta$ is always positive.

Then Wöhler's experiments appear to suggest a rule of the following kind, for the relation between K and $k_{max}$ :—

$$k_{max.} = \frac{\Delta}{2} + \sqrt{K^2 - n \Delta K} \quad . \quad . \quad . \quad (1)$$

If $\Delta = 0$, we get $k_{max} = K$, the load being then a steady one. Further, by choosing a suitable value for $n$, we can make the decrease of $k_{max}$ for increasing values of $\Delta$ correspond with the observed values in Wöhler's experiments. For ductile iron and steel the average value of $n$ appears to be about 1·5. For hard qualities $n = 2$.

The special cases most useful to consider are the following :—

Case A. The load is invariable ; then $\Delta = 0$.

Case B. The load is entirely removed and replaced. Then $k_{min.} = 0$ and $\Delta = k_{max}$

Case C. The stress is alternately a compressive and tensile stress of the same magnitude. Then $k_{max}$ and $k_{min.}$ are equal in magnitude and of opposite sign, and $\Delta = 2\,k_{max.}$

effect of continuous changes of load on a riveted girder. In Germany the breaking strength receives different names according to the conditions in which the piece is placed. The stress at which fracture occurs by a single application of a steady or very gradually applied load is called the *statical breaking strength* (Tragfestigkeit). If after each application of a load the bar reverts to its original condition, and if all stresses are in the same sense (that is, all tensions, compressions, or shears in one direction), then the greatest stress which can be sustained for a specified number of repetitions is termed the *primitive strength* (Ursprungfestigkeit). If the stresses are alternately of opposite senses, that is, alternately tensions and compressions, or shears in opposite directions, the stress which can be sustained is termed the *vibration strength* (Schwingungsfestigkeit). See Weyrauch, *Proc. Inst. Civil Engineers*, vol. lxiii.

For these cases the formula gives the following values :—

| | Greatest Stress. | Least Stress. | Range of Stress. $\Delta$ | |
|---|---|---|---|---|
| Case A. | $k_{max.}$ | $k_{max}$ | 0. | Then $k_{max.} = \text{K}$. |
| Case B. | $k_{max}$ | 0 | $k_{max.}$ | Then $k_{max.} = 2(\sqrt{n^2+1} - n)\,\text{K}$. |
| Case C. | $k_{max}$ | $-k_{max}$ | $2k_{max.}$ | Then $k_{max.} = \frac{1}{2n}\,\text{K}$. |

Putting $n = 1{\cdot}5$, we get, in Case A, $k_{max} = \text{K}$; in Case B, $k_{max} = 0{\cdot}605\dot{4}\,\text{K}$; in Case C, $k_{max} = \frac{1}{3}\,\text{K}$. In round numbers the stresses at which the bar breaks are in the proportion of 3 : 2 : 1 in these three cases.

From an examination of all the earlier series of experiments on repeated stress in which bars stood at least two or three million changes of load, the values of $n$ and K in formula (1) above have been deduced. Then from these values the breaking strengths given in the following table for each of the materials, and for different ranges of stress, were calculated. A fuller account of the experiments will be found in the author's treatise on 'The Testing of Materials of Construction.'

BREAKING STRESS FOR AN INDEFINITELY LARGE NUMBER OF CHANGES OF LOAD. (AT LEAST TWO OR THREE MILLIONS)

*Stresses in Tons per square inch*

| | Value of $n$ | Case A. Steady load K | Case B. $\Delta = f$max. | Case C. $\Delta = 2f$max | Authority |
|---|---|---|---|---|---|
| Phœnix iron . . . . | 1·33 | 22·8 | 15·25 | 8·6 | Wöhler |
| Krupp's axle steel . . | 1·83 | 52·0 | 20·5 | 14·05 | ,, |
| Untempered spring steel. | 2·14 | 57·5 | 25·5 | 13·5 | ,, |
| Wrought iron plate . . | 1·60 | 22·8 | 13·1 | 7·15 | Bauschinger |
| Bessemer steel . . . | 1·68 | 28·6 | 15·7 | 8·55 | ,, |
| Bar iron . . . . . | 1·67 | 26·0 | 14·4 | 7·85 | ,, |
| ,, . . . . . | 1·53 | 26·4 | 15·75 | 8·65 | ,, |
| Steel axle . . . . | 1·91 | 40·0 | 19·7 | 10·5 | ,, |
| Steel rail . . . . | 2·0 | 39·0 | 18·4 | 9·7 | ,, |
| Steel boiler plate . . | 1·53 | 26·6 | 15·8 | 8·65 | ,, |

In these earlier experiments the changes of stress were at the rate of under 100 per minute. More recently a very valuable investigation has been carried out by Dr. Stanton, at the National Physical Laboratory,[1] with reversals of stress at the rate of

[1] Resistance of Iron and Steel to reversals of direct stress. *Proc. Inst. Civil Eng.* vol. clxvi.

800 per minute. The results were quite similar to those given above, so far as it is possible to compare the materials used. But in some similar tests by Osborne Reynolds and Smith, with reversals at the rate of 2000 per minute, somewhat lower breaking stresses were found.[1]

From Dr. Stanton's results, in which the bars broke after about one million reversals of stress, the author has calculated the value of $n$ in Eq. 1, and the value of K was determined by test. Some of the values of $n$ are lower than in Bauschinger's tests, and this may be due to the fact that the number of repetitions before fracture was much less in Stanton's than in Bauschinger's tests.

Using these values of $n$ and K the breaking stresses for the three cases described above have been calculated and are given in the following table.

BREAKING STRESSES FOR ONE MILLION CHANGES OF LOAD. (STANTON.)

*Stresses in Tons per square inch*

| | Per cent. of Carbon | Yield Stress | Primitive elastic Limit | Value of $n$ | Case A Steady load K | Case B. $\Delta = f_{max.}$ | Case C. $\Delta = 2f_{max}$ |
|---|---|---|---|---|---|---|---|
| Swedish Bessemer Steel: | | | | | | | |
| No. 3 | 0·65 | 29·1 | 27·7 | 1·47 | 47·6 | 29·5 | 16·1 |
| No. 2 | 0·45 | 28·1 | 25·0 | 1·47 | 43·8 | 27·1 | 14·8 |
| No. 1 | 0·17 | 23·8 | 21·4 | 1·12 | 28·5 | 21·6 | 12·8 |
| Swedish Charcoal Iron | 0·04 | 14·5 | 12·9 | 1·07 | 19·6 | 15·2 | 9·2 |
| Piston-rod Steel | 0·45 | 22·3 | 19·6 | 1·57 | 43·9 | 25·4 | 14·1 |
| Steel Forging | 0·34 | 14·5 | 12·9 | 1·47 | 29·5 | 18·2 | 10·0 |
| Mild Steel, No. 2 | 0·33 | 15·8 | 14·3 | 1·09 | 28·3 | 22·0 | 13·0 |
| ,, ,, No. 1 | 0·07 | 13·4 | 10·7 | 1·10 | 21·9 | 16·6 | 9·4 |
| Wrought Iron, No. 2 | 0·20 | 14·7 | 13·4 | 1·28 | 25·6 | 17·4 | 9·9 |
| ,, ,, No. 1 | 0·03 | 16·5 | 14·3 | 1·24 | 23·8 | 16·7 | 9·5 |

Approximately, the primitive yield stress is the breaking stress for variations of stress from 0 to a maximum (Case B), and half this the breaking stress for variations from a maximum in tension to an equal stress in compression.

32. *Working Stress.*—It is now possible to give a reasonable estimate of safe working stress and to distinguish between real and apparent factors of safety. The real factor of safety, in pieces subject to varying stress, is the ratio of the breaking stress determined by Wöhler's law to the actual stress. In railway bridges the contingencies to be allowed for in the factor

[1] On a throw-testing machine for reversals of mean stress. *Phil. Trans.* vol. cxcix.

of safety are that from imperfect workmanship the distribution of the stresses may be somewhat different from that assumed in calculation, and that from imperfect conditions of the roadway there may be impacts and vibrations which cannot be estimated. It appears that in such cases a real factor of safety of three is sufficient. The same factor appears sufficient for machinery wherever the loading forces can be determined with a fair approximation. In other cases a larger factor should be taken.

33. *Effect of abrupt changes of section.* — It was shown strikingly in the earlier Wöhler tests, as well as in those of Dr.

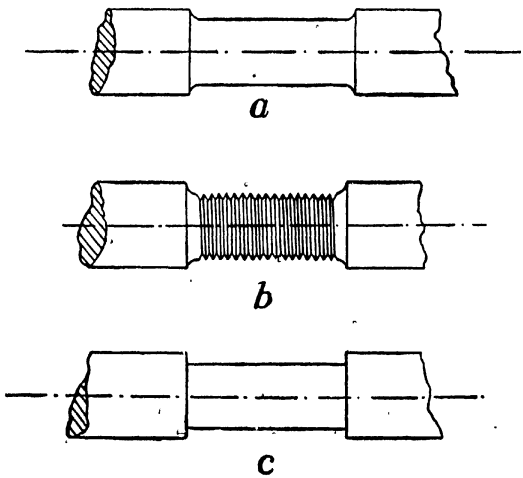



Fig. 7

Stanton, that any abrupt change of section very greatly reduced the endurance of bars subjected to repetition of loading. At such changes of section the distribution of stress is not uniform and the maximum stress is greater than the mean stress under the maximum load. Hence, comparing an ordinary bar with a bar having an abrupt change of section exposed to the same loading and with the same mean stress, the latter would be expected to break with fewer repetitions of loading. In Dr. Stanton's tests, the forms shown in fig. 7 were used, *b* being a Whitworth screw thread $\frac{3}{8}$ in. diameter; *a*, a $\frac{1}{2}$ in. bar turned down to 0·295, and with a very small fillet 0·062 in. radius at the corners; and *c*, a similar bar with the corners square. The

resistance of bars *a* and *b* was practically the same. Such bars sustained a million changes of load with a range of mean stress about 0·7 that sustained by an ordinary bar or bar with a large fillet at the corners. In the case of bar *c*, it broke down after a million changes of load with a range of mean stress varying from 0·48 for hard steel to 0·55 or 0·65 for wrought iron and mild steel of that sustained by an ordinary uniform bar.

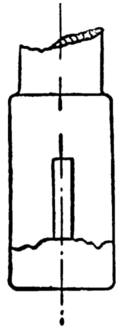


Fig. 8

It has long been known that sharp re-entrant angles and abrupt changes of section were dangerous in machine parts, such as railway axles subjected to constant variations of stress, and the breaking down of machine parts in which such features existed is very frequently referred to in Mr. Longridge's reports. For instance, rods with cotter holes very frequently break (fig. 8). In this case there is not only an abrupt change of section, but probably the un-uniform distribution of the stress is increased by imperfect seating or bending of the cotter. The following are cases, given in Mr. Longridge's report for 1889, of rods which broke at cotter holes.

| | Mean Stress at Cotter hole, tons per sq. in. | Total Range of Stress, tons per sq. in. | Millions of changes of load before fracture |
|---|---|---|---|
| Wrought iron rod | 5·14 | 10·28 | 43·5 |
| ,, ,, ,, | 3·58 | 7·16 | 115 |
| Steel rod | 3·80 | 7·60 | 46 |
| Wrought iron rod | 3·80 | 7·60 | 3·5 |

Mr. Longridge infers that the mean stress on the section through the cotter hole should not exceed 2 tons per sq. in.

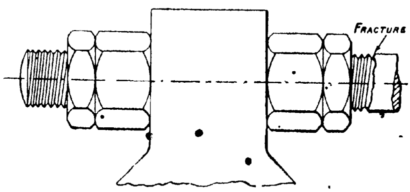



Fig. 9

The screwed ends of valve rods (fig. 9) not infrequently break, and also crank pins when they have a collar, as in fig. 10. In this

case the grip of the crank eye, when shrunk on, also increases the stress at the angle where the fracture occurs. The form in fig. 11 obviates this difficulty. Nearly always in such cases it is found that a fissure has existed some time before final fracture, and the gradual extension of this fissure causes the final disaster. A fine cut made by the turning tool in a shaft appears sometimes to be the cause of fracture, and Mr. Longridge observes (Report, 1907) that it is astonishing how slight a scratch appears to determine the position of a fracture in a piece of steel subjected to alternations of stress.

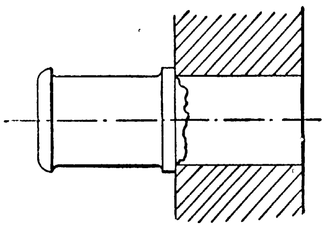


Fig. 10

34. *Case of parts subjected to shocks.*—In many cases materials are subjected to impulsive loads, and a gradual deterioration of strength is observed. Thus a crane chain sometimes breaks when carrying a load which it has often before carried safely, and which is within the ordinary limits of working stress. In part, this deterioration of strength may be due to the ordinary action of a live or repeated load; but it appears to be more often due directly to the gradual loss of the power of elongation, in consequence of the slow accumulation of the permanent set. Suppose a crane chain carrying a load, W, surges, so that the load falls a distance, $h$, and let the elongation of the chain under the action of this impulsive load be $l$. Then the work done by the load in falling is W $(h+l)$. The work absorbed in elongating the chain is $k\ p\ l$ A, where A is the area of the section of the chain, $p$ is the maximum intensity of the stress induced, and $k$ is a constant, which would be $\frac{1}{2}$ if the stress $p$ were within the elastic limit, but which lies between $\frac{1}{2}$ and 1, if $p$ exceeds the elastic resistance. Equating these, we get—

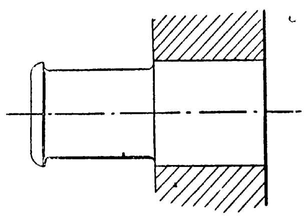


Fig. 11

$$\text{W}(h + l) = k\,p\,l\,\text{A}$$

$$p = \frac{\text{W}\,(h + l)}{k\,l\,\text{A}} \quad . \quad . \quad . \quad (2)$$

Hence, if $h$ is not small compared with $l$, the stress $p$ will be greater the smaller the elongation $l$ of the chain. In a new chain, $l$ will include the permanent set as well as the elastic elongation. In an old chain, which has already elongated permanently, and thus become less tough, the power of elongation before fracture is diminished. Hence the stress $p$, induced by a load W (capable of producing stresses somewhat beyond the elastic limit), increases for any given chain as the chain gets older, and may ultimately reach the breaking stress. A new chain under the action of impulsive loads gradually lengthens permanently with no other ill effect except that it has become more brittle and less capable of resisting additional impacts. Annealing a crane chain restores its power of elongation and its original power of resisting impulsive loads. Periodical reannealing of crane chains is prescribed in factory regulations.

35. *Straining Action due to Power transmitted.*—When HP horses' power are transmitted through a link or connecting rod moving with velocity $v$, in ft. per second, the straining force, parallel to the axis of the rod, due to the work transmitted, is

$$\text{P} = \frac{550\ \text{HP}}{v}\ \text{lbs.} \quad . \quad . \quad . \quad . \quad (3)$$

There will be in this case other straining actions, due to the reactions of the supports of the link, if the link is not moving parallel to its axis.

When HP horses' power are transmitted through a rotating piece, making $n$ revolutions per second, the twisting moment, about the axis of the piece, is given by the equation

$$\text{T} = \frac{550\ \text{HP}}{2\,\pi\,n} = 1050{\cdot}4\,\frac{\text{HP}}{n}\ \text{inch lbs.} \quad . \quad . \quad (4)$$

Or if N = revolutions per minute.

$$\text{T} = 63024\,\frac{\text{HP}}{\text{N}}\ \text{inch lbs.} \quad . \quad . \quad . \quad . \quad (5)$$

*Straining Actions due to Variations of Velocity.*—When a heavy body is accelerated or retarded, straining actions are produced, due to its inertia. If $w$ lbs. acquire an increase of

velocity $d\,v$, in the time $d\,t$, the stress due to acceleration estimated in lbs. is

$$-\frac{w}{g}\cdot\frac{d\,v}{d\,t} \quad \ldots \ldots \quad (6)$$

*Graphic Method of determining the Stress due to Inertia.*—In most cases the algebraic expression of the velocity of machine parts is rather complicated, but it is easy to draw a curve representing very approximately the velocity at any instant. Suppose such a velocity curve, having for abscissa the distance moved, and for ordinate the velocity, to have been drawn. Then the acceleration per unit of mass at any point of the path is proportional to the subnormal of the velocity curve at that point.

For instance, let $O\,x$, fig. 12, represent the stroke of a piston weighing $w$ lbs., and let $O\,b\,x$ be the velocity curve. When

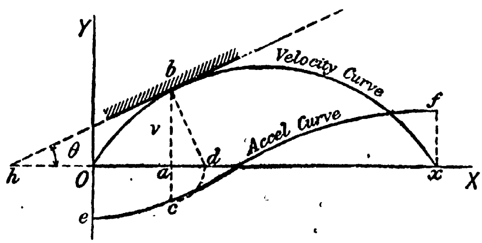



Fig. 12

the piston has moved a distance $O\,a=s$, the velocity is $a\,b=v$. Draw $b\,d$ the normal to the velocity curve at $b$. Then the acceleration at that point of the stroke is proportional to the subnormal $a\,d$. For

$$-\frac{w}{g}\cdot\frac{d\,v}{d\,t}=-\frac{w}{g}\cdot\frac{d\,v}{d\,s}\cdot\frac{d\,s}{d\,t}=-\frac{w}{g}\,v\,\frac{d\,v}{d\,s}.$$

But $dv/ds$ is the trigonometrical tangent of the angle $\theta$ between the tangent to the curve and the axis $O\,x$, or, what is the same thing, the angle $a\,b\,d$. Putting $v=a\,b$; $d\,v/d\,s=a\,d/a\,b=\tan\theta$, the acceleration is :—

$$\frac{d\,v}{d\,t}=v\,\frac{d\,v}{d\,s}=a\,d$$

The accelerating force acting on piston is

$$-\frac{w}{g}\,.\,a\,d \text{ lbs.}$$

$$W(h + l) = k\,p\,l\,A$$

$$p = \frac{W\,(h + l)}{k\,l\,A} \quad . \quad (2)$$

Hence, if $h$ is not small compared with $l$, the stress $p$ will be greater the smaller the elongation $l$ of the chain. In a new chain, $l$ will include the permanent set as well as the elastic elongation. In an old chain, which has already elongated permanently, and thus become less tough, the power of elongation before fracture is diminished. Hence the stress $p$, induced by a load W (capable of producing stresses somewhat beyond the elastic limit), increases for any given chain as the chain gets older, and may ultimately reach the breaking stress. A new chain under the action of impulsive loads gradually lengthens permanently with no other ill effect except that it has become more brittle and less capable of resisting additional impacts. Annealing a crane chain restores its power of elongation and its original power of resisting impulsive loads. Periodical reannealing of crane chains is prescribed in factory regulations.

35. *Straining Action due to Power transmitted.*—When HP horses' power are transmitted through a link or connecting rod moving with velocity $v$, in ft. per second, the straining force, parallel to the axis of the rod, due to the work transmitted, is

$$P = \frac{550\ \text{HP}}{v}\ \text{lbs.} \quad . \quad . \quad . \quad . \quad (3)$$

There will be in this case other straining actions, due to the reactions of the supports of the link, if the link is not moving parallel to its axis.

When HP horses' power are transmitted through a rotating piece, making $n$ revolutions per second, the twisting moment, about the axis of the piece, is given by the equation

$$T = \frac{550\ \text{HP}}{2\,\pi\,n} = 1050{\cdot}4\ \frac{\text{HP}}{n}\ \text{inch lbs.} \quad . \quad . \quad (4)$$

Or if $N$ = revolutions per minute.

$$T = 63024\ \frac{\text{HP}}{N}\ \text{inch lbs.} \quad . \quad . \quad . \quad . \quad (5)$$

*Straining Actions due to Variations of Velocity.*—When a heavy body is accelerated or retarded, straining actions are produced, due to its inertia. If $w$ lbs. acquire an increase of

the material. A bar of the length $l$ at the colder temperature would become $(1 + a t)\ l$ at the hotter, if unhindered. If an obstacle is opposed to the expansion, a stress is induced which may reach the value given by the relation

$$a\,t = f/\mathrm{E}$$
$$f = a\,t\,\mathrm{E} \quad . \quad . \quad . \quad . \quad (9)$$

Let $\mathrm{E}=29000000$ for iron or steel; then $f=313t$ in lbs. per sq. in., when $t$ is in degrees Centigrade; $f \doteq 200t$ in lbs. per sq. in., when $t$ is in degrees Fahrenheit.

## CHAPTER III

### RESISTANCE OF STRUCTURES TO DIFFERENT KINDS OF STRAINING ACTION

#### Physical Constants for Ordinary Materials

IT has already been shown that the *strain* or change of form or dimension in a body subjected to straining action is partly an elastic strain proportional to the stress, partly a permanent deformation or *set*. For any given condition of variation of straining action there are limits in compression and tension within which the strains are wholly elastic. Working stresses are well within these limits.

38. *Table of Strength and Coefficients of Elasticity.*—The table given on pp. 50, 51, shows the primitive elastic and ultimate strength of different materials, as determined by ordinary statical tests, when the stress is simple tension, pressure, or shearing stress. The values given are either average, or maximum, mean and minimum values, selected from the most trustworthy experiments. In different specimens of the same material, and even in different pieces of the same bar or plate, there are often differences of elasticity and strength, and the judgment of the engineer must be relied on in deciding how far average values of this kind are applicable in any given case. The first two columns of the table relate to direct stresses of pressure or tension produced by straining actions normal to the sections considered. The third relates to tangential or shearing stress produced by straining action parallel to the section. The elastic strength is that stress per unit of area at which the strains cease to be sensibly proportional to the stresses, and the values given are those obtained when a bar is first loaded with a gradually increasing stress of one kind only. The ultimate strength is the intensity of stress reckoned on the original section preceding rupture, and this depends in some degree on the manner of

TABLE I.—*Ultimate and Elastic Strengths of Materials and Coefficients of Elasticity, in lbs. per sq. in.*

| Material | Breaking Strength | | | Elastic Strength | | | Coefficient of Elasticity | |
|---|---|---|---|---|---|---|---|---|
| | Tension | Pressure | Shearing | Tension | Pressure | Shearing | Direct E | Transverse G |
| Cast iron | 30,500 | 130,000 | 12,000 | — | — | — | 23,000,000 | 7,600,000 |
| | 17,500 | 95,000 | 10,500 | 10,500[1] | 21,000[1] | 8,000[1] | 17,000,000 | 6,300,000 |
| | 10,800 | 50,000 | 8,700 | — | — | — | 14,000,000 | 5,000,000 |
| Wrought-iron bars | 67,000 | — | 49,000[1] | — | — | — | 31,000,000 | — |
| | 57,600 | 50,000 | 40,000 | 30,000 | 30,000 | 22,000 | 29,000,000 | 10,500,000 |
| | 33,500 | — | 22,400 | — | — | — | 27,000,000 | — |
| Iron ship plates \|\| | 49,000 | — | — | — | — | — | — | — |
| Iron boiler plates \|\| | 47,000 | — | 36,000 | 24,000 | 24,000 | 15,000 | 26,000,000 | 14,000,000 |
| " " ⊥ | 41,500 | — | — | — | — | — | 27,000,000 | — |
| Steel plates, $\frac{1}{4}$% carbon | 65,000 | — | 50,000 | 42,000 | 38,000 | — | | — |
| " " $\frac{1}{2}$% " | 78,000 | — | 56,000 | 47,000 | 49,000 | — | 31,000,000 | 13,000,000 |
| " " 1% " | 110,000 | — | 83,000 | 67,000 | 71,000 | — | | — |
| Steel boiler plates | 66,000 | — | 56,000 | 36,000 | — | — | 30,000,000 | 13,500,000 |
| Rivet steel | 65,000 | — | 55,600 | 46,000 | — | — | 30,000,000 | 13,000,000 |
| Cast steel untempered | 150,000 | — | — | — | — | — | — | — |
| | 120,000 | — | — | 80,000 | 80,000 | 64,000 | 30,000,000 | 12,000,000 |
| | 84,000 | — | — | — | — | — | — | — |
| Cast steel tempered | — | — | — | 190,000 | 190,000 | 145,000 | 36,000,000 | 14,000,000 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Steel castings | from | 80,000 | — | — | 34,000 | — | — | 30,000,000 | — |
| | to | 40,000 | — | — | 28,000 | — | — | 20,000,000 | — |
| Nickel steel | from | 70,000 | — | — | 50,000 | — | — | 29,700,000 | — |
| | to | 90,000 | — | — | 70,000 | — | — | — | — |
| Copper cast | from | 19,000 | — | — | — | — | — | 12,000,000 | — |
| | to | 23,000 | 58,000 | — | — | — | — | — | — |
| ,, rolled plates | | 31,000 | — | — | 5,600 | 4,000 | 3,000 | 15,000,000 | 5,600,000 |
| ,, annealed wire | | 45,000 | — | — | — | — | — | 16,000,000 | — |
| ,, hard drawn wire | | 58,000 | — | — | — | — | — | 17,000,000 | — |
| Brass | from | 17,500 | — | — | — | — | — | 13,500,000 | — |
| | to | 29,000 | — | — | — | — | — | — | — |
| Gun-metal or bronze | | 52,000 | — | — | — | — | — | — | — |
| | | 27,000 | — | — | 6,200 | — | 4,150 | 13,500,000 | — |
| | | 23,000 | — | — | — | — | — | — | — |
| Delta metal, cast | | 36,000 | — | — | 17,000 | — | — | 12,000,000 | — |
| ,, ,, rolled | | 74,000 | — | — | 51,000 | — | — | 13,000,000 | — |
| Phosphor bronze | | 58,000 | — | 43,000 | 19,700 | — | 14,500 | 14,000,000 | 5,250,000 |
| Muntz metal | | 49,000 | — | — | — | — | — | — | — |
| Cast zinc | | 7,500 | — | — | 3,200 | — | — | — | — |
| Lead | | 2,500 | 7,000 | — | 1,500 | — | — | 2,500,000 | — |
| Tin | | 4,700 | — | — | — | — | — | — | — |
| Wood pine | from | 6,700 | 3,400 | — | — | — | — | 1,000,000 | — |
| | to | 13,000 | 7,000 | 630[3] | — | — | — | 1,600,000 | — |
| ,, oak | | 15,000 | 10,000 | 2,240[3] | — | — | — | 1,450,000 | — |
| Leather | | 4,200 | — | — | — | — | — | 25,000 | — |

Cast iron has properly no elastic limit.

[2] The shearing resistance of wrought iron is less along the fibre than across it.

[3] These are along the fibres of the wood.

carrying out the experiment and the proportions of the bar tested. The more rapid the loading of the bar and the less vibration induced, the greater is the load carried before rupture ensues. Nevertheless, if the experiment is carried out with ordinary care, the breaking strength is a definite measure of the properties of the material. The elastic and breaking strengths are expressed in lbs. per sq. in.

A coefficient of elasticity is the ratio of the intensities of stress and strain of some given kind, when the elastic limit is not passed. Thus the *coefficient of direct elasticity* of a material is the ratio of the normal stress $p$, per unit of section of a bar, to the extension or compression, $e$, per unit of length, produced by the stress; that is, the coefficient of direct elasticity is,—

$$E = p/e$$

where $p$ is expressed in lbs. per sq. in. and $e$ in inches per inch of length. The bar is supposed to be free laterally. For ordinary metals the coefficient of direct elasticity may be taken to be the same for tension and pressure. The coefficient of transverse elasticity or *coefficient of rigidity* is the ratio of the shearing stress $q$ per unit of area to the distortion $n$; the distortion being measured by the tangent of the difference of the angles of an originally square particle before and after the stress is applied. Hence the coefficient of transverse elasticity is,—

$$G = q/n$$

The ratio $G/E$ for ordinary materials of construction is about $\frac{3}{8}$ to $\frac{2}{5}$.[1]

39. *Tables of Working Stress.*—It has been pointed out that the working stress of a material must be less than the elastic strength, to allow for straining actions which cannot be taken into account, for imperfections of workmanship, for the effects of varia-

[1] For some materials, such as cast iron, annealed copper, zinc, cement, and cement mortar, in which the strains increase faster than the stresses for practically all intensities of stress, the relation

$$E = p^n/e$$

is approximate for a considerable range of stress. E is a constant and $n$ is a constant rather greater than unity for metals and less than unity for stone and leather.

tion of straining action and for other sources of danger. TABLE II on pp. 54, 55, gives values of the ordinary working stress allowed in designing machinery. The safe working stress has been fixed with attention partly to practical experience, partly to the results of Wöhler's experiments. The cases for which the working stress is given are those discussed in § 31, p. 38.

The following data are supplementary to those given in Tables II. Subjected to bending hardened spring steel will carry 62,000 lbs. per sq. in. in case B. Hard wood, in case A, will carry 2,100 lbs. per sq. in., and soft wood 1,500 lbs. per sq. in.

A kind of mild steel, termed high tensile steel, has been produced which has a tenacity of 78,000 lbs. per sq. in. in normal condition and 75,000 lbs. when annealed. The elongation is 24 per cent. in 8 ins. in normal condition and 26 per cent. when annealed.

Table IIA gives the working stress very commonly adopted in building construction, in cases where the load is chiefly a dead load :—

TABLE IIA.—*Working Stresses in Building Construction*

| | Tension lbs. per sq. in. | Compression lbs. per sq. in. |
|---|---|---|
| Wrought iron | 14,000 | 14,000 |
| Cast iron | 3,360 | 10,080 |
| Oak | 1,400 | 940 |
| Pine | 1,120 | 800 |
| Brickwork, lime mortar | — | 36 to 72 |
| ,, cement mortar | — | 72 to 108 |
| Rubble masonry, lime mortar | — | 58 |
| ,, ,, cement mortar | — | 72 |
| Portland cement concrete | — | 100 |
| Pressed bricks in cement mortar | — | 114 to 128 |

For Reinforced Concrete the working stresses may be :—

| | lbs. per sq. in. |
|---|---|
| Concrete in compression, in beams | 600 |
| Concrete in compression, in columns | 500 |
| Concrete in shear, in beams | 60 |
| Adhesion of concrete to steel | 100 |
| Steel in tension | 15,000 to 17,000 |
| Coefficient of elasticity—steel | 30,000,000 |
| Coefficient of elasticity—concrete | 2,000,000 |

TABLE II.—ORDINARY WORKING STRESS

CASE A. *The straining action a steady or permanent one*

| Material | Kind of Stress | | | | |
|---|---|---|---|---|---|
| | Tension $f_t$ | Compression $f_c$ | Bending $f_b$ | Shear $f_s$ | Torsion |
| Cast iron | 4,200 | 12,000 | 6,000 to 8,000 | 4,000 | 4,000 to 6,000 |
| Wrought iron | | | | | |
| Bar or forged | 15,000 | 15,000 | 15,000 | 12,000 | 7,500 |
| Plate ‖ | 13,500 | — | — | — | — |
| Plate ⊥ | 12,000 | — | — | 10,000 | — |
| Mild steel | 13,000 to 17,000 | 13,000 to 17,000 | 13,000 to 17,000 | 10,000 to 13,000 | 8,000 to 12,000 |
| Cast steel | 17,000 to 21,000 | 17,000 to 21,000 | 17,000 to 21,000 | 13,000 to 17,000 | 12,000 to 16,000 |
| Steel castings | 8,000 to 12,000 | 12,000 to 16,000 | 10,000 to 14,000 | 7,000 to 12,000 | 7,000 to 12,000 |
| Phosphor bronze | 10,000 | — | — | 7,000 | 4,200 |
| Gun-metal | 4,200 | — | — | — | — |
| Rolled copper | 6,000 | — | — | 2,400 | — |
| Brass | 3,000 | — | — | — | — |

TABLE II.—ORDINARY WORKING STRESS—*continued*

CASE B. *Straining action producing stress of one kind only, varying from zero to a greatest value frequently*

| Material | Tension $f_t$ | Compression $f_c$ | Bending $f_b$ | Shear $f_s$ | Torsion |
|---|---|---|---|---|---|
| Cast iron | 2,800 | 8,500 | 4,000 to 5,300 | 2,800 | 2,600 to 4,000 |
| Bar iron | 10,000 | 10,000 | 10,000 | 8,000 | 5,000 |
| Plate iron ‖ | 9,000 | — | — | — | — |
| ,, ,, ⊥ | 8,000 | — | — | 6,500 | — |
| Mild Steel | 8,600 to 11,400 | 8,600 to 12,000 | 8,600 to 11,400 | 6,500 to 8,600 | 5,300 to 8,000 |
| Cast steel | 11,400 to 14,000 | 11,400 to 14,000 | 11,400 to 14,000 | 8,600 to 11,400 | 8,000 to 10,600 |
| Steel castings | 5,300 to 8,000 | 8,000 to 10,600 | 6,600 to 9,400 | 4,700 to 8,000 | 4,700 to 8,000 |
| Phosphor bronze | 6,600 | — | — | 4,600 | 2,800 |
| Gun-metal | 2,800 | — | — | — | — |
| Rolled copper | 3,000 | — | — | 1,600 | — |
| Brass | 2,000 | — | — | — | — |

CASE C. *Straining action producing equal stresses of opposite sign alternately*

| Material | Tension and Compression | Bending | Shear | Torsion |
|---|---|---|---|---|
| Cast iron | 1,400 | 2,000 to 2,700 | 1,400 | 1,300 to 2,000 |
| Bar iron | 5,000 | 5,000 | 4,000 | 2,500 |
| Mild steel | 4,300 to 5,700 | 4,300 to 5,700 | 3,300 to 4,300 | 2,700 to 4,000 |
| Cast steel | 5,700 to 7,000 | 5,700 to 7,000 | 4,300 to 5,700 | 4,000 to 5,300 |
| Steel castings | 2,700 to 4,000 | 3,300 to 4,700 | 2,300 to 4,000 | 2,300 to 4,000 |
| Gun-metal | 1,400 | — | — | — |

### Hardness of Materials

The relative hardness of different materials, measured by the resistance to indentation, is given in the following table:—[1]

| | |
|---|---|
| Cast steel | 554 |
| Mild steel | 144 |
| Copper, annealed | 62 |
| ,, unannealed | 105 |
| Brass, No. 1 | 221 |
| ,, ,, 2 | 246 |
| Zinc | 41 |
| Lead, cast | 4 |

Table III.—*Heaviness of Materials*

| | Lbs. per c. foot | Lbs. per c. inch |
|---|---|---|
| *Gaseous Bodies.* | | |
| Air at 32° and one atm. | ·0807 | — |
| Steam at 212° and one atm. | ·0378 | — |
| *Liquids.* | | |
| Pure water at 39°·1 F. | 62·425 | 0·0361 |
| ,, ,, 60° F. | 62·373 | — |
| River water (mean) | 63·0 | — |
| Sea water (mean) | 64·05 | 0·0371 |
| Mercury, at 32° | 849 | 0·491 |
| *Timber.* | | |
| Oak or teak | 45 to 55 | ·026 to ·032 |
| Pine or fir | 30 to 44 | ·017 to ·025 |
| Greenheart | 72 | ·042 |
| *Brick and Stone.* | | |
| Ordinary brick | 112 | 0·065 |
| Brickwork (mean) | 106 | 0·062 |
| Rubble masonry (mean) | 140 | 0·081 |
| Concrete (mean) | 150 | 0·087 |
| Ashlar, sandstone | 150 | 0·087 |
| ,, limestone | 165 | 0·096 |
| Granite | 175 | 0·102 |
| *Earths.* | | |
| Chalk | 150 | 0·087 |
| Moist clay | 163 | 0·094 |
| Mud or moist sand | 102 | 0·059 |
| Compact moist soil | 140 | 0·081 |
| *Metals.* | | |
| Wrought iron | 490 | 0·283 |
| Cast iron | 468 | 0·271 |
| Sheet lead | 711 | 0·411 |
| Copper | 555 | 0·321 |
| Zinc | 449 | 0·260 |
| Cast steel | 496 | 0·287 |
| Soft steel | 480 | 0·278 |
| Gun-metal | 546 | 0·316 |
| Cast brass | 518 | 0·299 |
| Rolled brass | 526 | 0·305 |

[1] Unwin, *The Testing of Materials of Construction*, p. 51.

## RESISTANCE TO STRESS

### Resistance to Simple Tension and Compression

40. A bar is in tension or compression when the load acts parallel to its axis, and the stress on any section of the bar is uniformly distributed or not, according as the line of action of the load does or does not pass through the centre of figure of that section. Cases in which the stress is a varying stress will be treated as cases of compound stress. At present only cases of uniformly distributed stress are considered.

Let fig. 13 represent a portion of a straight cylindrical or prismatic bar, loaded longitudinally. Then, apart from conditions near the ends, depending on the way in which the load is applied, if the resultant of the load acts in the axis of the bar, the stress on a cross-section, AB, normal to the axis, is a uniformly distributed normal or direct stress (tension or pressure). On an oblique section CD, the stress is also uniformly distributed and consists of a direct stress (tension or pressure) normal to the section and a shearing stress tangential to the section.

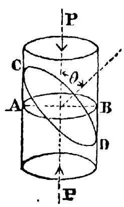


Fig. 13

*Simple Tension.*—When the straining action is a pull, let P be the load in lbs., $a$ the area of the normal cross-section AB in sq. ins. The stress on AB is a tension the intensity of which is—

$$f_t = P/a \text{ lbs. per sq. in.} \quad \quad (1)$$

Consider an oblique section, CD. Let $\theta$ be the angle between the normal to CD and the axis of the bar. Then the stresses on CD are

$$\left.\begin{array}{lll} \text{Normal tension } f_n = f_t \cos^2\theta & \text{lbs. per sq. in.} & . \\ \text{Shearing stress } f_s = f_t \sin\theta\cos\theta & \text{,,} \quad \text{,,} & . \end{array}\right\} \quad (2)$$

The tension on oblique sections is clearly always less than that on normal cross-sections, and the shearing stress is greatest if $\theta = 45°$, and is then equal to half the tension on a normal cross-section. Hence, only the stress on normal cross-sections needs to be attended to, unless the resistance to shearing of the material is less than half the resistance to tension.

To determine the required cross-section for a given load P, a value must be selected for the working stress $f_t$ in tension

(Table II, § 39), suitable for the material and the kind of loading. Then the section of the bar normal to the direction of P is

$$a = \mathrm{P}/f_t \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The effect of a pull or tensile load, on a bar of length $l$ and lateral dimension $d$, is to produce an elongation $\lambda$ and a lateral contraction $\delta$. From the definition of the coefficient of elasticity (§ 38), and provided that the stress does not exceed the limit of elasticity, the extension per unit length is—

$$e = \lambda/l = f_t/\mathrm{E} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where E is the coefficient of direct elasticity or Young's modulus (Table I, § 38).

Similarly the contraction per unit width is—

$$e_1 = \delta/d \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

Fig. 14 shows the bar, the original form being in full lines and the form after stretching in dotted lines.

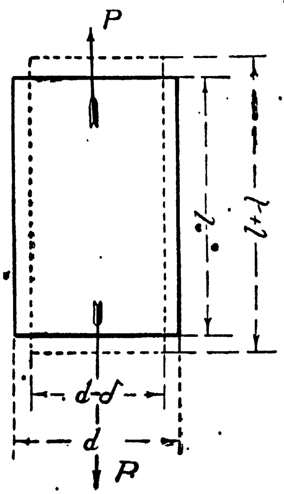



Fig. 14

*Ratio of lateral contraction to extension. Poisson's Ratio.*—Let

$$m = \frac{\lambda/l}{\delta/d} = \frac{e}{e_1} \quad . \quad (5)$$

be the ratio of the elongation per unit length to the contraction per unit of lateral dimension. The reciprocal $1/m$ is called Poisson's Ratio. For different materials $m$ varies from 3 to 4 and for metals is about 10/3. Hence in metals the lateral contraction per inch of width is about three-tenths of the longitudinal extension per inch of length.

41. *Simple Compression.* — If the straining action on a cylindrical bar is an axial thrust it produces a longitudinal compression and lateral extension. The intensity of pressure on normal cross-sections of area $a$ with a thrust P is $f_c = \mathrm{P}/a$. If $c$ is the longitudinal compression per unit length—

$$c = f_c/\mathrm{E} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where in general E has the same value as for tension.

When the bar is more than about five diameters in length, there is a tendency to bend or buckle laterally, and then the strength must be determined by the rules for short or long columns which take account of the bending.

A short prism compressed beyond the elastic limit gives way finally by shearing at planes inclined about 45° to the axis, if the material is brittle like cast iron. But if the material is ductile, it yields plastically, shortening and increasing in diameter. In that case the intensity of pressure, reckoned on the enlarged area, tends as the load is increased to a fixed value termed *the pressure of fluidity*. The increase of area balances the increase of load.

42. *Work done in Extending or Compressing a Bar.*—During the extension of the bar from $l$ to $l + \lambda$ the stress increases from zero to $f$ proportionately to the elongation. Hence the mean stress during the operation is $\frac{1}{2}f$, and consequently the work done in extending the bar is—

$$\text{W} = \tfrac{1}{2} f a \lambda = \tfrac{1}{2} f^2 a \frac{l}{\text{E}} \qquad (7)$$

That is, for a given intensity of stress the work varies as the volume of the bar. The same formula is applicable for compression. The bar is assumed to be uniform in section, and not strained beyond the elastic limit.

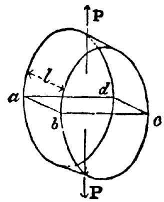


Fig. 15

43. *Resistance of thin Cylinders to an internal bursting Pressure.*—Consider a thin cylindrical shell of diameter $d$; length $l$, and thickness $t$, in inches, subjected to a uniform internal pressure of $p$ lbs. per sq. in. Let the cylinder be cut by a diametral plane $a\ b\ c\ d$, fig. 15. The resultant force P acting on either side of that plane $= p \times$ area $a\ b\ c\ d$. Hence, $\text{P} = p\ d\ l$. The molecular tensions which resist the bursting force act on the sections of the shell at $a\ b$ and $c\ d$, and are equal to the intensity of stress induced $\times$ sectional area of metal at $a\ b$ and $c\ d$, that is, $2\ t\ l$. Putting $f$ for the intensity of circumferential tensile stress, the total force resisting the bursting pressure is $2 f t l$. Equating the load and resistance

$$2 f t l = p\ d\ l$$

and the circumferential stress is

$$f = \frac{p\ d}{2\ t} \qquad (8)$$

If the cylinder is supposed cut by a section normal to the axis, the total force acting axially and normally to the section is $\frac{1}{4}\pi d^2 p$ and the area of metal cut by the section is $\pi d t$. Equating

$$\frac{1}{4}\pi d^2 p = \pi d t f$$

and the longitudinal stress is

$$f = \frac{p\,d}{4\,t} \quad . \quad . \quad . \quad . \quad . \quad (8a)$$

or half the circumferential stress. Similarly, in a spherical vessel of diameter $d$ and thickness $t$, the resultant pressure on a semisphere is $\pi d^2 p/4$ and the resistance of the metal cut by a diametral section is $\pi d t f$. Equating these—

$$f = \frac{p\,d}{4\,t} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

or equal to the longitudinal stress in a cylinder of the same diameter.

*Cylindrical Boiler Shells.*—If the cylinder consists of riveted plates, the sections $a\,b$ and $c\,d$ should be taken so as to pass through the rivet holes. Then the area of the rivet holes must be deducted from $2\,t\,l$, before equating the molecular and external forces.

The working stress for boiler shells may be taken at 12,000 lbs. per sq. in. for mild steel and 10,000 lbs. per sq. in. for wrought iron. Let $d$ be the diameter measured inside one of the outside rings of plates, $t$ the thickness of the plates, $p$ the steam pressure and $\eta$ the efficiency of the longitudinal riveted joints, or ratio of the strength of the joint to the strength of the solid plate. Then—

$$p = \frac{2\,t\,f\,\eta}{d} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

It is sometimes necessary to cut large oval manholes in boiler shells. Then since the stress is greater on longitudinal than on cross sections, it is safest to place the long axis of the oval transversely and the short axis longitudinally to the boiler axis.

44. *Thick Cylinders.*—If the cylinder is thick relatively to its internal diameter, the mean stress is unaltered but the inner layers are more severely strained than the outer layers. In that case the thickness must be determined by one of the following formulæ.

Lamé . . . $t = \frac{d}{2}\left\{\sqrt{\frac{f+p}{f-p}} - 1\right\}$ . . (11)

Grashof . . $t = \frac{d}{2}\left\{\sqrt{\frac{3f + 2p}{3f - 4p}} - 1\right\}$ . (11*a*)

If, as is frequently the case, $p/f$ is a small ratio—

$$t = \frac{d}{2} \cdot \frac{p}{f}\left(1 + \frac{5}{6}\frac{p}{f}\right) \text{ nearly} \quad . \quad (11b)$$

$t$ is the thickness of the cylinder in inches, $d$ its internal diameter, $f$ the working stress, and $p$ the excess of internal pressure over external pressure in lbs. per sq. in. The test pressure (usually double the working pressure) should not produce a stress greater

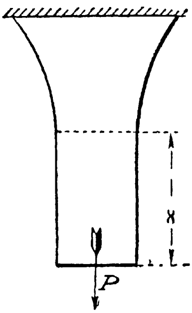



Fig. 16

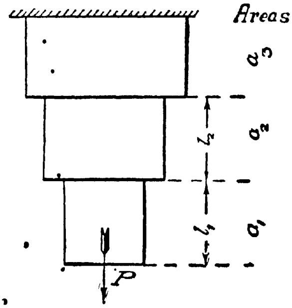



Fig. 17

than 9,000 lbs. per sq. in. for cast iron or 25,000 lbs. per sq. in. for steel castings. Using these values the last formula becomes—

Cast Iron . . . $t = \frac{dp}{9,000}\left(1 + \frac{p}{5,400}\right)$

Steel Casting. . . $t = \frac{dp}{25,000}\left(1 + \frac{p}{15,000}\right)$

where $p$ is the actual working pressure, taken at half the test pressure, and $t$ is the minimum safe thickness. Hydraulic pressures in extreme cases may amount to 3 tons per sq. in.

45. *Strength of Long Vertical Hoisting Cables when the Weight of the Cable is taken into Account.*—Colliery ropes in vertical shafts are long tie bars carrying a load at the bottom and strained at other cross-sections both by the load and the weight of the rope below the section. They are often tapered to reduce the weight, as in fig. 16.

Let P be the load lifted, $a_x$ the cross-section at $x$ from the

bottom, $f$ the safe working stress, $w$ the weight of the rope per cubic unit, $\epsilon = 2{\cdot}718$ the base of the natural system of logarithms. Then

$$a_x = \frac{P}{f} \epsilon^{\frac{wx}{f}}$$

$$\log a_x = \log P - \log f + 0{\cdot}434 \frac{wx}{f} \quad . \quad . \quad (12)$$

If the change of section is made in steps, as in fig. 17, let $l_n$ be the length of the $n$th section from the bottom and $a_n$ its cross-section

$$a_n = \frac{P f^{n-1}}{(f - w\,l_1)\,(f - w\,l_2)\,.\,.\,.\,(f - w\,l_n)}$$

## Resistance to Shearing

46. An action which causes sliding parallel to the section considered is termed a shearing action. Thus, the pressure

Fig. 18 Fig. 19

of the cutting edges of an ordinary shearing machine, fig. 18, induces a shearing stress in the plane $a\ b$. The mean intensity of the shearing stress is the shearing force P, divided by the area $a$ of the section $a\ b$ of the bar. In the case shown, a condition difficult to realise perfectly, the forces P P act exactly in the plane of the section, and the shearing stress is uniformly distributed, and at all parts of the section the shearing stress is—

$$f_s = P/a \quad . \quad . \quad . \quad . \quad (13)$$

But if the forces P P do not act exactly in the plane of the section, the bar bends as well as shears (fig. 19). The effect of this is to alter the distribution of the shearing action at all points between $a$ and $b$. Near the middle of the section the shearing stress is greater than the mean shearing stress, and at the upper and lower boundary of the section it becomes zero. A rivet connect-

ing two plates (fig. 20) is in shear, and a bolt is very often so. A cotter or key is similarly intended to resist shear. In these cases, the shearing forces P P do not act in the plane of the section *b c*, but along the centres of the plates connected. In consequence, however, of the rigidity and friction of the edges *a b* and *c d* of the plates, the points of application of the forces P P on the surface of the rivet may very nearly approach the plane *b c*, and then the shearing stress is approximately uniformly distributed on *b c*. If, however, the rivet fits very loosely in the rivet holes, fig. 21, the rivet bends and the distribution of stress becomes more or less unequal. For rivets, it is usual to assume that they fit their holes tightly, and that the shearing stress is simply $f_s = P/a$. But for bolts and cotters, it is safer to assume

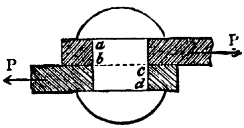


Fig. 20

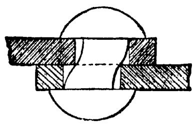


Fig. 21

that the stress is unequally distributed, and the maximum stress may then reach the values

$f_s = 1\cdot5 \frac{P}{a}$ if the section is rectangular and P perpendicular to one side;

$= 1\cdot33 \frac{P}{a}$ if the section is circular or elliptical;

$= 1\cdot59 \frac{P}{a}$ at halfway between the angle and centre, if the section is square and P acts parallel to a diagonal;

$= 2 \frac{P}{a}$ if the section is a ring of small thickness compared with the diameter.

The limits of working stress in shear are given in Table II, § 39.

In general, if there is shear on any set of parallel planes in a solid there is an equal shear on a set of planes at right angles to the former set. When the whole condition of stress can be reduced to a pair of equal shears at right angles, the body is said to be subjected to simple shearing. Let a cube of material (fig. 22) be subjected to equal shearing stresses, $s$ and $-s$, per

unit area, on two pairs of opposite faces. If $a$ is the area of a face, $S = sa$ is the total stress on a face, and the resultant stresses form pairs of equal couples, a system of forces in equilibrium. The cube is distorted into an oblique prism without change of volume. The distortion may be measured by the relative displacement $a\ a'$ of two opposite faces. The displacement per unit distance of the faces, $\sigma = a\ a'/a\ o$, is termed the sliding. The ratio $s/\sigma = G$ of the shearing stress to the sliding is termed the coefficient of transverse elasticity or *coefficient of rigidity* (Table I, § 38). Each right angle $a\ o\ c$ is altered by an amount $a\ o\ a' = \theta$ in radians. Since the deformation is small $\theta = \tan\theta = a\ a'/a\ o = \sigma$. Hence within the limit of elasticity—

$$s = G\,\sigma = G\,\theta \quad . \quad . \quad . \quad . \quad (14)$$

Clearly the material is lengthened in the direction $o\ b'$, and shortened in the direction $a'\ c$. It can be shown that a condition

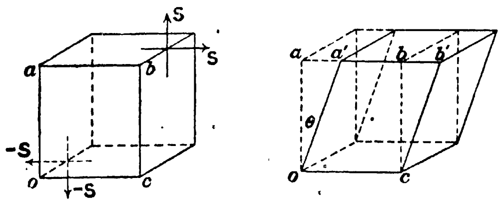



Fig. 22

of simple shear is equivalent to a thrust $-f_c$ and tension $f_t$ at right angles and at 45° with the shearing stresses. Also, $f_c = f_t = s$.

The relation between the coefficients of direct and transverse elasticity depends on Poisson's ratio $1/m$ (§ 40).

$$G = \frac{m\,E}{2\,(m+1)} \text{ and } E = \frac{2\,(m+1)}{m}\,G$$

For metals the average value is $m = 10/3$—

$$G = \frac{5}{13}\,E.$$

## Resistance to Bending

47. *Bending Moment and Shearing Force.*—A bar is subjected to simple bending when the following conditions are fulfilled:—(1) The axis of the bar is straight; the axis of the bar being a

line connecting the centres of figure of parallel transverse sections ; (2) The bar is symmetrical about a plane passing through the axis ; (3) All the external forces act in such a plane of symmetry, called the plane of bending, normally to the axis. If these conditions are not fulfilled, the action of the straining forces is more complex, and some cases in which this happens will be considered under the head of Compound Stress.

Consider the case represented in fig. 23, where, in the lower figure, the flexure is exaggerated for the sake of clearness. In this case, a bar originally straight, and having transverse sections symmetrical about the plane of the paper, in which the bending forces act, is subjected to flexure, under the action of two equal couples of forces applied to its ends. Then since

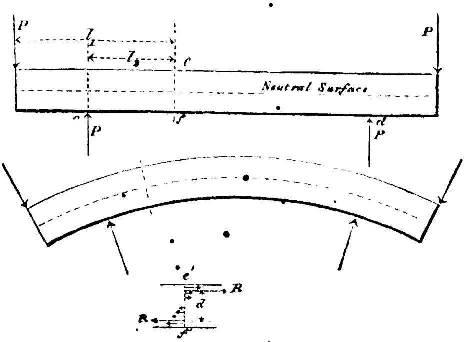



Fig. 23

the straining action is the same at every cross-section from $c$ to $d$, the curvature is circular and the effect of the bending is to lengthen the upper parts of the bar, and to shorten the lower parts. If the flexure is very small, so that the straining forces are sensibly parallel, a plane normal to the paper, through the axis of the bar, will divide the parts in tension from those in compression. The length of the bar measured along that surface will be unaltered by the flexure, and hence it is termed the neutral surface.

The amount of the bending action, at any section $e\ f$ of the bar, is measured by the moment of the resultant force or algebraic sum of the moments of all the forces on either side of that section, which is termed the bending moment. Taking the

forces to the left of $e\,f$, the bending moment is $P\,l_1 - P\,l_2$. The molecular stresses in the bar, developed by the external actions, form at any section a couple, whose moment is equal and opposite to the bending moment, and which is termed the moment of resistance of the section. The action of the molecular stresses is represented at $e'f'$. The tensions above and the compressions below the axis have resultants R, R, whose moment is $R\,d$. Equating this to the bending moment

$$P\,(l_1 - l_2) = R\,d \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In other cases the action is a little more complex. Suppose the force P acts at the end of a bar (fig. 24) solidly fixed at the other end, and let it be required to find the straining action at $e\,f$. Equilibrium is not disturbed, if we introduce two equal and opposite forces $P'\,P''$, in the direction $e\,f$. Then the action of P on the section $e\,f$ is equivalent to that of a couple, $P, P''$, and an unbalanced force $P'$. The couple has a moment $P\,l$, which produces simple bending, and is in equilibrium with a couple formed by molecular stresses at $e\,f$, normal to the section, precisely similar to those described in the previous case. The remaining force $P'$ produces a shearing stress on the section $e\,f$. The two actions are independent, and the bar must be strong enough to resist both the bending moment and the shearing action. In a large number of cases, the amount of material necessary to resist the bending moment is much more than sufficient to resist the shearing action, so that the latter may be left out of consideration.

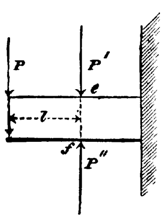


Fig. 24

If several forces act to the left of $e\,f$, we may take their resultant, and then proceed as if only a single force required to be dealt with.

48. *Bending Moment and Shearing Force Curves.*—Consider a beam A B (fig. 25), supported at the ends and loaded by a distributed load. Let $w$ be the intensity of loading at C distant $x$ from the left abutment, so that $w\,d\,x$ is the load on a small length $d\,x$ at C. Let R be the supporting reaction at A. If at each point along A B an ordinate is set up equal on any scale to the value of $w$ at that point, a curve D E is obtained which may be called the load line. The area A D E B represents the total load

on the beam and the resultant of the load acts through the mass centre of that area. The shearing force S at C is equal to the resultant of the forces to the left of C, that is—

$$S = R - \int_0^x w \, dx$$

Ordinates set up at each point along the beam equal to S at that point give a curve F G, which is termed the shearing force curve. Similarly the bending moment at C, which is the sum of the moments of the forces to the left of C, is—

$$M = \int_0^x S \, dx$$

And if ordinates are set up at each point along A B equal to M at that point a curve A H B is obtained called the bending moment curve. It is easy to see that the shearing force at C is equal

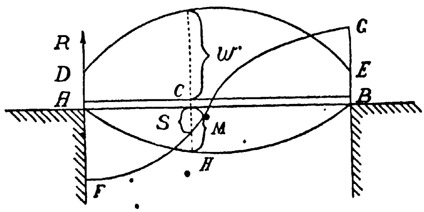



Fig. 25

to R—the area of the load curve over A C, and that the bending moment at C is equal to the area of the shearing force curve over A C. The bending moment is a maximum at the section at which the shearing force changes sign. The most common case is when $w$ is constant, then the shearing force curve is a straight line and the bending moment curve a parabola.

49. *Relation between the Bending Moment and the Bending Stress.*—It will alter nothing in the conditions of the stresses of the bar in fig. 23, if we suppose it to form part of a longer bar bent to a complete circle of the same curvature, by the action of the external forces. It can then be seen that fibres originally straight in the unstrained bar become coaxial circles in the strained bar, and plane transverse sections become plane radial sections, across which there is no shearing stress, but a bending moment only, the resultant of the tensions and pressures of the fibres.

Let $\rho$ (fig. 26) be the radius of the layer of fibres in the *neutral surface, which are neither extended nor compressed by the* bending of the bar. Then the length of all the fibres before bending was $2 \pi \rho$. After bending, a fibre at radius $\rho + y$ has the length $2 \pi (\rho+y)$, and its extension (or compression if $y$ is negative) is $2 \pi y$. Assuming the formulæ in §40, the stress $f$ due to this extension is given by the equation—

$$2 \pi y = \frac{f}{E} 2 \pi \rho.$$

Hence
$$f = \frac{E y}{\rho} . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The total stress on an element of area $a$, at radius $\rho + y$, in any cross-section normal to the axis, is therefore $f a = \frac{E y a}{\rho}$, and

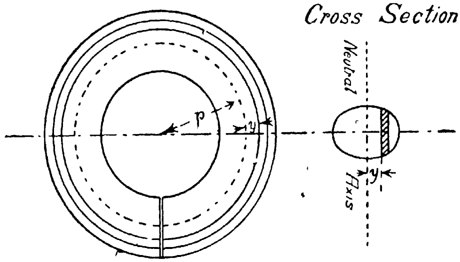



Fig. 26

the total stress on the whole section is $\Sigma \left(\frac{E y a}{\rho}\right)$. But since the pressures and tensions across the section form a couple,

$$\Sigma \left(\frac{E y a}{\rho}\right) = 0;$$

or since $E/\rho$ is a constant,

$$\Sigma y a \quad = 0.$$

This relation can only be satisfied if the distances $y$ are measured from a line passing through the centre of figure of the cross-section. This line is termed the *neutral axis* of the cross-section. But $f = 0$ when $y = 0$. Hence the neutral surface, which divides the compressed from the extended fibres, is a surface containing the neutral axes of all the cross-sections.

The moment of the stress $f\,a$ about the neutral axis of the section is $f\,a\,y = (E\,y^2\,a)/\rho$. The total moment of the couple formed by the molecular forces at the section, which is termed the moment of resistance of the section, is—

$$\Sigma\left(\frac{E\,y^2\,a}{\rho}\right) = \frac{E}{\rho}\,\Sigma\,a\,y^2 = \frac{E\,J}{\rho}$$

where $J = \Sigma\,a\,y^2$ is the quantity known as the *moment of inertia* of the section.

Now equating the bending moment $M$ of the external forces to the moment of resistance of the section

$$M = \frac{E\,J}{\rho} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

which expresses the relation between the bending moment and the curvature of the bar. Let $f$ be the stress at a distance $y$ from the neutral axis. Then since

$$f = \frac{E\,y}{\rho}$$

$$\left.\begin{aligned} M &= f\frac{J}{y} \\ f &= M\frac{y}{J} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (17)$$

which gives the relation between the stress at any point of a section, and the bending moment. Generally what is required is the greatest tension or pressure at any point of the section, because this must not exceed the safe limit of stress. The stress is greater the greater the value of $y$. Let $y_t$ be the distance from the neutral axis of the most distant fibre in tension, and $y_c$ that of the most distant fibre in compression. Then the greatest tension and pressure are

$$f_t = M\frac{y_t}{J} \text{ and } f_c = M\frac{y_c}{J}$$

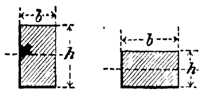
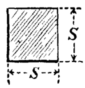
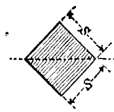
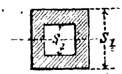
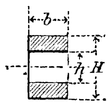
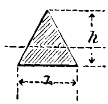
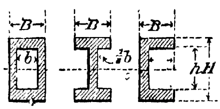
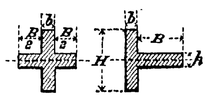
Let $z_t = y_t/J$ and $z_c = y_c/J$, then $z_t$ and $z_c$ are termed the *moduli* of the section with respect to tension and compression. Then—

$$M = f_t\,z_t = f_c\,z_c \quad . \quad . \quad . \quad (18)$$

For a bar to be safe $f_t = M/z_t$ must not exceed the safe working stress in tension nor $f_c = M/z_c$ the safe working stress in compression (Table II, § 39). If the section is symmetrical about the neutral axis, $y_t = y_c$ and $z_t = z_c$. Then it is only necessary

TABLE IV.—*Area, Moment of Inertia,*

The plane of bending is supposed parallel to the side of the page. The axis
tion. J is the moment of inertia about an axis through the centre of figure of the
for the modulus, Z′ applies to the fibres at the top edge and Z″ to the fibres at the

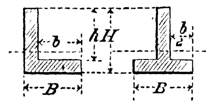
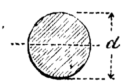
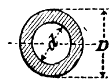
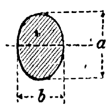
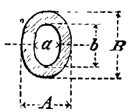
| Form of Section | Diagram of Section | Area of Section A |
|---|---|---|
| Rectangle |  | $bh$ |
| Square |  | $s^2$ |
| Square (axis diagonal) |  | $s^2$ |
| Hollow square |  | $s_1^2 - s_2^2$ |
| Pierced rectangle |  | $b\,(H - h)$ |
| Triangle |  | $\frac{1}{2}\,bh$ |
| Hollow rectangle, tee, or channel |  | $BH - bh$ |
| Cross |  | $bH + Bh$ |

*and Modulus of different Sections*

of moments is the horizontal dotted line through the centre of figure of the sec- section and at right angles to the plane of bending. Where two values are given bottom edge of the section.

| Moment of Inertia of Section $J$ | Square of radius of gyration of Section $\rho^2 = \frac{J}{A}$ | Modulus of Section $Z = J/y$ |
|---|---|---|
| $\frac{1}{12} bh^3$ | $\frac{1}{12} h^2$ | $\frac{1}{6} bh^2$ |
| $\frac{1}{12} s^4$ | $\frac{1}{12} s^2$ | $\frac{1}{6} s^3$ |
| $\frac{1}{12} s^4$ | $\frac{1}{12} s^2$ | $0{\cdot}118 s^3$ |
| $\frac{1}{12} (s_1^4 - s_2^4)$ | $\frac{1}{12} (s_1^2 + s_2^2)$ | $\frac{1}{6} \frac{s_1^4 - s_2^4}{s_1}$ |
| $\frac{b}{12} (H^3 - h^3)$ | $\frac{1}{12} (H^2 + h H + h^2)$ | $\frac{1}{6} \frac{b}{H} (H^3 - h^3)$ |
| $\frac{1}{36} bh^3$ | $\frac{1}{18} h^2$ | $\begin{cases} Z' = \frac{1}{24} bh^2 \\ Z'' = \frac{1}{12} bh^2 \end{cases}$ |
| $\frac{1}{12} (BH^3 - bh^3)$ | $\frac{1}{12} \frac{BH^3 - bh^3}{BH - bh}$ | $\frac{1}{6H} (BH^3 - bh^3)$ |
| $\frac{1}{12} (bH^3 + Bh^3)$ | | $\frac{1}{6H} (bH^3 + Bh^3)$ |

TABLE IV.—*Area, Moment of Inertia, and*

| Form of Section | Diagram of Section | Area of Section A |
|---|---|---|
| Angle iron |  | $BH - bh$ |
| Circle |  | $\frac{\pi}{4} d^2 = \cdot 785 d^2$ |
| Hollow circle |  | $\frac{\pi}{4}(D^2 - d^2)$ |
| Ellipse |  | $\frac{\pi}{4} ba$ |
| Hollow ellipse |  | $\frac{\pi}{4}(BA - ba)$ |

to consider the stress, tension or pressure, which the material is weakest to resist. If the bar is not symmetrical about the neutral axis, it is necessary to consider whether either $f_t$ or $f_c$ calculated by the equation above exceed the safe limit. If the straining action acts successively in opposite directions, as for instance on an engine beam, $y_t$ and $y_c$ when the load acts in one direction, become $y_c$ and $y_t$ when it acts in the other. There are then two values of $f_t$ and $f_c$, which need to be considered, if the section is not symmetrical about the neutral axis.

*Modulus of Different Sections*—continued.

| Moment of Inertia of Section $J$ | Square of radius of gyration of Section $\rho^2 = \frac{J}{A}$ | Modulus of Section $Z = J/y$ |
|---|---|---|
| $\frac{(BH^2 - bh^2)^2 - 4BHbh(H-h)^2}{12(BH - bh)}$ | | |
| $\frac{\pi}{64} d^4 = \cdot 0491\, d^4$ | $\frac{1}{16} d^2$ | $\frac{\pi}{32} d^3 = \cdot 0982 d^3$ |
| $\frac{\pi}{64}(D^4 - d^4)$ | $\frac{1}{16}(D^2 + d^2)$ If $D - d$ is small $= \frac{1}{8} D^2$ nearly | $\frac{\pi}{32} \frac{D^4 - d^4}{D}$ |
| $\frac{\pi}{64} ba^3$ | $= \frac{1}{16} a^2$ | $\frac{\pi}{32} ba^2$ |
| $\frac{\pi}{64}(AB^3 - ab^3)$ | $\frac{1}{16} \frac{AB^3 - ab^3}{AB - ab}$ | $\frac{\pi}{32} \frac{AB^3 - ab^3}{B}$ |

50. *Moments of Inertia.*—The moment of inertia of a body with reference to a straight line is the sum of the products of the elementary masses of which it consists by the squares of their distances from the line. By an extension of meaning the moment of inertia of a plane surface is the sum of the products of the elements of area by the squares of their distances from the axis considered. If $a$ is an element of the area at $y$ from the axis, the moment of inertia of the surface is

$$J = \Sigma a y^2$$

If A is the total area of the surface and J the moment of inertia about an axis through its centre of gravity, then

$$J/A = \rho^2$$

where $\rho$ is termed the radius of gyration of the section.

Table IV contains values of moments of inertia about an axis through the centre of figure and radii of gyration for the simpler sections.

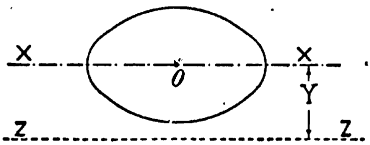
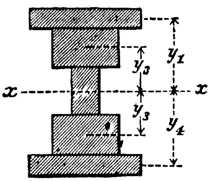



Fig. 27

*Moments of Inertia of a Surface about parallel axes.*—Let J be the moment of inertia of the surface shown in fig. 27, about an axis X X passing through O, its centre of figure or mass centre. Let $J_1$ be the moment of inertia about an axis Z Z parallel to X X at a distance Y.




| Areas. | Moments of inertia. |
|---|---|
| $a_1$ | $J_1$ |
| $a_2$ | $J_2$ |
| $a_3$ | $J_3$ |
| $a_4$ | $J_4$ |

Fig. 28

Let $a$ be an element of the area at $y$ from X X and A the total area of the surface. Then

$$\begin{aligned} J &= \Sigma a y^2 \\ J_1 &= \Sigma a (y + Y)^2 \\ &= \Sigma a y^2 + 2 Y \Sigma a y + Y^2 \Sigma a \end{aligned}$$

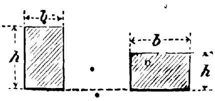
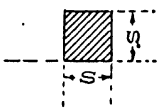
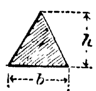
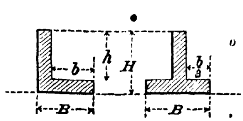
But $\Sigma a y = 0$, since $y$ is measured from X X which passes through the centre of gravity and $\Sigma a = A$

$$J_1 = J + A Y^2 \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Table IVA contains moments of inertia of some simple sections about an axis at the base or bottom edge of the section. This

result is convenient in finding the moment of inertia of complex sections which can be cut up into a series of rectangles. For instance, the **I**-section, fig. 28, in which $x\,x$ is an axis through the centre of figure of the whole section. Neglecting at present the small piece of the web in the middle, the areas $a_1\ a_2\ a_3\ a_4$ of the rectangles can be written down and the moments of inertia of these rectangles about axes through their centres of figure. Then the moment of inertia of the whole section about $x\,x$ is $J_1 + J_2 + J_3 + J_4 + a_1\,y_1^2 + a_2\,y_2^2 + a_3\,y_3^2 + a_4\,y_4^2$. The neglected part of the web can be treated in the same way or as two rectangles for which the moments of inertia can be written down at once from the value in Table IVA.

TABLE IVA.—*Area and Moment of Inertia of Sections about an Axis at the Base*

| | Section | Area of Section | Moment of Inertia about axis at the base. |
|---|---|---|---|
| Rectangle (axis at base) |  | $bh$ | $\frac{1}{3}\,bh^3$ |
| Square (axis at base) |  | $s^2$ | $\frac{1}{3}\,s^4$ |
| Triangle (axis at base) |  | $\frac{1}{2}\,bh$ | $\frac{1}{12}\,bh^3$ |
| Angle Iron (axis at base) |  | $BH - bh$ | $\frac{1}{3}\{(B-b)H^3 + b(H-h)^3\}$ |

For any plane figure there are two axes at right angles, passing through the centre of gravity of the surface about which the moments of inertia of the surface are one a maximum and the other a minimum, and these are called the principal axes of inertia.

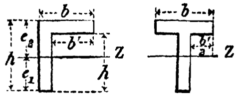


Fig. 29

It is convenient for some purposes to state the moment of inertia for an unsymmetrical angle-iron and T-iron section more fully.

Let z z be the axis through the centre of figure of the section (fig. 29), then

$$e_1 = \frac{b\,h^2 - b'\,h'^2}{2\,(b\,h - b'\,h')}\,;$$

$$e_2 = h - e_1$$

$$J = \frac{(b\,h^2 - b'\,h'^2)^2 - 4\,b\,h\,b'\,h'\,(h - h')^2}{12\,(b\,h - b'\,h')}\,;$$

and dividing this by the values of $e_1$ and $e_2$ we get the two moduli of the section.

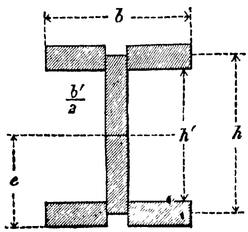


Fig. 30

The symmetrical I-section is most conveniently dealt with approximately. Suppose it reduced to the form shown in fig. 30, the small omitted parts being neglected and the heights measured to the centres of the flanges. Then—

$$z = \frac{J}{e} = b'\,h\,(h - h') + \tfrac{1}{6}h^2\,(b - b').$$

51. *Rolled joists of wrought iron.*—The strength of symmetrical I-shaped wrought-iron joists has been determined by experimental loading. If $w$ is the safe load in tons per foot of span, $l$ the span in feet, z the modulus of the section and $f$ the working stress in tons per sq. in., then $w\,l^2 = 8\,f\,z$. For any given working stress and section, $8\,f\,z$ is a constant. The following table gives some ordinary sections, their weight and the value of $8\,f\,z$ corresponding to about one-quarter of the breaking weight. If the tabular value of $8\,f\,z$ is divided by $l^2$, the safe load per foot run is found (factor of safety about 4).

| Depth and width of flanges in inches | Weight in lbs. per foot run | Suitable spans in feet | $8f'$ |
|---|---|---|---|
| $3 \times 3$ | 10 | 6 to 10 | 11·2 |
| $4 \times 1\frac{3}{4}$ | 7 | 6 to 14 | 10·9 |
| $4 \times 3$ | 12 | 6 to 14 | 18·9 |
| $6\frac{1}{4} \times 2$ | 11 | 6 to 18 | 24·4 |
| $6\frac{1}{4} \times 3\frac{1}{8}$ | 16 | 8 to 20 | 38·3 |
| $7 \times 3\frac{3}{4}$ | 20 | 8 to 24 | 55·7 |
| $8 \times 4$ | 22 | 10 to 24 | 77·4 |
| $8 \times 5$ | 29 | 8 to 28 | 92·1 |
| $9\frac{1}{2} \times 4\frac{1}{2}$ | 29 | 10 to 24 | 115·5 |
| $10 \times 5$ | 36 | 10 to 26 | 147·9 |
| $12 \times 5$ | 42 | 10 to 26 | 217·7 |
| $12 \times 6$ | 56 | 10 to 34 | 267·2 |
| $14 \times 6$ | 60 | 12 to 34 | 305·2 |
| $16 \times 6$ | 62 | 16 to 34 | 370·0 |
| $20 \times 8$ | 100 | 10 to 40 | 500·0 |

It should be noted that moments of inertia and section moduli for all standard rolled sections are given in Reports 4 and 6 published by the Engineering Standards Committee.

52. *Bending Moment and Shearing Force with various arrangements of Loading and Support.*—In many cases bars subjected to bending are of uniform section throughout. Then the stress will be greatest at the section at which the bending moment M is greatest, which may be called the dangerous section. When the cross-section of the bar varies, putting $f$ for the safe working stress and M and Z for the bending moment and section modulus of any section,

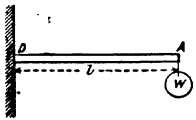
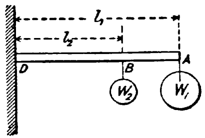
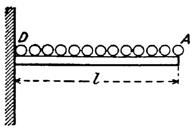
$$M/Z \mathrel{\overline{\overline{<}}} f$$

for all cross-sections. The dangerous section is that at which M/Z is greatest.

Table V gives for various loads and modes of support of beams or bars of uniform section, loaded transversely, the greatest bending moment, the greatest shearing force, the position of the dangerous section, the stress in terms of the load and the deflection.

The loads and shearing forces are in pounds (or tons); dimensions in inches; stresses in pounds (or tons) per sq. in.; E in pounds (or tons) per sq. in.; distributed loads are in pounds (or tons) per inch of span.

TABLE V.—*Bending Moment, Shearing Force, and Deflection corresponding to different Distributions of Load and different Modes of Support. Inch Units.*

| | Diagram of loading | Section at which shearing force is greatest | Greatest shearing force | Section at which the bending moment is greatest | Greatest bending moment | Normal stress due to load $f_b$ | Deflection $\delta$ | Remarks |
|---|---|---|---|---|---|---|---|---|
| | | BEAMS FIXED AT ONE END OR CANTILEVERS | | | | | | |
| I |  | Any section | W | At D | $W\,l$ | $\frac{W\,l}{Z}$ | At A $\frac{W\,l^3}{3\,E\,J}$ | Load at free end |
| II |  | From B to D<br>From B to A | $W_1 + W_2$<br>$W_1$ | At D | $W_1 l_1 + W_2 l_2$ | $\frac{W_1 l_1 + W_2 l_2}{Z}$ | | More than one load |
| III |  | At D | $w\,l = Q$ | At D | $\frac{w\,l^2}{2} = \frac{Q\,l}{2}$ | $\frac{w\,l^2}{2\,Z}$ | At A $\frac{Q\,l^3}{8\,E\,J}$ | Uniform load = $w$ per in. run |

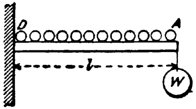
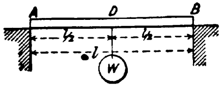
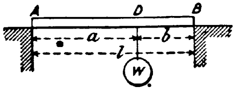
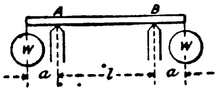
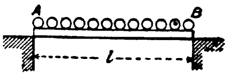
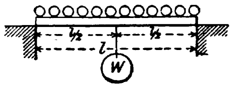
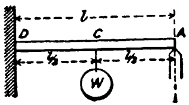
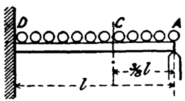
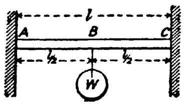
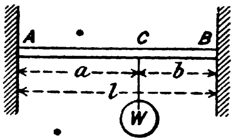
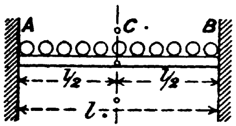
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| IV |  | At D | $W + wl$ $= W + Q$ | At D | $\frac{wl^2}{2} + Wl = (\frac{1}{2}Q + W)l$ | $\frac{wl^2 + 2Wl}{2Z}$ | At A $\frac{l^3}{EJ}\left(\frac{W}{3} + \frac{Q}{8}\right)$ | Uniform load = $w$ per in. and W at free end |
| | | | | | | | | |
| **Beams supported at Both Ends.** | | | | | | | | |
| V |  | Between A and D Between D and B | $\frac{W}{2}$ $-\frac{W}{2}$ | At D | $W\frac{l}{4}$ | $\frac{Wl}{4Z}$ | At D $\frac{Wl^3}{48EJ}$ | W at centre |
| VI |  | Between A and D Between D and B | $\frac{Wb}{l}$ $-\frac{Wa}{l}$ | At D | $W\frac{ab}{l}$ | $\frac{Wab}{lZ}$ | $\frac{Wa^2b^2}{3lEJ}$ | W not at centre |
| VII |  | At A or B Between A and B | W 0 | Between A and B | $Wa$ | $\frac{Wa}{Z}$ | At centre $-\frac{Wal^2}{8EJ}$ At ends $\frac{W}{EJ}\left(\frac{a^3}{3} + \frac{a^2l}{2}\right)$ | Equal couples at free ends. |
| VIII |  | At A or B | $\frac{wl}{2} = \frac{Q}{2}$ | At centre | $\frac{wl^2}{8} = \frac{Ql}{8}$ | $\frac{wl^2}{8Z}$ | $\frac{5l^3Q}{384EJ}$ | Uniform load $w$ per in span $wl = Q$ |

TABLE V.—*Bending Moment, Shearing Force, and Deflection corresponding to different Distributions of Load and different Modes of Support*—continued.

| | Diagram of loading | Section at which shearing force is greatest | Greatest shearing force | Section at which the bending moment is greatest | Greatest bending moment | Normal stress due to load $f_b$ | Deflection $\delta$ | Remarks |
|---|---|---|---|---|---|---|---|---|
| IX |  | At A or B | $\frac{W}{2} + \frac{wl}{2} = \frac{1}{2}(W + Q)$ | At centre | $(W + \frac{1}{2}Q)\frac{l}{4}$ | $\frac{(W + \frac{1}{2}Q)l}{4Z}$ | $(W + \frac{5}{8}Q)\frac{l^3}{48EJ}$ | W at centre and $w$ per in. span |
| | BEAMS FIXED AT ONE END AND SUPPORTED AT THE OTHER | | | | | | | |
| X |  | From D to C<br>From C to A | $\frac{11}{16}W$<br>$\frac{5}{16}W$ | At C<br>At D | $\frac{5}{32}Wl$<br>$-\frac{3}{16}Wl$ | $\frac{3Wl}{16Z}$ | At 0·45 $l$ from A $\frac{Wl^3}{107EJ}$ | W at centre. Reaction at A $= \frac{5}{16}W$ |
| XI |  | At A<br>At D | $\frac{3}{8}wl$<br>$\frac{5}{8}wl$ | At C<br>At D | $\frac{9}{128}wl^2$<br>$\frac{wl^2}{8}$ | $\frac{wl^2}{8Z}$ | At C $\frac{w}{192} \cdot \frac{l^4}{EJ}$ | $w$ per in. of span. Reaction at A $= \frac{3}{8}wl$ |

BEAMS FIXED AT BOTH ENDS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| XII |  | | | At A, B, or C | $\frac{W\,l}{8}$ | $\frac{W\,l}{8\,Z}$ | $\frac{1}{192} \cdot \frac{W\,l^3}{E\,J}$ | W at centre |
| XIII |  | | | At A<br>At C<br>At B | $\frac{W\,a\,b^2}{l^2}$<br>$-\frac{2Wa^2b^2}{l^3}$<br>$W\frac{b\,a^2}{l^2}$ | | | W not at centre |
| XIV |  | | | At A or B<br>At C | $\frac{w\,l^2}{12}$<br>$-\frac{w\,l^2}{24}$ | $\frac{w\,l^2}{12\,Z}$ | $\frac{1}{384} \cdot \frac{w\,l^4}{E\,J}$ | $w$ per in of span |

*Encastrement.*—A beam is fixed or encastre when it is so fixed that the direction of the axis at the support is not altered by the bending forces. A beam, fig. 31, loaded with P at $l$ from the support A is encastre if a couple

$$Rx = Pl$$

is applied beyond the point of support A.

53. *Continuous Beams.*—When a beam rests on more than two supports, the ordinary statical conditions of equilibrium do not suffice to determine the reactions of the supports. Recourse may then be had to a relation due to Clapeyron, which is termed the Theorem of three moments. Let fig. 32 represent two consecutive spans of a beam resting on several supports, at the same level; let $l_1$, $l_2$, be the lengths of the spans; $w_1$, $w_2$, the loads per unit of span; $M_1$, $M$, $M_2$, the bending moments over the supports. Then

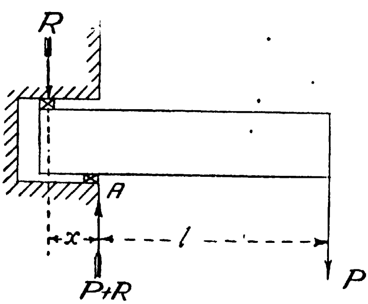



Fig. 31

$$8(l_1 + l_2)M + 4l_1 M_1 + 4l_2 M_2 = w_1 l_1^3 + w_2 l_2^3 \quad . \quad (20)$$

The equation assumes that the beam is of uniform section But it may be applied in cases where the beam is approximately a beam of uniform strength. This theorem furnishes, for a beam of $n$ spans, $n - 1$ equations. In addition to these, the condition that a beam simply supported at the ends has no bending moment at the ends, furnishes two additional equations, $M_0 = 0$, $M_n = 0$. There are then $n + 1$ equations, to determine the $n + 1$ bending moments at the points of support. By then reversing the ordinary process, the reactions can be found from the bending moments and loads. The following are some of

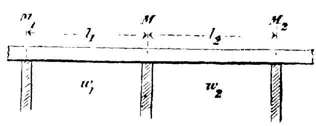



Fig. 32

the simplest results of applying this theorem to beams uniformly loaded with $w$ lbs. per inch run.

| | Reactions at supports |
|---|---|
| Beam of 2 equal spans | $\frac{3}{8} w l$; $\frac{5}{8} w l$; $\frac{3}{8} w l$. |
| „ 3 „ | $\frac{4}{10} w l$; $\frac{11}{10} w l$; $\frac{11}{10} w l$; $\frac{4}{10} w l$. |

54. *Relative Economy of different forms of Section.*—The weight of a bar is proportional to its sectional area, its resistance to bending to its section modulus. Of two bars of different forms, subjected to the same loading, that will be the more economical of material which, with a given value of the modulus of resistance Z, has the lesser sectional area A. Hence the more economical the form of the bar, the greater will be the ratio Z/A.

In a prismatic bar, of circular or rectangular section, only the material at the extreme top and bottom of the section is fully strained. Nearer the neutral surface the material is less strained, and at the neutral surface it is not strained at all by the direct stresses due to bending. Such a bar would be made stronger by removing some of the material from the neighbourhood of the neutral surface towards the top and bottom of the section. We thus arrive at the excellent form of section known as the I or double T section. The material is chiefly collected in the top and bottom flanges, which bear nearly the whole of the direct stresses due to bending; the remainder forms a vertical web, whose chief function is to resist the shearing action.

55. *Flanged Sections when both Flanges are strained to the Working Limit.*—In order that the stress at the stretched edge of the bar may be at the working limit of tension, and the stress at the compressed edge may be at the working limit of pressure, we must have

$$M = f_t \frac{y_t}{J} = f_c \frac{y_c}{J} = f_t Z_t = f_c Z_c \quad . \quad . \quad (21)$$

where $y_t$, $y_c$, are the distances from the neutral axis of the most strained fibres in tension and compression; $Z_t$, $Z_c$, the corresponding section moduli; $f_t$ and $f_c$, the safe working stresses in tension and compression. If $f_t = f_c$, then $Z_t = Z_c$, which will be the case if the section is symmetrical about the neutral axis. If $f_t$ and $f_c$ are not equal both edges of the beam cannot be fully strained when $Z_c = Z_t$, and the material is not then distributed in the most economical way.

*Unsymmetrical Sections.*—In some materials $f_t$ and $f_c$ are very unequal and then a section unsymmetrical about the neutral axis is economical.

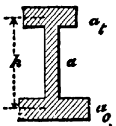


Fig. 33

Let $a_t$, fig. 33, be the area of the tension flange, $a_c$ the area of the compression flange, and $a$ the area of the web of a beam of I-shaped section. Let $h$ be the depth, measured from centre to centre of the flanges. Then if the area $a$ of the web is small compared with that of the flanges the required condition is nearly fulfilled, when

$$f_t a_t = f_c a_c$$

$$a_t = \frac{M}{f_t h} \text{ and } a_c = \frac{M}{f_c h} \quad . \quad . \quad . \quad (22)$$

Further, if F is the total shearing action, and $f_s$ the safe shearing stress, the strength of the web is approximately sufficient when

$$a = \frac{F}{f_s} \quad . \quad . \quad . \quad . \quad (22a)$$

For practical reasons, especially in cast beams, $a$ has often to be made of larger area than is given by this equation.

In the foregoing equations the resistance of the web to bending is neglected, and in cast-iron beams especially this introduces considerable error. If the web is taken into account we proceed as follows :

*To design a beam of unsymmetrical* I *section for a given bending moment.*—Cast-iron beams are very commonly of I-shaped section with unequal flanges, the ultimate resistance of cast iron to pressure being five or six times its resistance to tension. It is not usual, however, to proportion the beam with so great an inequality of flange section as it would have if the working stresses were proportional to the ultimate stresses in tension and compression. With great inequality of flange section the beam is difficult to cast. More commonly the limits of working stress are taken at $f_t = 2$ tons for tension and $f_c = 4$ tons or 5 tons per sq. in. for pressure. It is

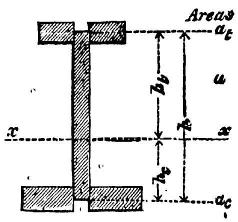


Fig. 34

also desirable to facilitate casting that the web should be of not greatly less thickness than the flanges.

Let fig. 34 represent a diagram of the section required. The depth $h$ of the beam between the centres of the flanges is assumed. It is often 1–10th to 1–15th of the span. Assume also from practical considerations a thickness for the web. Then the area $a$ of the web is known and the areas $a_c$ and $a_t$ of the pressure and tension flanges remain to be determined. The small omitted parts of the section may be treated as negligible for simplicity.

If $x\,x$ is a line through the centre of figure of the section, then the flanges will have the required stresses if

$$h_c : h_t : h :: f_c : f_t : f_c + f_t$$

$$\left.\begin{aligned} h_c &= \frac{f_c}{f_c + f_t} h \\ h_t &= \frac{f_t}{f_c + f_t} h \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (23)$$

Taking moments about $x\,x$

$$a_c\,h_c + a\,\frac{h_c^2}{2h} = a_t\,h_t + a\,.\,\frac{h_t^2}{2h}$$

$$a_c = a_t\,\frac{h_t}{h_c} + a\left(\frac{h_t^2 - h_c^2}{2\,h\,h_c}\right) = a_t\,\frac{h_t}{h_c} + a\,\frac{h_t - h_c}{2\,h_c}$$

Replacing by the relations above

$$\left.\begin{aligned} a_c &= a_t\frac{f_t}{f_c} + a\,\frac{f_t - f_c}{2\,f_c} \\ a_t &= a_c\frac{f_c}{f_t} - a\,\frac{f_c - f_t}{2\,f_t} \end{aligned}\right\} \quad . \quad . \quad . \quad (24)$$

Now taking the second moments of the areas about $x\,x$, the moment of inertia of the section is

$$\text{J} = a_c\,h_c^2 + a_t\,h_t^2 + \tfrac{1}{3}\,a\,\frac{h_c}{h}\,h_c^2 + \tfrac{1}{3}\,a\,\frac{h_t}{h}\,h_t^2$$

$$= a_c\,h_c^2 + a_t\,h_t^2 + \tfrac{1}{3}\,a\,(h_c^2 - h_c\,h_t + h_t^2)$$

Replacing $a_t$ by its value above

$$= a_c\,h\,h_c + \frac{a}{6}(h_c\,h_t - h_t^2 + 2\,h_c^2)$$

$$= a_c\,h\,h_c + \frac{a\,h}{6}\,(2\,h_c - h_t)$$

Then,
$$z_c = a_c\,h + \frac{a\,h}{6}\,(2 - \frac{h_t}{h_c}).$$

$$\left.\begin{aligned} Z_c &= h\left\{a_c + \left(2 - \frac{f_t}{f_c}\right)\frac{a}{6}\right\} \text{ for pressure} \\ Z_t &= h\left\{a_t + \left(2 - \frac{f_c}{f_t}\right)\frac{a}{6}\right\} \text{ for tension} \end{aligned}\right\} \quad . \quad . \quad (25)$$

From the known bending moment M and the relation

$$M = f_c Z_c = f_t Z_t$$

we can determine one modulus of the section, either $Z_c$ or $Z_t$. Then one of the equations (25) furnishes one of the flange areas, and one of the equations (24) the other flange area. Knowing the areas, the width and thickness of the flange can be decided by practical considerations. In completing the design of the section, the web may be tapered so as to be a little stronger at its junction with the larger flange, and the flanges may be tapered (without altering their area) so as to draw easily from the sand.

By applying the same process to sections of the beam at some distance from the centre, the amount of tapering of the flanges towards the ends can be determined. Some surplus strength is, however, provided at the ends to allow for inequality of loading and tendency to twist.

In consequence of the tendency in wrought-iron beams to a vertical or lateral buckling of the compressed flange, the working stress, in compression, is sometimes taken ⅜ths less than the working stress in tension.

56. *Beams of Uniform Resistance to the direct Stresses due to Bending.*—Except in one special case, the bending moment varies at different points in the length of the beam. At the point where the bending moment is greatest, the section must be designed for that maximum moment. For practical reasons, it is frequently necessary to make the beam, or bar, uniform, and then the section where the bending moment is greatest determines the section of the rest of the bar. In other cases, the section of the bar may be diminished in parts where the bending moment is less, and material is then economised. The best distribution of material, so far as the direct stresses are concerned, is that which fulfils the condition

$$f = M/Z = \text{constant}$$

for every transverse section, M being the bending moment at any section, and Z the modulus of that section. Beams so designed are often termed beams of uniform strength. The theoretical form thus obtained requires, in some cases, to be modified for practical reasons.

TABLE VI.—*Distribution of Bending Moment and Shearing Force*

In each case, in the Table below, the first figure gives the distribution of load, the second the bending moment diagram, and the third the shearing force diagram.

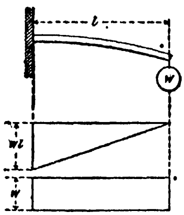
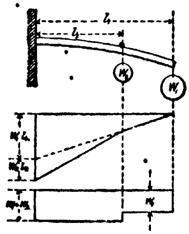
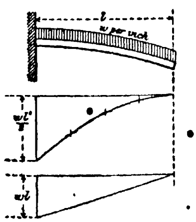
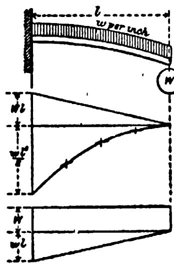
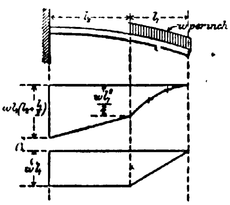
| | Loading | Diagram of load, and of bending moment and shearing force curves | Bending moment curve | Shearing force curve |
|---|---|---|---|---|
| | | BEAMS ENCASTRE AT ONE END | | |
| I. | Load at free end |  | Straight line | Straight line |
| II. | Two loads |  | Broken line | Broken line |
| III. | Uniform load |  | Parabola, vertex at free end, axis vertical | Straight line |

TABLE VI.—*Distribution of Bending Moment and Shearing Force*—*continued*

BEAMS ENCASTRE AT ONE END—*continued*

| | Loading | Diagram of load, and of bending moment and shearing force curves | Bending moment curve | Shearing force curve |
|---|---|---|---|---|
| IV. | Uniform load and load at free end |   | Obtained by combining the curves in I. and III. | |
| V. | Partial uniform load |   | Parabola, with vertex at free end and straight line | Broken line |

BEAMS SUPPORTED AT BOTH ENDS

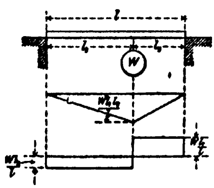
| | | | | |
|---|---|---|---|---|
| VI. | Single load |   | Broken line | Broken line |

TABLE VI.—*Distribution of Bending Moment and Shearing Force*—continued

BEAMS SUPPORTED AT BOTH ENDS—continued

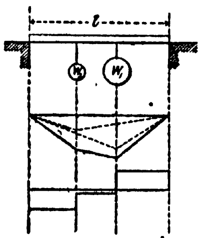
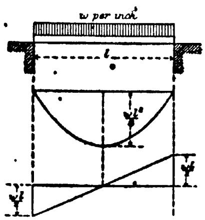
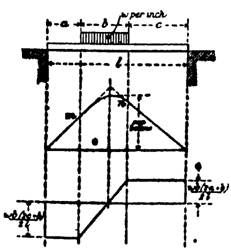
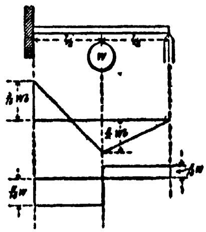
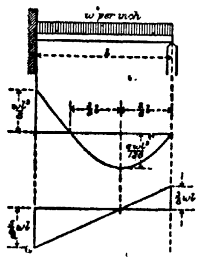
| | Loading | Diagram of load, and of bending moment and shearing force curves | Bending moment curve | Shearing force curve |
|---|---|---|---|---|
| VII. | Two loads |  | Obtained by adding ordinates due to each separate load | |
| VIII. | Uniform load |  | Parabola, vertex at centre, axis vertical | Straight line |
| IX. | Partial uniform load *w* per inch |  | Parabola, and its tangents | Broken line |

TABLE VI.—*Distribution of Bending Moment and Shearing Force*—continued

BEAMS FIXED AT ONE END.

| | Loading | Diagram of load, and of bending moment and shearing force curves | Bending moment curve | Shearing force curve |
|---|---|---|---|---|
| X. | Central load |  | Broken line | Broken line |
| XI. | Uniform load $w$ per inch |  | Parabola | Straight line |

BEAMS FIXED AT BOTH ENDS

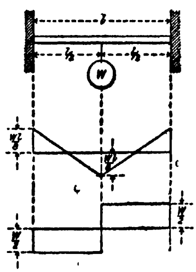
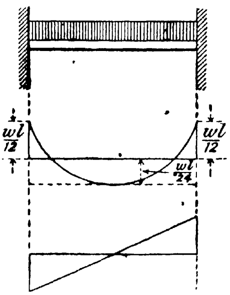
| | | | | |
|---|---|---|---|---|
| XII. | Loaded at centre |  | Broken line | Broken line |

TABLE VI.—*Distribution of Bending Moment and Shearing Force*—continued

BEAMS FIXED AT BOTH ENDS—continued

| | Loading | Diagram of load, and of bending moment and shearing force curves | Bending moment curve | Shearing force curve |
|---|---|---|---|---|
| XIII. | Uniformly loaded |  | Parabola | Straight line |

The cases given in this table should be compared with the corresponding cases in Table V.

In the following table, in some cases, approximate forms are given which nearly comply with the condition of uniform strength, as well as the forms which exactly comply with that condition. The former are simpler to execute in construction, and are often preferable in economy to forms of uniform section and to the exact forms in convenience.

TABLE VII.—*Forms of Beams of Uniform Strength*

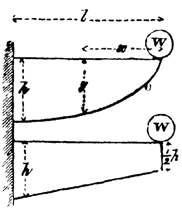
| | | Longitudinal elevation of beam | Form of transverse section | Bounding lines of elevation or plan | Equation for determining the dimensions |
|---|---|---|---|---|---|
| | | BEAMS FIXED AT ONE END | | | |
| I. | Load at free end |  | Rectangle of uniform breadth, $b$, and variable depth, $y$ | Straight line and parabola. Approximate form, a truncated pyramid | $y = \sqrt{\frac{6wx}{bf}}$ <br> $h = \sqrt{\frac{6wl}{bf}}$ |

TABLE VII.—*Forms of Beams of Uniform Strength*—continued

BEAMS FIXED AT ONE END—continued

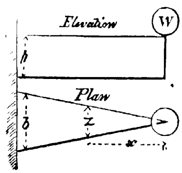
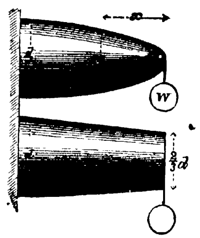
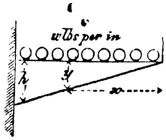
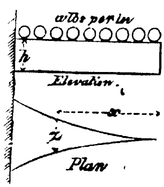
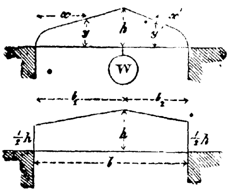
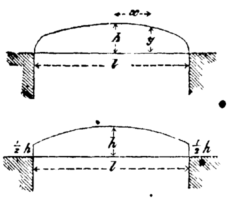
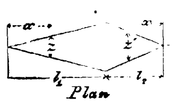
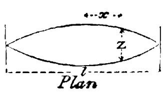
| | | Longitudinal elevation of beam | Form of transverse section | Bounding lines of elevation or plan | Equation for determining the dimensions |
|---|---|---|---|---|---|
| II. | Load at free end |  | Rectangle of uniform depth, $h$, and variable breadth, $z$ | Straight lines forming a wedge | $z = \frac{6\ w}{h^2 f} x$<br>$b = \frac{6\ wl}{f h^2}$ |
| III. | |  | Circle of variable diameter, $y$ | Cubic parabola. Approximate form, a truncated cone | $y^3 = \frac{32\ w}{\pi f} x$<br>$d = \sqrt[3]{\frac{32\ wl}{\pi f}}$ |
| IV. | Uniform load |  | Rectangle of uniform breadth, $b$, and variable depth, $y$ | Straight lines forming a wedge | $y = x\sqrt{\frac{3\ w}{f b}}$<br>$h = l\sqrt{\frac{3\ w}{f b}}$ |
| V. | Uniform load |  | Rectangle of uniform depth and variable breadth, $z$ | Parabolas with vertex at free end | $z = \frac{3\ w}{f h^2} x^2$<br>$b = \frac{3\ w}{f h^2} l^2$ |

TABLE VII.—*Forms of Beams of Uniform Strength*—continued

BEAMS SUPPORTED AT EACH END

| | | Longitudinal elevation of beam | Form of transverse section | Bounding lines of elevation or plan | Equation for determining the dimensions |
|---|---|---|---|---|---|
| VI. | Single load |  | Rectangle of uniform breadth, $b$, and variable depth, $y$ | Two parabolas and straight line. Approximate form, truncated pyramids | $y^2 = \frac{6 w l_2}{b f l} x$ <br> $y'^2 = \frac{6 w l_1}{b f l} x'$ <br> $h = \sqrt{\frac{6 w l_1 l_2}{b f l}}$ |
| VII. | Uniform load |  | Rectangle as above ($w$ lbs per inch uniformly distributed) | Ellipse and straight line. Approximate form, circular arc and straight line | $y^2 = \frac{3 w}{4 b f}(l^2 - 4 x^2)$ <br> $h = \sqrt{\frac{3 w l^2}{4 b f}}$ |
| VIII. | Single load |  | Rectangle of uniform depth, $h$, and variable breadth, $z$ | Straight lines | $z = \frac{6 w l_2}{h^2 f l} x$ <br> $z' = \frac{6 w l_1}{h^2 f l} x'$ |
| IX. | Uniform load |  | Rectangle as above. (Load, $w$ lbs. per inch, uniformly distributed) | Two parabolas | $z = \frac{3(l^2 - 4x^2)w}{4 f h^2}$ |

A beam, supported at each end, is equivalent to two beams encastre at the point where the bending moment is greatest. The forms given for beams encastre at one end, may be used for each segment of a beam supported at both ends.

### Resistance to Torsion

57. A bar is subjected to simple torsion when two equal and opposite couples act upon it in two planes perpendicular to its axis, instead of being, as in the case of bending, in the plane of the axis. When the bar is subjected to straining action of this kind, any two transverse sections rotate slightly relatively to each other, and on any one transverse section the stress is a simple tangential or shearing stress, varying in intensity as the distance from the centre of the bar, where it is zero, to the circumference, where it is greatest. Of the two couples, one, P P′, fig. 35, is usually due to motive forces applied to the bar. The other, P″ P‴, is due to the reaction of the parts to which the bar is attached, or to the resistances which are being overcome. Further, of the two forces P, P′, constituting the former couple, one of the two

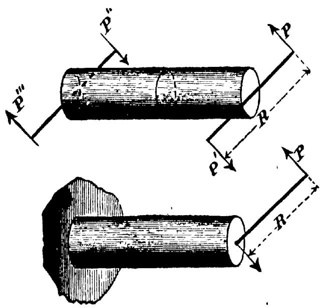


Fig. 35

forces, for instance, P′, may be due to the reaction of a support or bearing of the shaft, and it then acts at the centre of the shaft, as shown in the lower figure.

The amount of straining action at any cross section is measured by the moment of the couple on either side of the section. In this case that moment, termed the twisting moment, is $T=P\ R$. If several couples act on one side of the section, the algebraic sum of the moments of all those couples is to be taken, right-handed couples being considered positive, and left-handed couples negative.

When the bar is kept in rotation overcoming a resistance, and the amount of work transmitted is known, the twisting moment is easily found. Let H P be the number of horse power transmitted, N the number of revolutions of the bar per minute.

Then the work expended in inch lbs. per minute is $12 \times 33000 \times$ H P; and this is equal to the twisting moment T in statical inch lbs., multiplied by the angular motion $2\pi$ N of the bar in the same time. Hence

$$T = \frac{12 \times 33000 \times \text{H.P.}}{2\pi N} = 63024 \frac{\text{H.P.}}{N} \text{ inch lbs.} \qquad (26)$$

The moment of resistance of any section to twisting is proportional to the greatest shearing stress $f_s$ at any part of the section and to a function of the dimensions $z_t$ which is termed the modulus of the section with respect to torsion. If the elastic limit is not exceeded—

$$T = f_s z_t$$

For circular and elliptical sections the section modulus $z_t$ is the polar moment of inertia of the section. In other cases it has a somewhat different value. Values of the safe working stress in torsion $f_s$ are given in Table II, § 39.

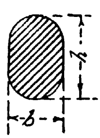
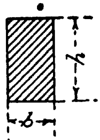
For cylindrical bars of diameter $d$—

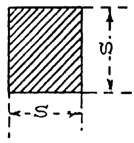
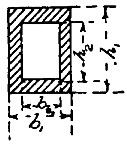
$$z_t = \frac{\pi}{16} d^3 = \frac{d^3}{5 \cdot 1} = 0 \cdot 196\, d^3 \qquad (27)$$

For hollow cylindrical bars with $d_1$, $d_2$, for outside and inside diameters—

$$z_t = \frac{\pi}{16} \frac{d_1^4 - d_2^4}{d_1} = 0 \cdot 196 \frac{d_1^4 - d_2^4}{d_1} \qquad (27a)$$

The following are values of $z_t$ for some other sections :—

| Section | — | Section Modulus $Z_t$ |
|---|---|---|
| Ellipse $h > b$ |  | $\frac{\pi}{16} b^2 h$ |
| Rectangle $h > b$ |  | $\frac{2}{9} b^2 h$ |

| Section | — | Section Modulus $Z_t$ |
|---|---|---|
| Square |  | $0{\cdot}208s^3$ |
| Hollow Rectangle $h > b$ <br> $h_1/h_2 = b_1/b_2$ |  | $\frac{2}{9}\,\frac{b_1^3h_1 - b_2^3h_2}{b_1}$ |

58. *Angle of Twist.*—The angle of twist in circular measure, reckoned per unit length of bar, in the case of bars of circular section and diameter $d$ is—

$$\theta = \frac{32}{\pi d^4}\,\frac{T}{G} = \frac{2f_s}{Gd} \quad . \quad . \quad . \quad . \quad (28)$$

where T is the twisting moment, $f_s$ the greatest shearing stress, and G the coefficient of rigidity (Table I, § 39). For some other sections the researches of Bach give the following values—

Square. Side $= s$. $\theta = 7{\cdot}2\,\frac{T}{Gs^4}$

Rectangle. Sides $h$ and $b$. $\theta = 3{\cdot}6\,\frac{b^2 + h^2}{b^3\,h^3}\,\frac{T}{G}$

If a shaft is $l$ inches long the total twist is—

$$\theta l \text{ in radians} = 57{\cdot}3\ \theta l \text{ degrees.}$$

### Resistance to Buckling. Struts and Columns

59. *Short and long columns.*—When a cylindrical or prismatic bar of a length not more than about five times the least lateral dimension is subjected to an axial thrust it gives way by direct crushing or yielding. Then (as shown in § 41), if $f_c$ is the safe working stress in compression and A the cross-section, the greatest working load is—

$$P_c = f_c A$$

This may be called the crushing formula. If the bar is of greater length it gives way by lateral bending or buckling with a much less load than its full compressive strength.

Rules for the ultimate resistance of long bars to compression were first obtained by Euler. He showed that in the case of a long bar subjected to an axial thrust there was a limiting load consistent with stability. For any less load, if the bar was laterally deflected by a very small amount it would recover itself when the deflecting force was removed. But for any greater load, the deflection once set up would increase till the bar broke by transverse bending.

Let E be the coefficient of elasticity of the material, A the area of section, and J the least moment of inertia about any axis through the centre of figure of the section (that is, J is the moment of inertia about an axis through the centre of figure of the section at right angles to the plane in which the column most easily bends). Let $\rho$ be the least radius of gyration of the section, so that $\rho^2 = J/A$, and let $\lambda$ be the length of an arc of the curved bar measured between two points of contrary flexure. Then the greatest load consistent with stability is

$$P = \pi^2 \frac{JE}{\lambda^2} = 10 \frac{JE}{\lambda^2} \text{ nearly} \qquad (29)$$

The greatest stress consistent with stability per unit of sectional area is

$$p = 10 \frac{\rho^2 E}{\lambda^2} \qquad (29a)$$

Table VIII gives the corresponding equations for variously fixed columns, the total length of which is $l$, and also values of the ratio $\lambda / l$.

TABLE VIII.—*Strength of Long Compression Bars. Greatest Load consistent with Stability.*

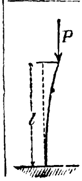
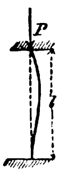
| | | Mode of fixing | Ratio $\lambda/l$ | Greatest load P consistent with stability | Greatest stress $p$ per unit of area consistent with stability |
|---|---|---|---|---|---|
| I. |  | One end free the other fixed. | $\lambda = 2l$ | $\frac{\pi^2}{4}\frac{EJ}{l^2}$ | $2\frac{1}{2}\frac{E\rho^2}{l^2}$ |

TABLE VIII.—*Strength of Long Compression Bars. Greatest Load consistent with Stability*—continued

| | | Mode of fixing | Ratio $\lambda/l$ | Greatest load P consistent with stability | Greatest stress $p$ per unit of area consistent with stability |
|---|---|---|---|---|---|
| II. |  | Both ends free, but guided in the direction of the load. | $\lambda = l$ | $\pi^2 \frac{EJ}{l^2}$ | $10\frac{E\rho^2}{l^2}$ |
| III. |  | One end fixed, the other free, and guided in direction of the load. | $\lambda = \frac{l}{\sqrt{2}}$ | $2\pi^2 \frac{EJ}{l^2}$ | $20\frac{E\rho^2}{l^2}$ |
| IV. |  | Both ends fixed in direction. | $\lambda = \frac{l}{2}$ | $4\pi^2 \frac{EJ}{l^2}$ | $40\frac{E\rho^2}{l^2}$ |

The greatest working load is found by dividing P by a factor of safety $k$. Hence the working load is $P/k$ and the working stress is $p/k$. The working stress should not in any case exceed the safe crushing resistance $f_c$ given in Table II, § 39, p. 54.

*Ordinary Values of the Factor of Safety k*

| Cast Iron | Wrought Iron and Mild Steel | Timber |
|---|---|---|
| 8 | 5 | 10 |

Where there is a doubt as to the axiality of the load (as when beams are attached to columns by brackets) these factors should be increased by one-half. Cases in which the load is definitely eccentric are treated later as cases of compound stress.

The following table gives the most useful values of the least moment of inertia $J_{min}$ and of the corresponding square of the radius of gyration $\rho^2 = J_{min}/A$. The moment of inertia is taken with reference to an axis through the centre of figure of the section and at right angles to the plane in which the bar most easily bends.

| | Least Moment of Inertia J | Square of least radius of gyration $\rho^2$ |
|---|---|---|
| Circular section (diameter $d$) | $\cdot 05\, d^4$ | $\frac{1}{16} d^2$ |
| Annular section (diameters D, $d$) | $\cdot 05\, (D^4 - d^4)$ | $\frac{1}{16} (D^2 + d^2)$ |
| Square section (side $s$) | $\frac{1}{12} s^4$ | $\frac{1}{12} s^2$ |
| Rectangular section (longer side $b$, shorter $h$) | $\frac{1}{12} b h^3$ | $\frac{1}{12} h^2$ |
| Thin square cell (side $s$) | | $\frac{1}{6} s^2$ |
| Thin rectangular cell (shorter side $h$, longer $b$) | | $\frac{h^2}{12} \cdot \frac{h+3b}{h+b}$ |
| Thin circular cell (diameter $d$) | | $\frac{1}{8} d^2$ |
| Equal angle iron (width $b$) | | $\frac{1}{24} b^2$ |
| Equal armed cross (width of arms $h$) | | $\frac{1}{24} h^2$ |

The following table gives the values of the working stress $f/k$ per sq. in. of section in lbs., calculated by Euler's formulæ or round ended columns, Case II.

*Working Stress on round ended columns, by Euler's formula*

| Value of $l/\rho$ | Working Stress in lbs. per square inch | |
|---|---|---|
| | Cast Iron E = 17,000,000 | Wrought Iron or Mild Steel E = 29,000,000 |
| 75 | 3,780 | 10,300 |
| 100 | 2,125 | 5,800 |
| 125 | 1,360 | 3,720 |
| 150 | 944 | 2,580 |
| 175 | 694 | 1,900 |
| 200 | 531 | 1,450 |
| 250 | 340 | 928 |

60. *Limit to the application of Euler's formulæ for columns.*—The case generally to be considered corresponds to Case B, Table II, § 39, and the safe working stress $f_c$ in simple compression may then be taken at 8,500 lbs. per sq. in. for cast iron; 10,000 for wrought iron; and 12,000 for steel, and Euler's formulæ should not be used if they give values of $p/k$ greater than these. If $x$ is put for the numerical constant in the last column of Table VIII, Euler's formulæ should be abandoned if

$$p/k = (x \, \text{E} \, \rho^2)/(kl^2)$$

is greater than the safe working stress in simple compression $f_c$. The limiting length $l_o$ below which Euler's equations cease to apply is—

$$l_o = \rho \sqrt{\frac{x \, \text{E}}{k f_c}} \quad . \quad . \quad . \quad . \quad (30)$$

The following short table gives values of $l_o/\rho$ for the values of $f_c$, E, and $k$ given above.

*Values of limiting ratio of length of column to radius of gyration of section*

| — | Cast Iron | Wrought Iron | Mild Steel |
|---|---|---|---|
| $f_c$ = | 8,500 | 10,000 | 12,000 |
| $k$ | 8 | 5 | 5 |
| E = | 17,000,000 | 29,000,000 | 29,000,000 |
| E/($kf_c$) | 250 | 580 | 483 |
| | *Values of $l_o/\rho$* | | |
| Case I, $x = 2\frac{1}{2}$ | 25 | 38 | 35 |
| Case II, $x = 10$ | 50 | 76 | 69 |
| Case III, $x = 20$ | 71 | 108 | 98 |
| Case IV, $x = 40$ | 100 | 152 | 139 |

The case of long columns which most frequently arises is that of a column with rounded or unfixed ends, Case II. For flat ended columns the equations for Cases III and IV are used, but it is doubtful whether practically it is safe to assume that in such cases the direction of the ends is quite as rigidly fixed as the theory requires.

61. *Columns or struts of moderate length. Gordon's and Rankine's formula.*—For columns of moderate length in proportion to their least lateral dimension the crushing formula gives

greater values than are safe, because the liability to buckling is not absent and at the same time Euler's formulæ are not strictly applicable. Various more or less empirical formulæ have been devised for bars in compression which are neither very short nor very long. The best known is one devised by Gordon and made more general by Rankine. It is of the nature of an interpolation formula which gives results agreeing nearly with the crushing formula, when the length of the column is very short and with Euler's when the column is very long. It should be remembered that such empirical formulæ are only approximately true within limits.

Let $p$ be the mean breaking or buckling stress per sq. in. of section of column, $l$ the length of the column, and $\rho$ its radius of gyration with reference to the axis about which it most easily bends. Then Rankine's rule is—

$$\left.\begin{aligned} p &= \frac{f_0}{1 + 4c\dfrac{l^2}{\rho^2}} \quad \text{Ends rounded} \\ &= \frac{f_0}{1 + c\dfrac{l^2}{\rho^2}} \quad \text{Ends flat} \end{aligned}\right\} \quad . \quad . \quad (31)$$

where $f_0$ and $c$ are constants having the following values—

| | $f_0 =$ lbs. per sq. in. | $c =$ |
|---|---|---|
| Cast iron | 80,000 | $\frac{1}{6400}$ |
| Wrought iron | 36,000 | $\frac{1}{36000}$ |
| Mild steel | 48,000 | $\frac{1}{30000}$ |
| Hard steel | 70,000 | $\frac{1}{20000}$ |
| Timber | 7,200 | $\frac{1}{3000}$ |

The working safe stress is $p/k$ where $k$ is a factor of safety values for which are given above. The greatest safe load for a section A is

$$P_w = p\,A/k$$

The following table gives values of the working stress $p/k$ for different values of $l/\rho$, calculated by Rankine's formula for round ended columns.

*Working Stress on round ended columns, by Rankine's formula. Lbs. per sq. in.*

R = 8 for cast iron and 5 for wrought iron and steel.

| Values of $l/\rho$ | Cast iron | Wrought iron | Mild steel |
|---|---|---|---|
| 5 | — | 7,178 | 9,572 |
| 10 | 9,408 | 7,120 | 9,477 |
| 15 | 8,770 | 7,025 | 9,322 |
| 20 | 8,010 | 6,930 | 9,140 |
| 30 | 6,410 | 6,545 | 8,570 |
| 40 | 5,000 | 6,100 | 7,930 |
| 50 | 3,910 | 5,630 | 7,220 |
| 75 | 2,210 | 4,420 | 5,485 |
| 100 | 1,380 | 3,400 | 4,120 |
| 125 | 926 | 2,630 | 3,120 |
| 150 | 667 | 2,057 | 2,400 |

In the case of cast iron the safe crushing resistance assumed above is exceeded, if $l/\rho < 20$. For wrought iron and steel Rankine's formula with the constants usually adopted gives lower values than the crushing formula, even for $l/\rho = 5$.

62. *Tetmajer's Formula.*—From careful experimental researches Tetmajer deduced the following empirical formula for columns with both ends rounded or unfixed in direction. It is more trustworthy than Euler's if $l/\rho$ is less than 80 for cast iron or about 100 for wrought iron or mild steel. The mean stress per sq. in. on the section when the column breaks or buckles is

$$p = \text{K}\left(1 - \frac{al}{\rho} + \frac{bl^2}{\rho^2}\right) \quad . \quad . \quad . \quad (32)$$

and the safe working stress is $p/k$, where $k$ has the values given above. The greatest safe load is $\text{P}_w = p\,\text{A}/k$. The constants have the following values.

| — | $k$ | K | K$/k$ | $a$ | $b$ |
|---|---|---|---|---|---|
| Cast iron . . . | 8 | 110,000 | 13,750 | 0·0155 | 0·00007 |
| Wrought iron . . | 5 | 43,000 | 8,600 | 0·00426 | 0 |
| Mild steel . . . | 5 | 44,020 | 8,804 | 0·00368 | 0 |
| Medium steel . . | 5 | 45,580 | 9,116 | 0·00361 | 0 |
| Timber . . . | 10 | 4,160 | 832 | 0·00662 | 0 |

The following table gives the working stress for various values of $l/\rho$ calculated from Tetmajer's formula for round

ended columns with a factor of safety of 8 for cast iron and 5 for wrought iron and mild steel :—

*Working Stress on round ended columns by Tetmajer's formula. Lbs. per sq. in.*

| Values of $l/\rho$ | Cast Iron | Wrought Iron | Mild Steel | Medium Steel |
|---|---|---|---|---|
| 10 | 11,000 | 8,230 | 8,474 | 8,790 |
| 20 | 9,840 | 7,869 | 8,149 | 8,463 |
| 30 | 8,190 | 7,500 | 7,832 | 8,135 |
| 40 | 6,740 | 7,138 | 7,506 | 7,843 |
| 50 | 5,500 | 6,768 | 7,207 | 7,478 |
| 75 | 3,170 | 5,857 | 6,371 | 6,657 |
| 100 | — | 4,936 | 5,562 | 5,827 |
| 125 | — | 4,025 | 4,752 | 5,006 |

To compare with the results calculated by the empirical formulæ of Rankine and Tetmajer the following results selected by Prof. Claxton Fidler from a comparison of available experimental data may be used. The factor of safety is 8 for cast iron and 5 for wrought iron and steel. The table contains results both for round ended and flat ended columns.

*Working Stress for Columns. Claxton Fidler. Lbs. per sq. in.*

| Values of $l/\rho$ | Cast Iron | Wrought Iron | Mild Steel | Hard Steel |
|---|---|---|---|---|
| | | *Columns with rounded ends* | | |
| 20 | 9,050 | 7,040 | 9,340 | 13,400 |
| 40 | 6,350 | 6,520 | 8,540 | 11,700 |
| 60 | 3,750 | 5,680 | 7,[illegible]00 | 9,100 |
| 80 | 2,200 | 4,640 | 5,660 | 6,600 |
| 100 | 1,460 | 3,640 | 4,300 | 4,740 |
| 120 | 1,040 | 2,820 | 3,280 | 3,500 |
| 160 | 613 | 1,760 | 2,020 | 2,080 |
| 200 | 401 | 1,180 | 1,340 | 1,370 |
| | | *Columns with ends fixed in direction* | | |
| 20 | 9,700 | 7,160 | 9,480 | 13,700 |
| 40 | 8,480 | 6,980 | 9,140 | 13,200 |
| 60 | 6,840 | 6,680 | 8,660 | 12,100 |
| 80 | 5,250 | 6,220 | 7,980 | 10,700 |
| 100 | 3,750 | 5,680 | 7,200 | 9,090 |
| 120 | 2,650 | 5,060 | 6,200 | 7,480 |
| 160 | 1,570 | 3,840 | 4,500 | 5,000 |
| 200 | 1,040 | 2,820 | 3,280 | 3,500 |

## Laminated and Helical Springs

The steel of which springs are made has a tenacity (untempered) of 100,000 to 140,000 lbs. per sq. in. and an elastic limit of 70,000

to 90,000 lbs. per sq. in. After tempering in oil, its tenacity is 140,000 to 220,000 lbs. per sq. in. and its elastic limit 110,000 to 140,000 lbs. per sq. in. The safe working stress may be taken at $f$=80,000 lbs. per sq. in. in springs subjected to bending and 60,000 in springs subjected to torsion. The coefficient of direct elasticity, E, may be taken at 30,000,000 lbs. per sq. in., and the coefficient of rigidity, G, at 12,500,000 lbs. per sq. in. In the following calculations it is assumed that the load on a spring is gradually applied. If quite suddenly applied the stress and deflection would be momentarily double that due to a gradually applied load.

63. *Laminated Springs.—General Case.* Suppose a spring, fig. 36, composed of $n$ straight leaves or laminæ so arranged that the mutual pressure is confined to the ends. Let $p_1$, $p_2$ . . . be the pressures at the ends acting at $l_1$, $l_2$ . . . from the

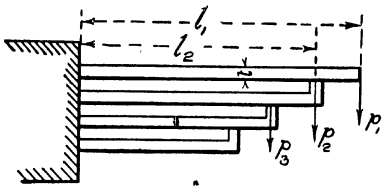



Fig. 36

support and producing stresses $f_1$, $f_2$,. . . in the successive leaves. Let $b$ be the common width, $t$ the common thickness of the leaves, $\rho_1$, $\rho_2$ the radii of curvature at the support when deflected. The bending moment on the first leaf is $p_1 l_1 - p_2 l_2$. Then by the bending equations—

$$p_1 l_1 - p_2 l_2 = \frac{\text{E J}}{\rho} = \frac{\text{E} bt^3}{12\rho_1}$$

. . . . . . . . . .

$$p_n l_n = \frac{\text{E} b t^3}{12 \rho_n} \quad . \quad . \quad . \quad (a)$$

$$p_1 l_1 - p_2 l_2 = \frac{b t^2 f_1}{6}$$

. . . . . . . . . .

$$p_{n-1} l_{n-1} - p_n l_n = \frac{b t^2 f_{n-1}}{6} \quad . \quad . \quad . \quad . \quad (b)$$

In a perfect spring two conditions should be satisfied, (1) the stresses $f_1$, $f_2$ should all be equal so that all the laminæ are equally

strained; (2) if the leaves are in contact initially instead of merely touching at the ends they should remain in contact after deflection, or the radii of curvature $\rho_1$, $\rho_2$, . . . should be the same. Hence—

$$p_1 - p_2 = p_2 - p_3 \ldots = \Delta p \quad \ldots \quad (c)$$

$$p_1 l_1 - p_2 l_2 = p_2 l_2 - p_3 l_3 \ldots = \frac{b t^2 f}{6} \quad \ldots \quad (d)$$

where $\Delta p$ is the common difference of load at the ends of two adjacent leaves and $f$ the common stress in all the leaves. Add together $n$ equations $(d)$ and also $k-1$ of the equations $(d)$.

$$p_1 l_1 = \frac{n b t^2 f}{6}$$

$$p_1 l_1 - p_k l_k = (k-1)\frac{b t^2 f}{6}.$$

Adding the first $k-1$ equations $(c)$

$$p_k = p_1 - (k-1)\,\Delta p$$

Eliminating $f$ and $p_k$—

$$l_k = l_1 \frac{1 - (k-1)/n}{1 - (k-1)\dfrac{\Delta p}{p_1}}$$

Now $\Delta p/p_1$ may be zero, but cannot exceed $1/n$, or the lower leaves would be longer than the upper.

Let $\Delta p/p_1 = 1/yn$, where $y$ is greater than unity.

$$l_k = l_1 \frac{1 - (k-1)/n}{1 - (k-1)/yn}.$$

If $y = 1$, all the leaves are of equal length, $l$, and the spring is rectangular in elevation. The difference $\Delta p$ is constant and equal to $p_1/n$. The bending moment on each leaf is $l\,\Delta p$, the stress in all is the same, and all being similarly strained the curvature when bent is the same. If $y = \infty$,

$$l_k = l_1 \{1 - (k-1)/n\}$$
$$l_{k+1} = l_1 \{1 - k/n\}$$
$$l_{k+1} - l_k = l_1/n$$

That is, the difference of length of the leaves is constant and equal to $\Delta l = l_1/n$. Then the elevation of the spring is triangular. Also

$$\Delta p = 1/yn = 0, \text{ or } p_1 = p_2 = \quad \ldots \quad = p$$

Hence all the radii of curvature are equal when deflected. Each lamina is subjected to a couple $p\,\Delta l$ at the end and the curvature

of all the leaves is circular. Also the stress on all the leaves is the same and equal to $(6\,p\,\Delta\,l)/b\,t^2$. These are the conditions to be satisfied in a perfect spring.

The general expression for the deflection is

$$\delta = \frac{f\,l_1^2}{\mathrm{E}\,t}\left(1 - \frac{1}{3y}\right)$$

$$\text{Rectangular spring, } y = 1,\ \delta = \frac{2}{3}\frac{f\,l_1^2}{\mathrm{E}\,t}$$

$$\text{Triangular spring, } y = \infty,\ \delta = \frac{f\,l_1^2}{\mathrm{E}\,t}.$$

Simple laminated springs are subjected to bending only. They are usually loaded at the centre where the leaves are confined by a buckle and supported at the ends. Three cases may be considered.

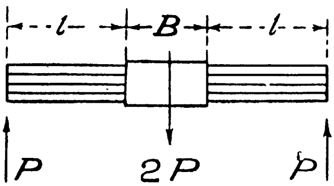


Fig. 37

(1) *Rectangular laminated spring* (fig. 37).—Let the spring consist of $n$ leaves of width $b$ and thickness $t$, of the same length $l$ measured from the buckle, so that the total length between supports is $\mathrm{L} = 2\,l + \mathrm{B}$. Let $2\,\mathrm{P}$ be the central load, and $f$ the safe working stress. Then from what precedes—

$$2\,\mathrm{P} = (n\,f\,b\,t^2)/(3l) \quad . \quad . \quad . \quad (33)$$

and the deflection under load is—

$$\delta = \frac{4\mathrm{P}l^3}{\mathrm{E}\,n\,b\,t^3} = \frac{2}{3}\frac{fl^2}{\mathrm{E}\,t} \quad . \quad . \quad . \quad (33a)$$

(2) *Triangular laminated spring* (fig. 38).—A triangular spring may be conceived to be formed by cutting a lozenge-shaped plate into $2n$ strips of equal breadth $b/2$ and piling them in pairs.

$$2\,\mathrm{P} = (n\,f\,b\,t^2)\,/\,(3\,l) \quad . \quad . \quad . \quad (34)$$

$$\delta = \frac{6\,\mathrm{P}\,l^3}{\mathrm{E}\,n\,b\,t^3} = \frac{f\,l^2}{\mathrm{E}\,t} \quad . \quad . \quad . \quad (34a)$$

(3) *Laminated railway springs.*—Such springs are made of superposed leaves (fig. 39), and are loaded at the centre and supported at the ends. The leaves are often initially curved, but this

makes little difference if the projected lengths are used in the equations. The lower contour generally approximates to an

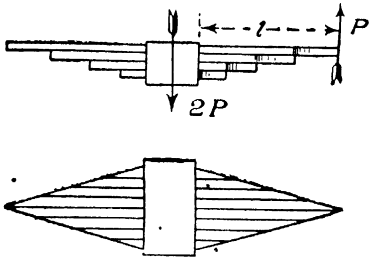



Fig. 38

ellipse. Usually about one-fourth of the leaves are of the full length of the spring. Let $n$ be the number of leaves, $l$ the

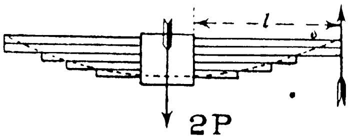



Fig. 39

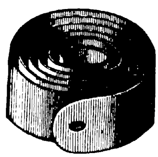


Fig. 40

length of the spring on each side of the buckle, $b$ the width, $t$ the thickness of the leaves, and $f$ the working stress.

$$2\mathrm{P} = \frac{f n b t^2}{3 l} \quad . \quad . \quad . \quad . \quad . \ (35)$$

and approximately,—

$$\delta = 5{\cdot}5 \frac{\mathrm{P} l^3}{\mathrm{E} n b t^3} = 0{\cdot}92 \frac{f l^2}{\mathrm{E} t} \quad . \quad . \ (35a)$$

64. *Spiral Springs.*—Spiral springs are subjected to simple bending (fig. 40). Suppose the section rectangular, of width $b$ and thickness $t$, and let the greatest outside radius be $r$, and the length of the spring $l$. Then the tangential load at the extremity of the spring, when the stress is $f$, is—

$$\mathrm{P} = (f b t^2)/6r \quad . \quad . \quad . \quad . \quad . \ (36)$$

and the angle through which the extremity turns in circular measure is—

$$\theta = \frac{12\,\text{P}\,l\,r}{\text{E}\,b\,t^3} = \frac{2fl}{\text{E}\,t} \qquad . \quad . \quad . \quad (36a)$$

65. *Springs subjected to torsion.*—(1) *Simple straight bar.* Let a force P act at a radius R on a bar of length $l$. The twisting moment is P R inch lbs. (Fig. 41.)

If the bar is cylindrical and of diameter $d$ the stress is—

$$f = \frac{\text{P R}}{\cdot 196\, d^3} \qquad . \quad . \quad . \quad . \quad (37)$$

the angle through which the bar is twisted in circular measure is

$$\omega = \frac{32\,\text{P R}\,l}{\pi\, d^4\, \text{G}} \qquad . \quad . \quad . \quad . \quad (37a)$$

If the bar is of rectangular section, of width $b$ and thickness $t\,(b > t)$

$$\left.\begin{aligned} f &= \frac{4{\cdot}5\,\text{P R}}{b\,t^2} \\ \omega &= 3{\cdot}6\,\text{F R}\,l\,\frac{b^2 + t^2}{\text{G}\,b^3\,t^3} \end{aligned}\right\}\text{nearly} \qquad . (37b)$$

66. (2) *Helical Springs.*—A helical spring, fig. 42, is formed by coiling a bar of uniform section in a helix. It is loaded axially either in tension or compression. Let P be the load, G the coefficient of rigidity of the material, $\theta$ the angle of twist of the bar reckoned per

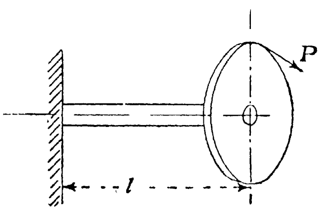



Fig. 41

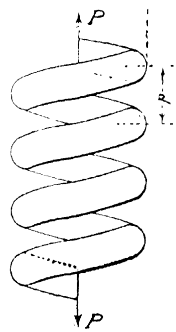



Fig. 42

unit length, $\delta$ the elongation or compression of the spring, $l$ the length of the bar. Very commonly the bar is of circular section. Consider this case, and let the diameter of the bar = $d$, the radius of the coils to the centre of the bar = R, the length

of the coiled bar $= l$, the pitch of the coils $= p$, the modulus of the section for torsion $= z_t$, and the number of coils $= n$.

$$l = n\sqrt{(2\pi R)^2 + p^2} = 2\pi R n \text{ nearly.}$$

On any cross-section of the bar normal to the axis the twisting moment is P R. There is also some bending, but this is negligi-

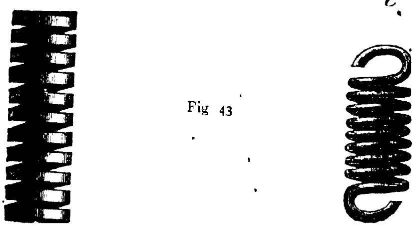


Fig 43

ble, if the coils are fairly close and R is large compared with $d$. From the torsion formulæ, the shearing stress is—

$$f_s = \frac{16\,P\,R}{\pi d^3} = \frac{P\,R}{z_t} \quad . \quad . \quad . \quad . \quad . \quad (38)$$

The angle of twist per unit length is—

$$\theta = \frac{2 f_s}{G\,d} = \frac{32\,P\,R}{G\,\pi d^4} = \frac{2\,P\,R}{G\,d\,z_t} \quad . \quad . \quad . \quad . \quad (38a)$$

The total elongation or compression of the spring is—

$$\delta = R\,\theta\,l = \frac{2\,R\,l f_s}{G\,d} = \frac{32\,P\,R^2 l}{G\,\pi d^4} = \frac{2\,P\,R^2 l}{G\,d\,z_t} \quad . \quad . \quad (38b)$$

The work done in elongating or compressing the spring, by a gradually applied load, is—

$$W = \tfrac{1}{2}\,\delta\,P = \frac{\pi d^2 f_s^2 l}{16\,G} = \frac{16\,P^2 R^2 l}{G\,\pi d^4} = \frac{P^2 R^2 l}{G\,d\,z_t} \quad . \quad . \quad (38c)$$

The work of compression or extension reckoned per unit volume of the spring is—

$$\frac{4W}{\pi d^2 l} = \frac{f_s^2}{4G} \quad . \quad . \quad . \quad . \quad . \quad . \quad (38d)$$

If the section of the bar is not circular, the equations in which $z_t$ appears may be used, inserting the values of $z_t$ already given under torsion, § 57. Thus for a square section with side $= s$, $z_t = 0{\cdot}208\,s^3$; for rectangular sections, breadth $= b$, height $= h$, $z_t = \frac{2}{9}\,b^2 h$.

Fig. 43 shows ordinary forms of helical spring. If $d$ is the diameter or $s$ the side of a square bar coiled to a radius R, then

$R/d$ or $R/s$ should not be less than $1\frac{1}{2}$ and is generally not less than $2\frac{1}{2}$. The greatest stress when the coils are compressed close so that the height of the spring is $nd$ or $ns$ should not exceed the elastic limit of the material. In helical springs the greatest working stress may vary from 80,000 lbs. per sq. in. for bars $\frac{1}{4}$ inch in diameter or length of side to 50,000 for 1 inch in diameter or side. For tension springs, the stress should not exceed two-thirds of these values. The coefficient of rigidity may be taken at $G = 12500000$. In the following table the stress $f_s$ corresponding to a load $P$, and the deflection are given in terms of the dimensions of the spring, the units being pounds and inches. The numerical values of the constants are those calculated by Mr. A. E. Young (Proc. Inst. Civil Engineers, ci. p. 277), and are more accurate than the values generally given.

*Helical Springs, Loaded Axially*

| Section of Wire | Stress $f$ | Deflection $\delta$ |
|---|---|---|
| Circular, diameter $d$ | $\dfrac{16\,PR}{\pi d^3}$ | $\dfrac{64\,PR^3 n}{G d^4}$ |
| Elliptical, diameters $D$, $d$ | $\dfrac{16\,PR}{\pi D d^2}$ | $\dfrac{32\,n\,PR^3}{G\,D^3 d^3}(D^2 + d^2)$ |
| Square side $s$ | $\dfrac{4{\cdot}79\,PR}{s^3}$ | $\dfrac{44\,n\,PR^3}{G s^4}$ |
| Rectangle sides $b$ and $h$: | | |
| $b = 2h$ | $\dfrac{4{\cdot}06\,PR}{b h^2}$ | $\dfrac{27\,n\,PR^3}{G b h^2}$ |
| $b = 3h$ | $\dfrac{3{\cdot}75\,PR}{b h^2}$ | $\dfrac{24\,n\,PR^3}{G b h^2}$ |
| $b = 5h$ | $\dfrac{3{\cdot}44\,PR}{b h^2}$ | $\dfrac{22\,n\,PR^3}{G b h^2}$ |
| $b = 10h$ | $\dfrac{3{\cdot}21\,PR}{b h^2}$ | $\dfrac{20\,n\,PR^3}{G b h^2}$ |
| $b = 100h$ | $\dfrac{3\,PR}{b h^2}$ | $\dfrac{19\,n\,PR^3}{G b h^2}$ |

## Strength of Hollow Cylindrical Vessels subjected to External Pressure

67. *Mean Circumferential Crushing Stress.*—When a cylindrical vessel is exposed to an external fluid pressure, a circumferential compressive stress is induced, the mean intensity of

which is found by the same formula as that for bursting, § 43. Let $d$ be the external diameter and $t$ the thickness in inches; let $p$ be the excess of external over internal pressure in lbs. per sq. in. Then the circumferential stress is—

$$\left.\begin{aligned} f &= \frac{pd}{2t} \\ p &= 2f\frac{t}{d} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (39)$$

In any case $f$ should be less than the working stress in compression, Table II, § 39. When $t/d$ is small the variation of stress in the shell is small and $f$ is sensibly equal to the maximum stress.

*Lamé's formula for thick tubes.*—A modified formula in which the variation of stress is taken into account is due to Lamé.

The stress is greater at the outside and less at the inside of the cylinder. Let $p$ be the external pressure, R the external radius, $r$ the internal radius and $f$ the greatest intensity of stress. Then—

$$f = 2p\frac{R^2}{R^2 - r^2}$$

But $R = d/2$ and $r = d/2 - t$. Inserting these values—

$$\left.\begin{aligned} p &= 2f\frac{t}{d}\left(1 - \frac{t}{d}\right) \\ f &= \frac{pd}{2t}\,\frac{d}{d-t} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (40)$$

That is, when the variation of stress is taken into account $p$ is less for a given value of $f$, or $f$ greater for a given value of $p$, than in Eq. 39. But the difference is not great if $t/d$ is small.

*Values of $p$*

| $t/d$ | Formula 39 | Formula 40 |
|---|---|---|
| 0·2 | 0·4$f$ | 0·32$f$ |
| 0·1 | 0·2$f$ | 0·18$f$ |
| 0·05 | 0·1$f$ | 0·095$f$ |

Hence in practical cases where $t/d$ is less than about 0·05 Eq. 39 may be used instead of the more complicated Eq. 40.

The following are values of $f$ in Eq. 39 deduced from some experiments by Prof. R. T. Stewart (Trans. Am. Soc. Mechanical Engineers, 1906, p. 730) on lap-welded Bessemer steel tubes. Only the thickest tubes in which $t/d$ was largest have been selected, for which the failure was most likely to be due to simple crushing.

*Crushing Strength of moderately thick Tubes.* (*Stewart*)

| Length $l$ | Diameter $d$ | Thickness $t$ | Crushing pressure $p$ | $\frac{t}{d}$ | $f = \frac{pd}{2t}$ |
|---|---|---|---|---|---|
| ins. | ins. | ins. | lbs per sq in | | |
| 240 | 2·99 | ·143 | 2,962 | ·048 | 31,100 |
| ,, | 3·00 | ·188 | 4,095 | ·063 | 32,700 |
| ,, | 4·03 | ·212 | 3,170 | ·053 | 30,100 |
| ,, | 4·01 | ·327 | 5,560 | ·082 | 34,000 |
| 56 | 8·65 | ·306 | 2,073 | ·035 | 29,300 |
| 26 | 8·65 | ·311 | 2,397 | ·036 | 33,300 |
| | | | Mean . . . | | 31,400 |

Probably some of these tubes were not quite thick enough for simple crushing, and it will be safe to take $f = 32{,}000$.

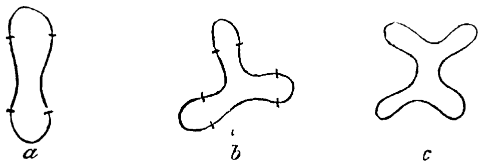


Fig. 44.

Hence the external crushing pressure for moderately thick tubes is at least

$$p = 64{,}000\ (t/d) \text{ lbs. per sq. in.}^{1}$$

Taking a factor of safety of five, the safe external working pressure for such tubes is

$$p_w = 12{,}800\ (t/d) \text{ lbs. per sq. in.} \quad . \quad . \quad (41,$$

68. *Thin tubes of moderate length, when the length influences the strength.*—The flues of Cornish and Lancashire boilers are such tubes as are here considered, and they are rigidly supported in the boiler end-plates and sometimes at intermediate points. In such tubes, since $t/d$ is small, the material tends to buckle under a circumferential thrust, just as in the case of long columns there is lateral bending. Such cylinders exposed to an external pressure give way suddenly by *collapse*. Fig. 44 shows the very characteristic forms of the cross-section of collapsed

[1] The author gave this formula with a constant 58,800 in his paper of 1876, for tubes in which $t/d > 0{\cdot}278$ (Trans. Inst. Civil Engineers, xlvi.).

tubes. The occurrence of some serious boiler explosions in which the flues collapsed led Sir W. Fairbairn to undertake an experimental investigation. Tests were made of thin tubes of tinplate, which had rigid ends and were collapsed by water pressure (Trans. Royal Soc. 1858). The remarkable result was found that with such thin tubes and within the range of the experiments the collapsing pressure varied inversely as the length of the tube, other things being the same.

The author re-examined these results in 1876 (Proc. Inst. Civil Engineers, xlvi.). He found that there was an absolutely clear rule for the whole of these experiments connecting the number of lobes into which a tube collapsed and the ratio $l/d$ of length to diameter of tube. Thus calling $a$, fig. 44, a curve of four segments, between points of contrary flexure, $b$ a curve of six segments, and so on—

| | | | |
|---|---|---|---|
| For $l/d =$ | 15 to $7\frac{1}{2}$ | $7\frac{1}{2}$ $4\frac{1}{4}$ | $3\frac{3}{4}$ $2\frac{1}{2}$ |
| The number $n$ of segments into which the tube collapsed was | 4 | 6 | 8 or 10 |

It is obvious that a ring of the thin cylinder under compression is in a condition quite similar to that of the long columns in Table VIII, and that the thrust with which the tube gives way depends on the length of the segments between points of contrary flexure. The segment length corresponds to $\lambda$ in Table VIII, § 59.

The reason why in such tubes the length influences the strength is that it determines the number of segments into which the tube collapses. From the formula above, Eq. 39, the circumferential thrust in the tube is—

$$f = (p\,d)/(2\,t)$$

Considering one inch length of the tube, the greatest thrust consistent with stability in a long column of length $\lambda$ (Case II, Table VIII) is—

$$f = 10\,\frac{E\,t^2}{12\,\lambda^2}$$

If $n$ is the number of segments between points of contrary flexure in the collapsed tube, $\lambda = \pi\,d/n$, then

$$p = \frac{E\,n^2\,t^3}{6\,d^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

which is a rational formula for the collapsing pressure. This

formula was given in the author's paper of 1867, and two limits to its application were pointed out. (1) If $t/d$ was greater than a limit provisionally fixed at 0·0278, the long column formula ceased to apply and the failure would be by crushing. (2) As the tube cannot collapse into fewer than four segments, when the length reached the value at which collapse into four segments occurred the strength will have reached its least value for the given diameter and the thickness, and will not further decrease with increase of length. In that case, putting $n=4$ in the equations above, the collapsing pressure for all lengths above the limit, which is about $l/d = 7$, is,[1]—

$$p = \frac{8}{3} \mathrm{E} \left(\frac{t}{d}\right)^3 \quad . \quad . \quad . \quad . \quad . \quad (43)$$

Since the precise conditions assumed in theory, exactness of cylindrical form, uniformity of thickness, &c., cannot be completely satisfied in actual tubes, these rational formulæ require to be controlled by experiment.

A comparison of the rational formulæ above with the results of Fairbairn's experiments shows that, just as in the case of moderately long columns, the resistance to buckling does not increase so fast as $t^3$, and that small deviations from the true circular form in actual tubes sensibly reduce the strength.

From an examination of Fairbairn's results and such other data as were available, the author obtained in 1876 the following empirical equations for tubes not exceeding the limit within which the length has an influence on the collapsing pressure.

For tubes with a longitudinal lap-joint,

$$p = 7{,}363{,}000 \frac{t^{2 \cdot 1}}{l^{0 \cdot 9} d^{1 \cdot 16}} \quad . \quad . \quad . \quad (\mathrm{i})$$

For tubes with a longitudinal butt-joint,

$$p = 9{,}614{,}000 \frac{t^{2 \cdot 21}}{l^{0 \cdot 9} d^{1 \cdot 16}} \quad . \quad . \quad . \quad (\mathrm{ii})$$

[1] Prof. Love (*Mathematical Theory of Elasticity*) subsequently gave the equation

$$p = \frac{2 \mathrm{E}}{1 - m^2} \left(\frac{t}{d}\right)^3$$

in which $1/m$ is Poisson's ratio. This is theoretically more exact in form, but does not much differ numerically from Eq. 43.

For tubes with longitudinal and cross-joints like ordinary boiler flues,

$$p = 15{,}547{,}000 \frac{t^{2\cdot35}}{l^{0\cdot9} d^{1\cdot16}} \quad . \quad . \quad . \quad \text{(iii)}$$

where $t$ is the thickness, $d$ the external diameter, and $l$ the length between rigid ends, in inches, and $p$ is the collapsing pressure in lbs. per sq. in. The limits within which the formulæ are applicable will be determined presently. The safe working pressure may be taken at one fourth to one-sixth of the collapsing pressure.

69. *Simplified Formula for Moderately Long Thin Tubes.*—The equations above are inconvenient and a simplified equation is accurate enough for most practical purposes. Let the collapsing pressure be—

$$p = \text{C} \frac{t^{2\cdot25}}{l\,d} \quad . \quad . \quad . \quad . \quad . \quad (44)$$

then C is a constant, values of which will be deduced presently.

*Fairbairn's Experiments.*—These tests of very thin tubes of tinplate are all within the limits for which Eq. 44 is applicable.

*Moderately Long Thin Tubes*

| Length $l$ | Diameter $d$ | Thickness $t$ | Collapsing Pressure $p$ | $l/d$ | $\text{C} = \frac{p\,l\,d}{t^{2\cdot25}}$ |
|---|---|---|---|---|---|
| ins. | ins. | ins. | lbs. per sq. in. | | |
| 38 | 4 | ·043 | 65 | 9·5 | 11,730,000 |
| 30 | 4 | ,, | 93 | 7·5 | 13,250,000 |
| 20 | 4 | ,, | 140 | 5·0 | 13,290,000 |
| 19 | 4 | ,, | 153·5 | 4·75 | 13,850,000 |
| 15 | 4 | ,, | 147 | 3·75 | 10,470,000 |
| 30 | 6 | ,, | 55 | 5·0 | 11,760,000 |
| 29 | 6 | ,, | 47 | 4·85 | 9,712,000 |
| 60 | 8 | ,, | 22 | 7·5 | 12,540,000 |
| 39 | 8 | ,, | 32 | 4·85 | 11,850,000 |
| 40 | 8 | ,, | 31 | 5·0 | 11,780,000 |
| 30 | 8 | ,, | 37·5 | 3·75 | 10,690,000 |
| 50 | 10 | ,, | 19 | 5·0 | 11,280,000 |
| 30 | 10 | ,, | 33 | 3·0 | 11,760,000 |
| 60 | 12 | ,, | 12·5 | 5·0 | 10,690,000 |
| 30 | 12 | ,, | 22 | 2·5 | 9,404,000 |
| | | Mean . . . . . | | | 11,604,000 |

These experiments are important as showing, when tubes of the same diameter are compared, the unmistakable influence of the length in the case of tubes of certain proportions of $l/d$.

*Prof. R. T. Stewart's Experiments.*—There are really only four of Prof. Stewart's tests which are probably within the limits, and these confirm the view that within these limits the length materially influences the strength.

| $l$ | $d$ | $t$ | $p$ | $\frac{l}{d}$ | $c = \frac{p\,l\,d}{t^{2\cdot25}}$ |
|---|---|---|---|---|---|
| 56·4 | 8·66 | ·180 | 592 | 6·5 | 13,710,000 |
| 26·5 | 8·66 | ·176 | 977 | 3·5 | 11,150,000 |
| 56·3 | 8·66 | ·212 | 907 | 6·5 | 14,370,000 |
| 26·5 | 8·66 | ·212 | 1314 | 3·5 | 9,837,000 |
| | | | Mean . . . | | 12,267,000 |

A result differing little from that deduced from Fairbairn's tests of much thinner tubes.

There are very few trustworthy tests of riveted tubes falling within the limits for this class. The following, however, are useful.

| $l$ | $d$ | $t$ | $p$ | $\frac{l}{d}$ | $c = \frac{p\,l\,d}{t^{2\cdot25}}$ |
|---|---|---|---|---|---|
| *Lap joint* | | | | | |
| 61 | 18·9 | ·25 | 420 | 3·3 | 10,830,000 |
| *Butt joint* | | | | | |
| 37 | 9 | ·14 | 378 | 4·1 | 10,500,000 |
| *Longitudinal and Cross lap joints.* | | | | | |
| 60 | 14·6 | ·125 | 125 | 4·1 | 11,790,000 |

Hence, allowing a factor of safety of four, the safe external working pressure for tubes of this class may be taken at—

$$p_w = 2{,}900{,}000\ \frac{t^{2\cdot25}}{ld}\ \text{lbs. per sq. in.}$$

and for riveted tubes a little less.

*Least length for which Eq. 44 is applicable.*—The equations 39 and 44 give the same value for the pressure at which failure occurs, if—

$$2f\frac{t}{d} = c\,\frac{t^{2\cdot25}}{ld}$$

Taking $f = 32{,}000$ and $c = 12{,}000{,}000$,

$$l = 188\,t^{1\cdot25}$$

for less lengths Eq. 39 applies and for greater Eq. 44—

$$t = \tfrac{1}{16} \quad \tfrac{1}{8} \quad \tfrac{1}{4} \quad \tfrac{3}{8} \quad \tfrac{1}{2} \quad \tfrac{5}{8} \text{ ins.}$$
$$l = 6 \quad 14 \quad 33 \quad 55 \quad 79 \quad 105 \text{ ins.}$$

70. *Collapsing pressure of thin tubes so long that the length ceases to influence the strength.*—When the length reaches the limit at which collapse into four segments occurs, the strength ceases to decrease with increase of length. The length is the distance between cross-sections of the tube which are rigidly supported against alteration of form. Let the limiting length be $l = x\,d$. Then for tubes of this and greater length—

$$p = c\,\frac{t^{2\cdot5}}{x\,d^2} = c_1\,\frac{t^{2\cdot5}}{d^2} \quad . \quad . \quad . \quad . \quad (45)$$

where $c_1$ is a new constant. This is the empirical formula corresponding to the rational formula Eq. 43.

*American Experiments.*—Very valuable tests on long tubes have been made by Prof. R. T. Stewart (Trans. Am. Soc. Mechanical Engineers, 1906) and by Prof. A. P. Carman (Bulletin of the University of Illinois, 1906). The following is a reduction of these, groups of tubes of the same diameter and thickness being averaged. The following table gives Prof. Stewart's tests of tubes which appear to have collapsed and not crushed, and which were longer than the limit stated above.

*Long Tubes (Stewart)*

| $l$ | $d$ | $t$ | $p$ | $l/d$ | $c_1 = \dfrac{p\,d^2}{t^{2\cdot5}}$ |
|---|---|---|---|---|---|
| 240 | 3·00 | 0·109 | 1733 | Greater than 15 | 2,281,000 |
| 240 | 3·00 | 0·188 | 4095 | | 1,591,000 |
| 240 | 6·02 | 0·128 | 519 | | 1,922,000 |
| 240 | 6·03 | 0·166 | 924 | | 1,913,000 |
| 240 | 6·03 | 0·264 | 2381 | | 1,731,000 |
| 240 | 8·64 | 0·185 | 530 | | 1,782,000 |
| 240 | 8·67 | 0·267 | 1438 | | 2,106,000 |
| 240 | 8·67 | 0·354 | 2028 | | 1,579,000 |
| 180 to 240 | 12·79 | 0·511 | 2196 | | 1,638,000 |
| 216 to 240 | 13·04 | 0·244 | 463 | | 1,885,000 |
| | | Mean . . . . . | | | 1,843,000 |

There is a fair agreement in the values of $c_1$, and the divergencies are irregular and independent of $d$ or $t$.

Mr. Carman's tubes were long, seamless, cold-drawn tubes of

comparatively small diameter. The tubes of nearly the same dimensions have been grouped and averaged.

*Long Tubes (Carman)*

| $l$ | $d$ | $t$ | $p$ | $l/d$ | $C_1 = \frac{pd^2}{t^{2.5}}$ |
|---|---|---|---|---|---|
| — | 1·5 | ·096 | 4167 | — | 1,830,000 |
| — | 2·0 | ·099 | 2953 | — | 2,150,000 |
| — | 2·5 | ·103 | 1912 | — | 1,990,000 |
| — | 2·5 | ·119 | 2077 | — | 1,570,000 |
| — | 3·0 | ·098 | 1160 | — | 1,950,000 |
| — | 3·5 | ·094 | 953 | — | 2,390,000 |
| | | Mean | . . . . | | 1,980,000 |

The working safe external pressure for such tubes as these, taking a factor of safety of four, is

$$p_w = 460{,}000 \text{ to } 490{,}000 \frac{t^{2.5}}{d^2} \text{ lbs. per sq. in.}$$

There are very few tests of long riveted boiler flues and these are not very satisfactory. The following is a reduction of some:—

| $l$ | $d$ | $t$ | $p$ | $\frac{l}{d}$ | $C_1 = \frac{pd^2}{t^{2.5}}$ |
|---|---|---|---|---|---|
| 420 | 42 | ·375 | 97 | 10 | 1,552,000 |
| 300 | 42 | ·375 | 127 | 7 | 2,032,000 |
| 360 | 33·5 | ·34 | 99 | 11 | 1,256,000 |
| | | Mean | . . . . . | | 1,610,000 |

*Limits to the application of Eq.* 45.—Equations 44 and 45 give the same value for the collapsing pressure if

$$x = \frac{l}{d} = \frac{12{,}000{,}000}{1{,}800{,}000} = \text{about } 7$$

If the length is less than 7 $d$, Eq. 44 should be used, if greater Eq. 45. But also, if the thickness exceeds a certain limit, the crushing formula Eq. 39 should be used. Equations 39 and 45 give the same value of $p$, if

$$2f\frac{t}{d} = C_1 \frac{t^{2.25}}{d^2}$$

Taking $f = 64{,}000$ and $C_1 = 1{,}800{,}000$,

$$d = 28\, t^{1.25}$$

and if $d$ is less than this or $t/d$ greater Eq. 39 is to be used.

| | | | | | | |
|---|---|---|---|---|---|---|
| $t = \frac{1}{16}$ | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{2}$ | $\frac{5}{8}$ | ins. |
| $d = 0·88$ | 2·1 | 5 | 8 | 12 | 16 | ins. |
| $t/d = 0·07$ | ·06 | ·05 | ·045 | ·042 | ·040 | |

71. *Practical Calculations on Tubes Subjected to External Pressure.*—By introducing collapse rings, the length of riveted boiler flues can generally be brought within the limits to which the crushing formula, Eq. 41, applies. But considering the special risks of corrosion and heating to which such flues are exposed, the working pressure is usually not greater than

$$p_w = 4{,}000 \text{ to } 6{,}000\ (t/d) \quad . \quad . \quad . \quad (46)$$

For welded tubes the constant may be 8,000.

For tubes of length not less than ·188 $t^{1·25}$ or greater than about 7 diameters the collapse formula for tubes of moderate length, Eq. 44 may be used. For tubes of length greater than 7 diameters and diameter greater than 28 $t^{1·25}$, the long tube collapse formula, Eq. 45, may be used. The factor of safety should be six, if heating and corrosion are to be provided for.

72. *Corrugated Boiler Furnaces.*—Mr. Fox first introduced furnace tubes corrugated circumferentially, so that the stiffness against collapse is very greatly increased. Other furnaces with modified forms of corrugation have been introduced by Morison & Purves. The Board of Trade have made careful tests of such flue or furnace tubes, crushing them by external hydraulic pressure. Putting $t$ for the actual thickness of the plate and $d$ for the least outside diameter, the crushing pressure is given by the equation $p = c\,t/d$, where $c = 70{,}000$ to 80,000. The Board of Trade allows for the working pressure—

$$p = 14{,}000\ t/d$$

Lloyds rule for the working pressure is—

$$p = 20{,}000\ (t - 0·125)/d$$

the greatest thickness permitted being $\frac{5}{8}$ inch by the Board of Trade and $\frac{3}{4}$ inch by Lloyds.

*Collapse Rings.*—When a riveted boiler flue is so long and thin that collapse may occur, it may be greatly strengthened by rigid rings riveted to it at intervals. Fig. 45 shows some form of such collapse rings. At $a$ is the T-iron ring, first used by Sir W. Fairbairn; this is spaced off the flue by ferrules to prevent overheating of the plates. At $c$ is another mode of applying a T-iron ring. At $b$ is the collapse ring introduced by Mr. Adamson.

Mr. Adamson's ring is a solid forged ring $\frac{3}{8}$ to $\frac{1}{2}$ in. thick. The flue plates are flanged out and riveted to this on each side. At *d* is an angle-iron ring spaced from the flue by ferrules through which the rivets pass. The rivets may be 6 or 7 inches apart. The angle-iron ring is made in halves, and these are connected by wrappers at the joints. This form of collapse ring answers

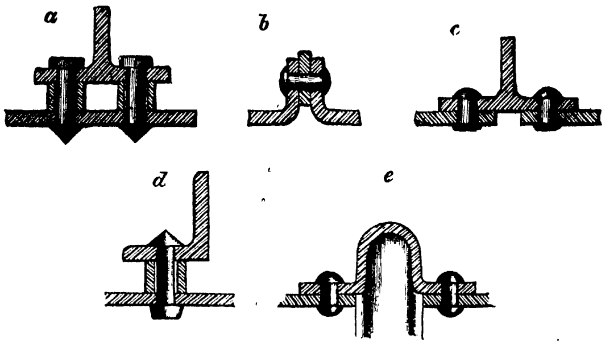



Fig. 45

well. At *e* is the Bowling collapse ring, which is rolled so as to be without a weld. It serves as an expansion joint also, springing a little if the flue expands more than the shell of the boiler. When collapse rings are applied at short intervals of about 8 feet, the effective length of the flue may be taken to be the length between two collapse rings.

### Compound Stress

73. When a bar is subjected to a simple tension or thrust, the safe limit of straining action is fixed by the intensity of direct stress on cross-sections normal to the direction of straining. In such cases there is a strain proportional to the stress, and the safe limit might be defined by fixing the amount of extension or compression per unit of length as well as by fixing the tension or pressure per unit of area. Further, in certain cases a bar stretched or compressed gives way at a section oblique to the axis and not at the section on which the direct stress is greatest. This suggests that the resistance to shearing determines the ultimate resistance, and a limit of shearing stress might be fixed as the limit of safety. But in cases of simple straining action this

would make no practical difference, because the stresses on sections normal and oblique to the axis have a fixed ratio.

When, however, cases of compound straining action are considered, the case is different. By the theorems of the ellipse or ellipsoid of stress and of strain, the greatest normal stress, the greatest shearing stress, and the greatest deformation due to any compound straining action within the limit of elasticity can be found ; but these no longer have a fixed ratio, as in cases of simple straining action, and it therefore comes to be of importance to determine on which of these quantities the safe limit of straining depends. Unfortunately, it is not at present known with certainty whether safety is secured in two different cases of combined straining action by fixing the limit of direct stress,

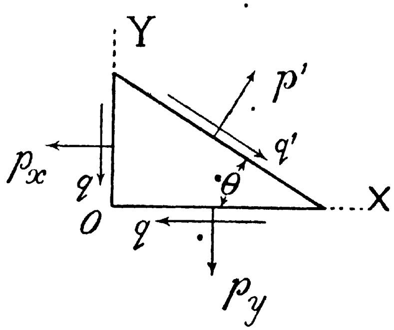



Fig. 46

of shearing stress, or of deformation. There is a tendency now to believe that in *ductile* materials safety depends on limiting the shearing stress. Ordinary tensile test bars of a ductile material such as mild steel give way, chiefly, by shearing at surfaces inclined at about 45° to the axis, forming cup-shaped fractures. It is only brittle materials such as cast iron which break in tension normally to the stress.

In the majority of cases in practice, the stresses are parallel to one plane or are uniplanar. Only such cases are here considered.

74. *Given the stresses on two planes at right angles to find the stresses on a third plane.*—Consider a triangular prism of material, fig. 46, and let the stresses on the faces at right angles be normal stresses $p_x$ and $p_y$, and shearing stresses $q$, which from the nature

of shearing stress must be of equal intensity. It is required to find the stress on a third plane making the angle $\theta$ with O X. Let $p'$ be the normal stress and $q'$ the shearing stress on the third plane.

$$\left.\begin{aligned} p' &= \frac{p_x + p_y}{2} + \frac{p_y - p_x}{2} \cos 2\theta + q \sin 2\theta \\ q' &= \frac{p_x - p_y}{2} \sin 2\theta + q \cos 2\theta \end{aligned}\right\} \quad . \quad . \quad (47)$$

If $p_x$ and $p_y$ are stresses of the same kind they are of the same sign, if different they are of opposite signs.

It can be shown that $p'$ has a maximum or minimum value for

$$\tan 2\theta = \frac{2q}{p_y - p_x}$$

an equation which gives two directions at right angles. The value of $p'$ is then

$$p_{\text{max. or min.}} = \frac{p_x + p_y}{2} \pm \tfrac{1}{2} \sqrt{\{4q^2 + (p_x - p_y)^2\}} \quad . \quad . \quad (48)$$

These maximum and minimum stresses are called *principal stresses.*

The maximum shearing stress is found on two planes at right angles, making angles of 45° with the planes on which the principal stresses act. These planes are given by the relation

$$\tan 2\theta = \frac{p_x - p_y}{2q}$$

$$q_{\text{max.}} = \pm \tfrac{1}{2} \sqrt{\{4q^2 + (p_x - p_y)^2\}} \quad . \quad . \quad . \quad . \quad (49)$$

A frequent case in practice is when one of the normal stresses $p_y = 0$. Then the principal stresses are

$$\left.\begin{aligned} p_1 &= \tfrac{1}{2}\{p_x + \sqrt{(4q^2 + p_x^2)}\} \\ p_2 &= \tfrac{1}{2}\{p_x - \sqrt{(4q^2 + p_x^2)}\} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (50)$$

75. *Principal Stresses.*—The principal stresses may be found more simply. It can be shown directly that it is always possible to reduce the normal and shearing stresses, on any two planes at right angles, to a pair of normal stresses on two planes at right angles on which there are no shearing stresses.

Consider the triangular prism A B C, fig. 47, having normal stresses $p_n$ $p'_n$ on the faces A B, B C, and equal shearing stresses $q$ on these faces. Let A C be a plane making the angle $\theta$ with

A B on which the stress is a wholly normal stress $p$. Resolving parallel to B C

$$p \text{ A C} \cos\theta = p_n \text{ A B} + q \text{ B C}$$

$$p - p_n = q \tan\theta \quad . \quad . \quad . \quad . \quad (a)$$

Similarly resolving parallel to A B—

$$p - p'_n = q \cot\theta \quad . \quad . \quad . \quad . \quad (b)$$

Hence,

$$p_n - p'_n = 2\,q \cot 2\,\theta$$

$$\tan 2\,\theta = \frac{2\,q}{p_n - p'_n}$$

which determines the position of the two planes at right angles

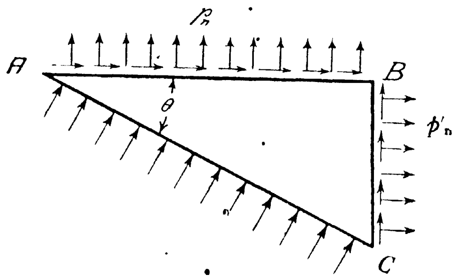



Fig. 47

on which the stresses are normal or principal stresses. Multiplying the two equations ($a$) and ($b$)

$$(p - p_n)\,(p - p'_n) = q^2$$

a quadratic the roots of which are the principal stresses $p_1$ and $p_2$.

In a large number of practical cases, there is one normal stress $p_n$ and a shearing stress $q$, and $p'_n = 0$. Then

$$\tan 2\,\theta = 2\,q/p_n$$

$$p\,(p - p_n) = q^2$$

$$p = \frac{p_n}{2} \pm \sqrt{(q^2 + \tfrac{1}{4}\,p_n^2)} \quad . \quad . \quad . \quad (51a)$$

The greater of the two values of $p$ is of the same sign as $p_n$. The greatest intensity of shearing stress is on planes inclined at 45° to the planes of principal stress, and the greatest shearing stress is

$$q_{max.} = \sqrt{(q^2 + \tfrac{1}{4}\,p_n^2)} \quad . \quad . \quad . \quad (51b)$$

## COMBINED BENDING AND TENSION OR THRUST

76. *Bending and tension or compression.*—Let a force P (fig. 48) act at a cross-section $a a'$ normally to the section and parallel to the axis of the bar, but at a distance $r$ from the centre of figure of the section. The conditions are the same as if a force P acted at the centre of figure and a couple P and $-$P, forming a couple of moment $M = P r$. Let the plane of the bending couple be a plane of symmetry of the section. Adding the normal stresses due to the two actions, the resultant normal stresses at the edges of the section are—

$$f = \frac{P}{A} \pm \frac{M}{Z} \quad . \quad . \quad . \quad . \quad . \quad (52)$$

where A is the area of the section and Z its modulus, about an axis through the centre of figure and perpendicular to the plane of bending. The greater stress is of the same kind as P, the other

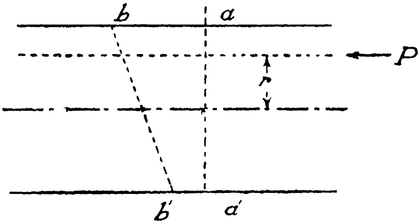



Fig. 48

may be of the same kind or not according to the magnitude of $r$.

The greatest working stress in tension or compression is given in Table II, § 39.

The distribution of normal stress is shown by the trapezium $a\,a'\,b\,b'$ where $a\,b$ and $a'\,b'$ are the two stresses at the edges.

For circular sections, diameter $d$—

$$f = \frac{4\,P}{\pi\,d^2}\left(1 \pm \frac{8r}{d}\right) \quad . \quad . \quad . \quad . \quad (53a)$$

For rectangular sections, $h$ in the plane of bending and $b$ at right angles—

$$f = \frac{P}{bh}\left(1 \pm \frac{6r}{h}\right) \quad . \quad . \quad . \quad . \quad (53b)$$

For a rectangular section the smaller value of $f$ is zero, if $r = h/6$, and is negative if this ratio is less. Some special cases may be considered.

*Case I.*—Let the bar be loaded as in fig. 49, the greatest stress is at the point of support. The equation 53*a* above gives for a circular section—

$$d^3 - \frac{4\,P\,d}{\pi f} - \frac{32\,P\,r}{\pi f} = 0$$

And for a rectangular section, Eq. 53*b* gives—

$$h^3 - \frac{P\,h}{n\,f} - \frac{6\,P\,r}{n\,f} = 0$$

where $n = b/h$.

These cubic equations are of the form

$$x^3 - M\,x - N = 0$$

It is generally possible from experience or from inspection

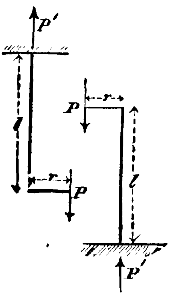


Fig. 49

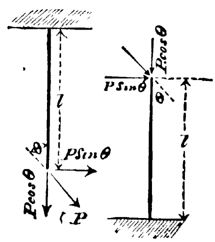


Fig. 50

of the numerical values of $M\,x$ and $N$ to assume an approximate root $a$ of this equation. Then a more approximate value is—

$$a - \frac{a^3 - M\,a - N}{3\,a^2 - M}$$

and the approximation may be repeated if necessary.

*Case II.*—The force P acts obliquely (fig. 50) at the end of the bar in a plane of symmetry of the section at the support. Let $l$ be the distance of the point where P cuts the axis from the point of support. Resolve P into $P \cos\theta$ along the axis producing a simple tension or compression, and $P \sin\theta$ producing bending, the bending moment at the support being $P\,l \sin\theta$. The greatest stress is—

$$f = \left(\frac{\cos\theta}{A} + \frac{l \sin\theta}{Z}\right) P.$$

## Combined Twisting and Bending

77. *Twisting and bending or wrenching.*—Let the loading force P act in a plane normal to the axis of the bar, fig. 51, parallel to an axis of symmetry of the section at which the stress is required. Let $r$ be the distance from the axis and $l$ that from the section. P produces an opposite reaction $P_1$. It will not alter the conditions if opposite forces $P', -P_1'$ are introduced equal and parallel to P. Then the action of P is equivalent to that of a twisting couple $P\,P'$, and a bending couple $P_1\,P_1'$. The twisting moment is $T = Pr$ and the bending moment is $M = Pl$.

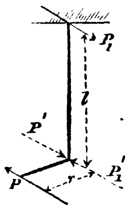


Fig. 51

The normal stress due to bending is for a circular section of diameter $d$—

$$f_n = \frac{32}{\pi d^3} M$$

and the shearing stress due to torsion is—

$$f_s = \frac{16}{\pi d^3} T$$

When these are combined, as in Eq. 51*a*, the maximum or minimum normal stress is—

$$p = \tfrac{1}{2} f_n \pm \tfrac{1}{2} \sqrt{(4 f_s^2 + f_n^2)} \quad . \quad . \quad . \quad . \quad (54)$$

and the maximum shearing stress from Eq. 51*b* is—

$$q = \tfrac{1}{2} \sqrt{(4 f_s^2 + f_n^2)} \quad . \quad . \quad . \quad . \quad (55)$$

*Equivalent Bending and Twisting Moments.*—It is convenient for purposes of calculation to reduce the moments M and T to a single bending or twisting moment, equivalent to the two in this sense, that it is a moment which would produce the same maximum stress. Inserting in these equations the values of $f_s$ and $f_n$ given above—

$$p = \frac{16}{\pi d^3} \left\{ M \pm \sqrt{(M^2 + T^2)} \right\}$$

Comparing this result with the expressions for the stress due to a simple bending or twisting moment it can be seen that

$$M_e = \tfrac{1}{2} M + \tfrac{1}{2} \sqrt{(M^2 + T^2)} \quad . \quad . \quad (56a)$$

is an equivalent bending moment and

$$T_e = M + \sqrt{(M^2 + T^2)} \quad . \quad . \quad . \quad (56b)$$

an equivalent twisting moment which would produce the same

maximum *normal* stress as the actual moments M and T. It is these equations which have hitherto been generally used in calculating the effect of combined bending and twisting moments, and in applying them in determining dimensions, the safe working stress has been taken at such values as are given for tension in Table II, § 39. In that case the two equations give the same result.

On the continent it has been usual to consider the safety of a bar subjected to a wrenching moment to depend on the greatest tensile strain or stretch. Then if Poisson's ratio $=0{\cdot}25$, and for a circular section,

$$M_e = \tfrac{3}{8}\, M + \tfrac{5}{8} \sqrt{(M^2 + T^2)} \qquad . \quad . \quad (56c)$$

78. *Shearing stress considered.*—Experiments by Mr. J. J. Guest (Proc. Phys. Soc. XVII.) and by Mr. Scoble (Phil. Mag. 6th ser., v. xii, 1906) on ductile material subjected to combined tension and shear appear to show that failure begins when the shearing stress attains a definite value.

In Mr. Scoble's tests the point at which yielding commenced was noted. The following are the results. In Group I, bending only or twisting only was applied till yielding occurred. In Group II, a fixed bending moment was applied and then twisting gradually increased up to yielding. In Group III, a fixed twisting moment was applied, and bending increased up to yielding. The bars were mild steel bars $\frac{3}{4}$ inch in diameter and 36 inches long. The principal normal stresses and the greatest shearing stress have been calculated for each of the cases.

*Experiments on Combined Bending and Twisting (Scoble)*

Inch pound units

| Group | M | T | Tension due to M | Shear due to T | Principal Stresses Maximum | Principal Stresses Minimum | Maximum Shearing Stress |
|---|---|---|---|---|---|---|---|
| I | 2,660 | 0 | 64,600 | 0 | 64,600 | 0 | 32,300 |
| | 0 | 2,400 | 0 | 29,170 | 29,170 | 29,170 | 29,170 |
| II | 667 | 2,280 | 16,220 | 28,250 | 37,500 | −21,300 | 29,400 |
| | 1,331 | 2,120 | 32,350 | 25,750 | 48,200 | −15,800 | 32,000 |
| | 2,000 | 1,899 | 48,600 | 23,050 | 57,800 | −9,200 | 33,500 |
| | 2,420 | 1,171 | 58,750 | 14,240 | 61,980 | −3,229 | 32,600 |
| | 2,000 | 1,720 | 48,600 | 20,900 | 56,740 | −8,140 | 32,440 |
| III | 2,558 | 645 | 62,100 | 7,840 | 63,080 | −980 | 32,030 |
| | 2,310 | 1,335 | 56,100 | 16,220 | 60,450 | −4,350 | 32,400 |
| | 1,454 | 2,033 | 35,330 | 24,700 | 48,060 | −12,740 | 30,400 |

While the maximum tension at which yielding occurs varies considerably the maximum shearing stress in the different tests is nearly constant. The matter requires further investigation, but it appears that in mild steel safety depends on the maximum shearing stress due to the combined moments.

Inserting in the value of the greatest shearing stress, Eq. 55, the values of $f_s$ and $f_n$ found before for a circular section—

$$q = \frac{16}{\pi d^3} \sqrt{(M^2 + T^2)}$$

The equivalent bending or twisting moment which would produce this shearing stress is—

$$M_e = T_e = \sqrt{(M^2 + T^2)} \quad . \quad . \quad (57)$$

In applying this equation the safe working stress should be the working limit of stress in shear. But the yield point in shear is badly determined. It appears to be about half the yield point in tension. Further, the working stress in shear in cases of combined stress is not well known from experience. Hence, pending further investigation it would appear to be best at present, if $M > T$, to calculate the normal stress due to bending from $M_e$, and limit the stress to the value of working stress in tension. But if $T > M$, to calculate the shearing stress from $T_e$, and to limit the working stress to the value for shear. (See Table II, § 39.)

### Strength of Flat Plates

79. The theory of flat plates supported or fixed at the edges is not very satisfactory. Assumptions have to be made as to the dangerous section or the position of the reactions at the supports. The formulæ usually relied on are due to Grashof and Bach.

Let $t$ be the thickness of plate in inches.

$f$ greatest normal stress in lbs. per sq. in.

$E$ the coefficient of elasticity.

*Case I*—A circular plate of radius $r$ is supported at the edges, fig. 52, and uniformly loaded with $p$ lbs. per sq. in. In this case the theoretical formula is well established.

$$f = p \left(\frac{r}{t}\right)^2$$

In most cases in practice the plate is more or less fixed at the edges (fig. 53). For such cases, let—

$$f = kp\left(\frac{r}{t}\right)^2$$

Then according to Bach—

For cast iron, more or less completely fixed at the edges, $k = 0{\cdot}8$ to $1{\cdot}2$. For mild steel fixed at the edge, $k = 0{\cdot}5$. For mild steel supported at the edge, $k = 0{\cdot}75$.

*Case II.*—Circular plate of radius $r$, supported at the edge and loaded with P lbs. distributed over a circle of radius $r_0$ (fig. 54).

$$f = k\left(1 - \frac{2}{3}\frac{r_0}{r}\right)\frac{P}{t^2}.$$

For cast iron Bach gives $k = 1{\cdot}43$. As $r_0$ is diminished $f$ tends to the value $k\,P/t^2$.

*Case III. Stayed surface.*—An indefinitely large plate is uniformly loaded with $p$ lbs. per sq. in. and supported by stays

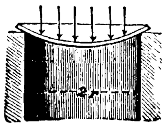



Fig 52

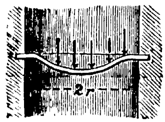



Fig. 53

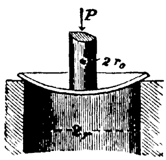



Fig. 54

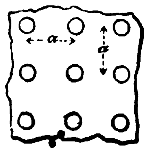



Fig. 55

at the corners of square areas. Distance apart of stays centre to centre $= a$ (fig. 55). For each single area—

$$f = 0{\cdot}228\,\frac{a^2}{t^2}\,p$$

*Case IV.*—*Rectangular plate* of sides $a$ and $b$, supported at the edges, and uniformly loaded with $p$ lbs. per sq. inch—

$$f = 0{\cdot}5\,k\,\frac{a^2 b^2}{a^2 + b^2}\,\frac{p}{t^2}$$

For a square plate, $b = a$—

$$f = 0{\cdot}25\,k\,\frac{a^2}{t^2}\,p$$

For cast iron $k = 0{\cdot}75$ to $1{\cdot}13$.

*Case V.—Rectangular plate* supported at the edges and loaded at the centre with P lbs.

$$f = 1{\cdot}5\,k\,\frac{a\,b}{a^2 + b^2}\,\frac{P}{t^2}$$

For cast iron $k = 1{\cdot}75$ to $2{\cdot}0$

The Board of Trade rule for the flat stayed surfaces of marine boilers is as follows :—

Let $p$ be the safe working pressure in lbs. per sq. in. ; $t$ the thickness of the plate in inches ; $s$ the area of surface supported by one stay in sq. ins.—

$$p = \frac{c\,(16\,t + 1)^2}{s - 6}$$

The constant $c$ has the following values :—

| | Plates not exposed to fire | Plates with steam on one side and fire on the other | Plates with water on one side and fire on the other |
|---|---|---|---|
| Stays with nuts and washers three times the diameter of the stay, and $\frac{2}{3}$rds the plate thickness | 100 | 60 | — |
| Stays with nuts only | 90 | 54 | — |
| Stays screwed into plate and nutted | — | — | 80 |
| Stays screwed into plate and riveted | — | 36 | 60 |

Sometimes stays give great trouble by breaking after being in use four or five years. The cause is fatigue due to continual bending, induced by the expansion and contraction of the more heated of the two connected plates. The liability to this is lessened by making the water space wider, or by reducing the diameter of the stays and using a greater number.

# CHAPTER IV

## ON FASTENINGS

### Riveted Joints

80. The simplest fastening is the rivet, employed to unite wrought iron, soft steel or copper plates. A rivet is virtually a bolt, with the head, body and nut in one piece. It is a permanent fastening, only removable by chipping off the head. Bolts are generally used with the straining force parallel to the axis, so that the bolt is in tension; but rivets are almost always placed at right angles to the straining force, so as to be in shear. They are not reliable in tension.

A rivet is formed of round bar, and, when ready for use, has the form shown in fig. 56. It is parallel for about half its length, and very slightly tapers for the remainder. The head is cup-shaped, or, more often, pan-shaped, as shown.

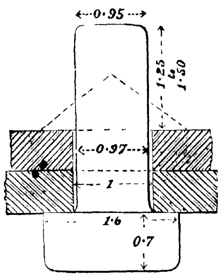


Fig. 56

Rivets are made in rivet-making machines of various kinds, being pressed, while red hot, in suitable dies. When used, the rivets are again heated to red heat, placed in the rivet hole in the plates to be connected, and then the second head is formed by hand, or by machine. In hand riveting, the tail of the rivet is held up, while the head is formed by two riveters working with hammers, and the head is either made conical by the hammers alone or finished by the aid of a cup-shaped die, called a snap. In machine riveting, the rivet is pressed between two dies, actuated by a lever, or by steam or hydraulic pressure. In machine riveting it is of importance that the plates should be well bolted together during riveting, or collars are formed on the

rivet between the plates. Machine riveting causes the rivet to fill up the holes more perfectly than hand riveting, and makes a closer and stronger joint, especially if the plates are thick. The pressure on the rivet should not exceed 100 or 120 tons per square inch of rivet section, or the lateral pressure of the rivet may crack the plate. Steel of 25 to 30 tons tenacity and 25 per cent. elongation in eight diameters is now generally used for rivets. Small rivets can be closed cold.

*Punching and drilling rivet holes.*—Rivet holes are most commonly made by punching. This somewhat rough process is objectionable on two grounds. The spacing of the rivet holes is not perfectly accurate, so that when two plates are brought together the holes are not in perfect register. Next the metal round the hole is injured by lateral flow of the metal under the pressure of the punch. The injury due to punching may be reduced, if the punch has a slanting or spiral face, so that the metal is sheared progressively.

The intensity of the pressure on the punch increases as the plates are thicker, and hence the pressure of fluidity of the plates is more likely to be reached and the consequent lateral flow is more likely to occur the thicker the plates which are punched. The injury done by punching to the material may, however, be entirely removed in two ways. If the hole is punched $\frac{1}{16}$ inch smaller than is required and rymered out to size, the strained material is removed and the plate is then found to be uninjured. Or, if the plate is annealed after punching, the injury is obviated. On the whole, steel plates, especially thick steel plates, have been found to be more injured by punching than wrought-iron plates. Plates less than $\frac{1}{2}$-inch thick are generally punched, and are not annealed after punching. Plates more than $\frac{1}{2}$-inch thick should be annealed after punching, or the holes punched small and rymered out. Plates of an inch or more in thickness are better drilled. In the best boiler work all holes are drilled, and most are drilled with the plates in place.

The hole made by punching is slightly conical because the diameter of the hole in the die block or bolster is slightly larger than the punch. The hole in die block is about equal to the diameter of rivet + one-fifth thickness of plate. Sometimes this conicity is entirely removed by rymering out the holes before riveting. If this rymering is done after the plates to be riveted are brought together, it insures the perfect agreement of the

corresponding holes. In drilling, the metal is removed by a cutting tool, and the plate round the hole is uninjured, also the holes are more accurate in size and spacing. But the sharp edge of a drilled hole is slightly unfavourable to the resistance of the rivet.

Sometimes the arris at the edge of rivet holes is removed by a countersink tool, and occasionally the countersink is of sensible depth, as in fig. 59. Riveting of this kind is sometimes used in shipbuilding, and it has the advantage that the rivet head is less likely to break off.

When the riveting is done at red heat, the contraction of the rivet in cooling nips the plates powerfully, and causes considerable tension on the rivet. In very long rivets, this may cause fracture of the rivet, and to prevent this the tail end is cooled before placing it in the rivet hole. In ordinary riveting, the contraction is advantageous in securing staunchness of the joint. Further, the contraction creates frictional resistance to

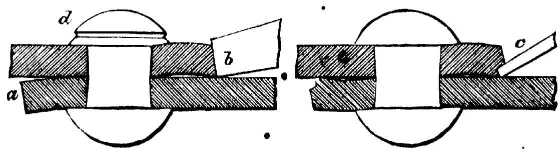



Fig. 57

slipping between the plates, which enables the joint to sustain a considerable force, even when the rivets do not fit the holes.

If the thickness of the plates, through which a rivet passes, is 6 inches or more, it is better to use bolts instead of rivets.

The staunchness of the joint, or its power of resisting the tendency to leak, when subjected to steam or water pressure, depends on the nearness of the rivets to the edge of the plate, and their nearness together. The metal between two rivets is in the position of a beam subjected to uniform pressure, and tending to spring or deflect. If the joint is not naturally staunch, it may be rendered so by caulking, that is, burring down a narrow strip at the edge of the plate by a chisel. In fig. 57 the condition of the plates before caulking is shown at *a*. At *b* is a fullering tool used to close up the plates ; at *c*, a caulking tool used to burr down the edge of the plate. A caulking tool is sometimes not used, but only a fullering tool like *b*, but having a small projection, which indents the plate at the middle of its

thickness. In the best boiler work the plates are planed on the edges with a slight bevel, before riveting, and this much facilitates the closing of the joints by fullering or caulking. It is a point of importance in boiler work to arrange the joints so that they can be caulked. The rivet heads are sometimes caulked also, as shown at *d*, fig. 57.

It has been found that the magnetic oxide on the surface of plates as they come from the rolling mill diminishes the staunch-

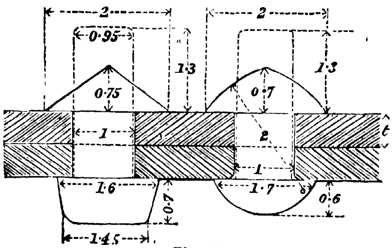



Fig. 58

ness of the joint. To obviate this, the surfaces of the joint are sponged with a solution of sal ammoniac before riveting. The plates then adhere more closely and require less caulking.

81. *Forms of rivets.*—Fig. 58 gives the proportions of the rivets commonly used in hand-riveting, the heads formed by

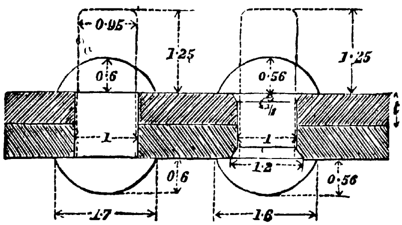



Fig. 59

the riveter being of conical form. Fig. 59 gives proportions for the rivets generally used in machine riveting. In boiler work the rivet head is rather larger than for girder work. Fig. 60 shows a countersunk rivet, which is only used when the surface of the plate must be fair and without projections. Countersunk rivets weaken the plate more, and are less reliable than ordinary rivets. The proportions of the head vary from 1·75 *d* in diameter

and 0·75 $d$ in height to 1·6 $d$ in diameter and 0·6 $d$ in height. The conical heads shown in fig. 58 are formed entirely by hand hammers, and are not finished with a snap. They are most used where there is restricted space for hammering. The cup-shaped head may be formed in hand riveting by a die or snap, which requires the use of a sledge hammer.

The numbers on the figures are proportional to the diameter of the rivet, and give good ordinary proportions, although it must be remembered that the sizes used by different engineers vary more or less. To fill the rivet hole and form the head, a length equal to about ⅘ of the diameter is required in countersunk riveting, and 1·3 to 1·7 times the diameter in ordinary riveting.

*Pitch, margin, and overlap.*—The distance from centre to centre of two rivets in the same row is termed the pitch. The distance from edge of rivet hole to edge of plate is called the margin. The distance between the edges of two plates which overlap is termed the overlap.

*Lap and butt riveting.*—When one plate is made to overlap the other, and one or more lines of rivets are put through the two, the riveting is lap riveting. When the plates are kept in the same plane, and a cover plate, or butt strap, is put over the joint and riveted to each, the riveting is butt riveting. In boiler work a cover is placed on each side of the joint.

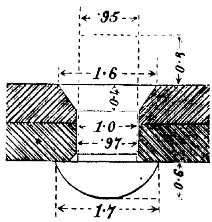


Fig. 60

*Single and double riveting.*—If there is one line of rivets in lap riveting, or one line on each side of the joint in butt riveting, the joint is single riveted. If there are two lines in lap, or two lines on each side of the joint in butt riveting, the joint is double riveted. Treble and quadruple riveting are also used.

*Single and double shear joints.*—When the plates are so arranged that they tend to shear the rivets in a single plane, the joint is a single shear joint. If the plates tend to shear the rivet in two or more planes, the joint is a double or multiple shear joint. In calculating the shearing area of the rivets, one section of each rivet is taken in single shear joints and two sections of each rivet in double shear joints.

*Combined lap and butt joint.*—A form of joint intermediate

between a lap and a butt joint has been used for locomotive boilers. It is shown in fig. 61. It consists of a lap joint with a cover plate outside the joint. There are three rows of rivets, the middle row having twice as many rivets as the outside rows. The ordinary proportions for $\frac{3}{8}$-inch plates are: rivets $\frac{13}{16}$ inch diameter; pitch of middle row, 2 inches; pitch of outside rows, 4 inches.

82. *Size of rivets for plates of different thicknesses.*—In plates with punched holes the operation of punching fixes a minimum rivet diameter. In plates with drilled holes it is not so, but practically about the same sizes of rivets are most convenient. Let $t$ = thickness of plate, $d$ = diameter of rivet, $f_s$ = resistance of plate to shearing, $f_c$ = resistance of punch to crushing. Then

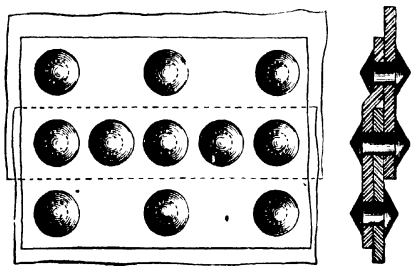


Fig 61

the area sheared in punching a rivet hole is $\pi\, d\, t$, and the area of punch is $\pi\, d^2/4$. Hence the punch will crush if

$$\pi\, d\, t f_s > \pi\, d^2 f_c/4$$

If $f_c = 4 f_s$ the diameter $d$ of the rivet must not be less than $t$, the thickness of the plate.

In practice the rivet diameter ranges from

$$d = 1{\cdot}2\sqrt{t} \text{ to } 1{\cdot}4\sqrt{t} \quad . \quad . \quad . \quad . \quad (1)$$

The larger size being preferable for steel and single riveted joints, the smaller for iron and for multiple riveted joints. In practice, the real rivet diameter when closed in the joint is about 4 per cent. greater than its nominal diameter. This may be allowed for, however, in choosing the working shearing stress.

*Diameter of Rivets for different thicknesses of Plates*

| Thickness of plates in inches, $t$ | Diameter of rivets $d = 1{\cdot}2\sqrt{t}$ | | Diameter of rivets $d = 1{\cdot}4\sqrt{t}$ | |
|---|---|---|---|---|
| | Nominal | Actual when closed | Nominal | Actual when closed |
| $\frac{1}{4}$ | $\frac{19}{32}$ | 0·62 | $\frac{11}{16}$ | 0·71 |
| $\frac{5}{16}$ | $\frac{11}{16}$ | 0·71 | $\frac{3}{4}$ | 0·78 |
| $\frac{3}{8}$ | $\frac{3}{4}$ | 0·78 | $\frac{7}{8}$ | 0·91 |
| $\frac{7}{16}$ | $\frac{13}{16}$ | 0·84 | $\frac{7}{8}$ | 0·91 |
| $\frac{1}{2}$ | $\frac{7}{8}$ | 0·91 | 1 | 1·04 |
| $\frac{9}{16}$ | $\frac{7}{8}$ | 0·91 | $1\frac{1}{16}$ | 1·10 |
| $\frac{5}{8}$ | $\frac{15}{16}$ | 0·97 | $1\frac{1}{8}$ | 1·16 |
| $\frac{3}{4}$ | $1\frac{1}{16}$ | 1·10 | $1\frac{3}{16}$ | 1·23 |
| $\frac{7}{8}$ | $1\frac{1}{8}$ | 1·17 | $1\frac{5}{16}$ | 1·36 |
| 1 | $1\frac{1}{4}$ | 1·30 | $1\frac{3}{8}$ | 1·42 |
| $1\frac{1}{8}$ | $1\frac{1}{4}$ | 1·30 | $1\frac{1}{2}$ | 1·56 |
| $1\frac{1}{4}$ | $1\frac{5}{16}$ | 1·36 | $1\frac{9}{16}$ | 1·60 |
| $1\frac{3}{8}$ | $1\frac{13}{32}$ | 1·47 | $1\frac{5}{8}$ | 1·69 |
| $1\frac{1}{2}$ | $1\frac{1}{2}$ | 1·57 | $1\frac{3}{4}$ | 1·81 |

## General Arrangement of Riveted Joints

83. *Overlap of plates and minimum pitch of rivets.*—If a rivet hole is too near the edge of the plate, or if two rivet holes are too near together, the plate may be cracked in punching, or in closing the rivets. A practical limit is thus fixed for the minimum overlap and pitch. Experience shows that the distance from rivet hole to edge of plate or margin and the distance between the edges of rivet holes should in no case be less than the diameter of the rivet. We thus get the proportions in fig. 62, as the *minimum* proportions of overlap and pitch. The unit is the diameter of the rivet. Frequently these proportions are exceeded. The pitch is determined by considerations of strength, and for convenience in forming the rivet heads is made not less than $2\frac{1}{4}\,d$ to $2\frac{1}{2}\,d$. In a double riveted joint, the minimum possible diagonal pitch is $2\,d$, which makes the distance between two rows of rivets $1{\cdot}4\,d$. But it is in practice usually 0·6 to 0·8 times the pitch.

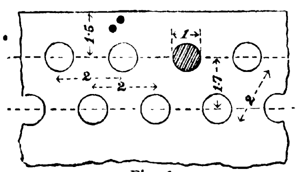
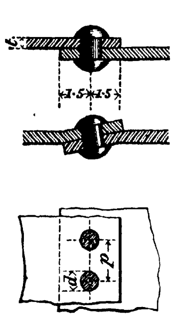


Fig. 62

84. *Proportions of various riveted joints.*—Fig. 63 shows an ordinary single riveted lap joint, and fig. 64 a similar butt joint.

The pitch $p$ is decided by the rules given below. The other dimensions are given by the proportional figures, the unit being the diameter of the rivet. An objection to a lap joint is that the straining force in one plate is not directly opposed to that in



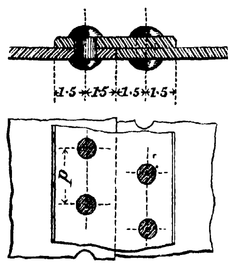
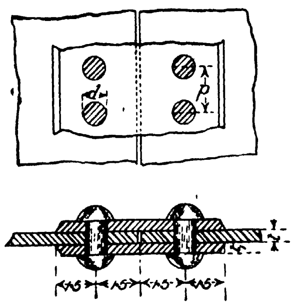




Fig. 63 Unit $= d$ Fig. 64

the other, but forms with it a couple tending to bend the joint each time the stress is applied, and rendering it weaker in consequence of the bending. Grooving of boiler plates is in some cases indirectly due to this bending. One of the sections in fig. 63 shows the plates bent before riveting so as to diminish




Unit $= d$

Fig. 65

the tendency of the joint to deform when strained. The butt joint, fig. 64, with a single butt-strap is subject to nearly the same action of bending as the lap joint.

The double shear butt joint with two cover plates (fig. 65) is free from bending action, and consequently is stronger. Butt

joints are preferable to lap joints for the longitudinal seams of boilers, and in that case should have double cover plates

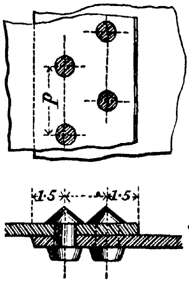



Fig. 66

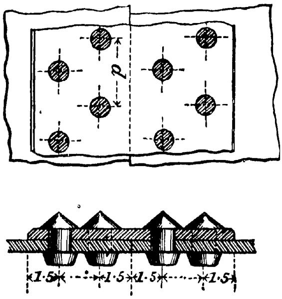



Fig. 67

If $t_1$ is the thickness of the cover plates in butt joints and $t$ the thickness of the plates, then in single cover joints $t_1=1\frac{1}{8}\,t$, and in double cover joints $t_1=\frac{5}{8}$ to $\frac{3}{4}\,t$.

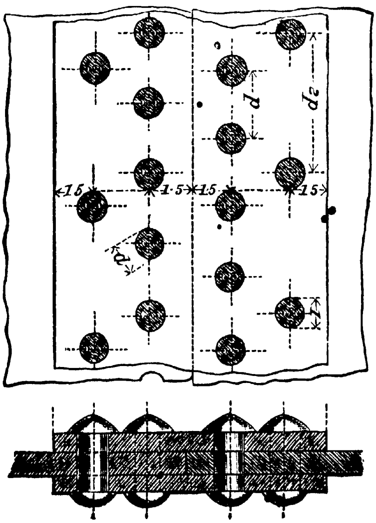



Fig. 68

Figs. 66 and 67 show a double riveted lap joint and a butt joint with single cover plate. The rivets are usually placed

zigzag, but sometimes directly behind each other, and then the riveting is called chain riveting. The distance between the rows of rivets should not be less than 0·6 $p$ in diagonal riveting, or 0·8 $p$ in chain riveting.

Fig. 68 shows a form of double riveted butt joint with double cover plates. The pitch of the rivets in the outer rows is double that in the inner row.

A special form of double riveted joint is shown in fig. 69, and a similar treble riveted joint in fig. 70. In these joints

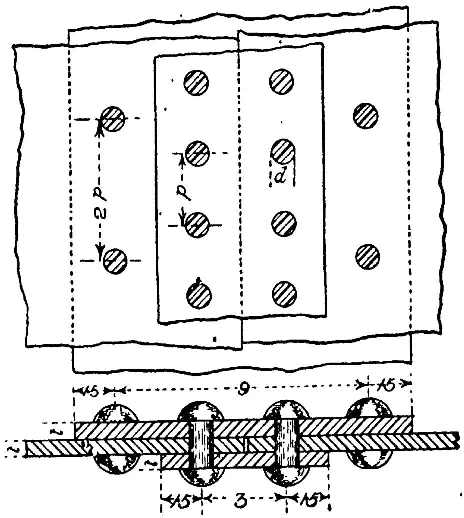



Fig. 69

part of the rivets are in single and part in double shear. The outside butt-strap is narrower than the inside strap.

Fig. 71 shows one form of triple riveted lap joint. Another form is shown in fig. 72, where the outer rows of rivets have double the pitch of the inner rows. The plate is less weakened than in the ordinary form of joint.

## Stresses in Riveted Joints when Tested to Destruction

85. A very large number of experiments have been made on the ultimate strength of riveted joints, with a view of deter-

mining the proportions which make the breaking strength greatest. In England this basis of design is generally adopted.

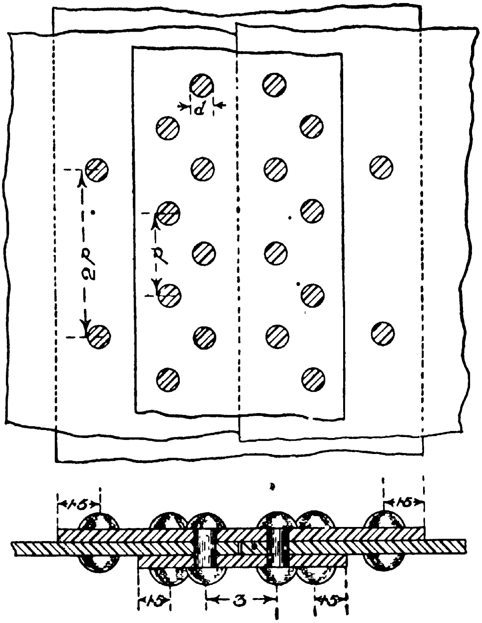



Fig 70

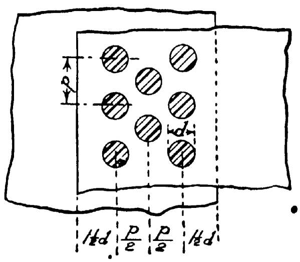



Fig. 71

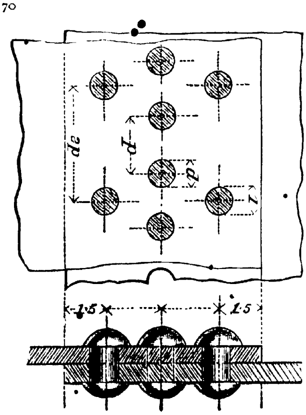



Fig. 72

In this section the straining action is expressed in tons and the stresses in tons per square inch. Dimensions are all in inches.

Let $d$ = diameter of rivets.
$p$ = pitch of rivets.
$t$ = thickness of plates.
$l$ = semi-overlap, or distance from centre of rivet to edge of plate.
$m$ = margin.
$f_t$ = tenacity of material of plates.
$f_b$ = resistance of plate or rivet to plastic flow, under the crushing pressure between rivet and plate, reckoned per sq. in. of projected area of rivet.
$f_s$ = shearing resistance of rivets.
T = resistance of a strip of the joint of width $p$.

*Modes of fracture of riveted joints.*—Consider, for simplicity, a simple single riveted lap joint, subjected to tension. The

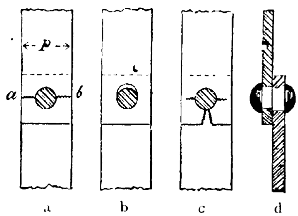



Fig. 73

plates may tear ($a$ $b$, fig. 73, a); the rivet may shear (fig. 73, d); the rivet hole may be enlarged by plastic flow (fig. 73, b); or, lastly, the plate may break across in front of the rivet (fig. 73, c). Confining attention to a strip of the joint of width $p$—that is, the width corresponding to one rivet—a longitudinal straining action T acts on the strip, producing a tension in each plate, and is transmitted from one to the other through the rivet.

(1) Either plate may give way by tearing. The section is $(p-d)\,t$. The tensile stress is $f_t = T/(p-d)\,t$.

(2) The rivet is in single shear and its section is $0{\cdot}785\ d^2$. The average shearing stress is $f_s = T/(0{\cdot}785\ d^2)$.

(3) The projected area of a rivet is $d\,t$. The crushing or bearing stress between rivet and plate tending to indent the plate is $f_b = T/d\,t$.

(4) The plate in front of the rivet may break across. But experience shows that this does not occur if the margin $m = d$.

86. *Bearing resistance of rivets.*—In testing joints to destruction in the testing machine, it is found that if the pressure between the plate and rivet exceeds a certain value, the metal of plate and rivet yields plastically or flows, and the rivet holes become oval, before the breaking strength of rivet or plate is fully exerted. When this markedly occurs the joint always gives way with a lower tearing or shearing stress than joints in which the crushing pressure is less. It is necessary, therefore, in some cases, chiefly in the joints of riveted structures, like bridges, to attend to the bearing pressure of the rivet on the plate, and to limit it to an intensity at which plastic flow does not occur. From experiments on indentation it is known that the resistance to indentation of a plastic material does not much depend on the form of the indenting body, but only on its projected area normal to the direction of indentation. Hence it is not an arbitrary rule, but one based on experiment, to take the resistance to the indentation of a plate of thickness $t$ by a rivet of diameter $d$ to be proportional to the projected area $d\,t$. If $f_b$ is the limit of bearing pressure at which plastic deformation begins, and $f_s$ is the resistance of the rivet to shearing, per square inch, then for safety against deformation of the rivet or plate, for a rivet in single shear—

$$f_b\, d\, t = \text{or} > \frac{\pi}{4} d^2 f_s$$

$$d = \text{or} < 1{\cdot}27\, t \frac{f_b}{f_s} \quad . \quad . \quad . \quad (2)$$

Similarly, for a rivet in double shear—

$$d = \text{or} < 0{\cdot}635\, t \frac{f_b}{f_s} \quad . \quad . \quad . \quad (2a)$$

From the results of experiments on the fracture of riveted joints, it appears that probable values of the limit at which deformation of the joint begins may be fixed as follows:

| | Stress in tons per sq. in. $f_s$ | $f_b$ | $f_b/f_s$ |
|---|---|---|---|
| Single shear, iron rivets and plates | 18·0 | 30·0 | 1·06 |
| " " steel " " | 23·0 | 40·0 | 1·74 |
| Double shear, iron " " | 16·0 | 36·0 | 2·25 |
| " " steel " " | 20·0 | 50·0 | 2·50 |

Putting these values in the equations above, for plate thickness $t$ the greatest permissible size of rivet is :

| | $d$ not greater than single shear | $d$ not greater than double shear |
|---|---|---|
| Iron rivets and plates . . . . . | $2{\cdot}10\,t$ | $1{\cdot}43\,t$ |
| Steel ,, ,, . . . . . . . | $2{\cdot}20\,t$ | $1{\cdot}59\,t$ |

With the ordinary proportion of rivet diameter to plate thickness, it is only in some joints in double shear that the bearing pressure needs to be attended to. In bridge work the safe limit of bearing pressure $f_b$ is by some engineers taken to be $7\frac{1}{2}$ to 10 tons per sq. in.

87. *General relation of shearing and tearing strength of riveted joints in tension.*—When tested to destruction, a riveted joint

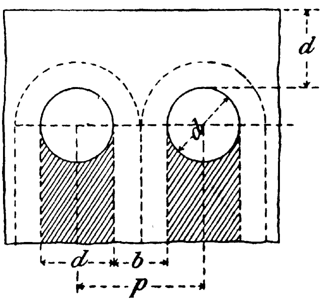



Fig. 74

usually gives way by shearing the rivets or by tearing the plate along a section of minimum resistance through the rivet holes. Consider a single riveted joint (fig. 74). To each rivet in shear corresponds a strip of plate of width $b = p - d$, which is in tension. The shaded areas of the plate are superfluous as regards resistance to tearing, and because there are such superfluous areas the joint is weaker than the solid plate. The pressure between rivet and plate may be considered to be taken up by a strip of plate of width $b/2$, passing round the rivet. Let $f_s$ be the resistance of the rivet to shearing, and $f_t$ the resistance of the plate to tearing, in tons per sq. in. of section, and let $d$ be the diameter of rivet and $t$ the thickness of plate. The joint will be most economically arranged, if the tearing and shearing

resistances are equal—that is, for rivets in single shear—when

$$\frac{\pi}{4} d^2 f_s = b\, t f_t$$

$$b = 0{\cdot}785 \frac{d^2 f_s}{t f_t}$$

$$\therefore\ p = 0{\cdot}785 \frac{d^2}{t} \frac{f_s}{f_t} + d$$

Similarly, if the single riveted joint has rivets in double shear, as in a butt joint with double covers—

$$\frac{\pi}{2} d^2 f_s = b\, t f_t$$

$$b = 1{\cdot}57 \frac{d^2}{t} \frac{f_s}{f_t}$$

$$p = 1{\cdot}57 \frac{d^2}{t} \frac{f_s}{f_t} + d\,.$$

For a double riveted single shear joint (fig. 75), to each pair of rivets in shear corresponds a width of plate $2b = p - d$. Clearly

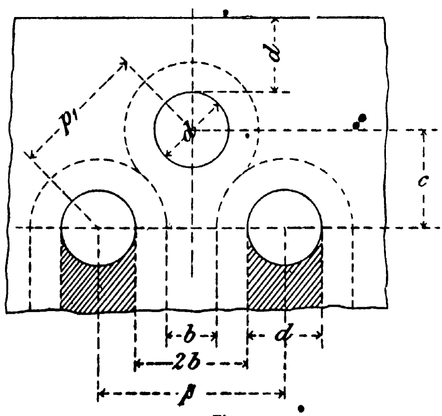



Fig. 75

the proportion of superfluous plate is less than with single riveting, and therefore the efficiency of the joint is greater.

Equating, as before, the shearing and tearing resistance. for rivets in single shear—

$$\frac{\pi}{2} d^2 f_s = 2b\, t\, f_t$$

$$2b = 1{\cdot}57 \frac{d^2}{t} \frac{f_s}{f_t}$$

$$\therefore p = 1{\cdot}57 \frac{d^2}{t} \frac{f_s}{f_t} + d$$

For a double riveted joint in double shear, as in a butt joint with two covers, four sections of rivet correspond to a strip of width $2b$

$$\pi\, d^2 f_s = 2b\, t\, f_t$$

$$2b = 3{\cdot}14 \frac{d^2}{t} \frac{f_s}{f_t}$$

$$\therefore p = 3{\cdot}14 \frac{d^2}{t} \frac{f_s}{f_t} + d$$

88. *Tenacity and shearing resistance of iron and steel used for riveted work.*—When broken by tension in the testing machine, ordinary test bars give the following results :

| | Stress at yield point, tons per sq in | Tenacity, tons per sq in. | Elongation in 8 in. per cent. |
|---|---|---|---|
| Iron, Lowmoor plate | 14 | 24 to 29 | — |
| ,, boiler plate [1] | 14 to 17 | 21 to 26 | 18 to 27 |
| ,, ,, ,, [2] | 13 to 15 | 19 to 21 | 8 to 15 |
| ,, ordinary plate [1] | 14 to 15 | 21 | 10 |
| ,, ,, ,, [2] | 13 to 14 | 18½ | 3 |
| ,, for rivets | 15 | 28 | 27 |
| Steel, boiler plate | 15 to 17 | 27 to 29 | 22 to 30 |
| ,, for rivets | 20 | 27 to 29 | 25 to 30 |
| ,, plates under ½ in. thick | 17 to 24 | 24 to 31 | 9 to 25 |

[1] In direction of rolling. [2] Across direction of rolling

Tests of material for shearing resistance are not so numerous or so satisfactory as tension tests. In iron the shearing resistance varies a good deal in different directions relatively to the direction of rolling. Steel is more uniform in resistance in different directions. For rivet iron and rivet steel, the shearing resistance is about four-fifths of the resistance of the same materials in tension. That is, the shearing resistance of rivet iron or rivet steel is about 23 tons per square inch. In the case of iron rivets in iron plates the rivet iron is usually of higher

strength than the plates, and hence, apart from some modifying circumstances to be mentioned, the resistance of rivets and plate are equal, if the shearing area of the rivets is equal to the tearing area (net section) of the plate. For steel rivets in steel plates the resistance of rivets and plate are equal, if the shearing area of the rivet is one and a quarter times the tearing area of the plate.

89. *Apparent tenacity and shearing resistance in tests of riveted joints.*—The tenacity and shearing resistance, calculated from tests of riveted joints, differ more or less from the tenacity and shearing resistance determined by testing simple test bars in the testing machine. This arises in some cases from alteration of the material by punching, in others from the complex distribution of stress in riveted joints. If $\omega_t$ is the minimum section of plate normal to the straining action, $\omega_s$ the section of rivets in shear in the plane of the joint, and P the load at which the joint gives way when tested, then $P/\omega_t$ and $P/\omega_s$ may be termed the *apparent* tenacity and shearing resistance. The *real* stress at which the joint gives way differs from the apparent stress, because of bending and other actions neglected in calculating the apparent stress.

*Tenacity of drilled plates.*—If a row of holes is drilled in a plate it breaks, when tested, along the line of holes where the section is least. The tenacity per sq. in. of plate section between the holes is found to be greater than that of an undrilled plate. The reason of this is that from the form of the metal between the holes, the section of fracture is adjacent to less strained sections, and the contraction which is formed in ordinary test bars is partially suppressed. The hindering of the contraction makes the resistance to tearing greater. Experiment shows that plates drilled at distances usual in riveted work are 10 to 12 per cent. stronger than undrilled plate.

*Tenacity of punched plates.*—The experiments on the tenacity of punched plates are extremely discordant. A number of experiments show a loss of tenacity after punching varying from 5 per cent. up to 20 per cent. in iron plates, and from 8 per cent. to 35 per cent. in steel plates. With steel plates the loss increases with the thickness of the plate; with both steel and iron it is diminished by making the hole in the die block greater in diameter than the punch; and the loss completely disappears, and the original tenacity is restored, if the plate is annealed after

punching; or if a small ring, 0·04 to 0·08 inch in thickness, is rymered out of the punched hole.

There can be no doubt that, in punching, a portion of metal is squeezed laterally into the plate, and a condition analogous to that produced by cold hammering is induced in the metal immediately surrounding the hole. It is this hardened ring which is got rid of by annealing or rymering.

*Values of apparent tenacity and shearing resistance of riveted joints.*—When a portion of riveted joint is tested, other conditions affect the breaking stress. The friction of the plates resists the straining action, though probably, since slip occurs before the breaking point is reached, it does not much affect the breaking stress. In most forms of joint the resultant of the straining action does not pass through the centre of figure of the section of fracture, the distribution of stress is altered by bending, and the apparent or mean stress on the section is less than the real stress causing fracture. Similar considerations apply to the rivets.

A large number of tests of riveted joints were examined to find the values of the apparent tenacity and shearing resistance. The following short tables give the mean results obtained in this examination. Tests of ordinary test-bars of the plates gave for the mean tenacity of the iron plates 20½ tons per sq. in., and for that of the steel plates 28 tons per sq. in.

| | Apparent tenacity of joint, tons per sq. in. Iron Plates | Steel Plates |
|---|---|---|
| Single-riveted, drilled | 18·1 | 27·6 |
| ,, punched | 15·8 | 25·0 |
| Double-riveted, drilled | 19·5 | 29·0 |
| ,, punched | 17·5 | 27·6 |
| Treble-riveted, drilled | 20·0 | 30·0 |

| | Apparent shearing resistance of rivets, tons per sq. in. |
|---|---|
| Iron rivets, in punched holes | 20·5 |
| ,, in drilled holes | 19·0 |
| Steel rivets, in punched holes | 23·5 |
| ,, in drilled holes | 22·0 |

90. *Ratio of apparent tenacity of plates to shearing resistance of rivets.*—Using the values of the apparent tenacity and shearing resistance given above, the following values of the ratio of resistance to tearing and shearing in different kinds of joints are obtained.

*Ratio of shearing to tearing resistance $f_s/f_t$, deduced from experiments on riveted joints*

| | Iron Plates, Iron Rivets | | Steel Plates, Steel Rivets | |
|---|---|---|---|---|
| | Drilled or punched and reamered or annealed | Punched | Drilled or punched and reamered or annealed | Punched |
| Single riveted . . . | 1·06 | 1·30 | 0·79 | 0·95 |
| Double riveted . . . | 0·98 | 1·18 | 0·75 | 0·85 |
| Treble riveted . . . | 0·95 | — | 0·74 | — |

## Strength and Proportions of Riveted Joints in Ordinary Working Conditions.

### JOINTS IN TENSION

91. *Ratio of section of plates to section of rivets.*—A joint will be strongest when the resistance to tearing and shearing are equal, so that there is no excess either of plate section or rivet section. Let $\omega_t$ be the section of plate in the line of fracture through the rivet holes, and $\omega_s$ the section of rivets which would be sheared if the joint gave way by shearing. Also let $f_t$, $f_s$ be the tearing and shearing resistance per sq. in. Then, in the most economical joint—

$$\omega_t f_t = \omega_s f_s,$$

$$\frac{\omega_s}{\omega_t} = \frac{f_t}{f_s},$$

that is, the sections are inversely as the resistances. Generally, it has been assumed in this country that a riveted joint should be designed so that its breaking strength is greatest, and joints have been proportioned on the basis of data obtained by testing joints to destruction, such as those given above. If a joint is intended to have the greatest ultimate strength, then the values of $f_s$ $f_t$ in the table are those which will secure the greatest breaking strength.

92. *Limits of working stress in boiler joints.*—It is a principal condition in boiler joints that they should be free from liability to leakage. Hence the workmanship is of high quality. Ordinary practice does not recognise great refinement in proportioning riveted joints, and the older view that the ultimate strength

should be a maximum is rejected in favour of the view that the working stresses should not exceed limits shown by experience in similar cases to be safe. For boilers, the working tensile stress of net section of plate may be taken to be on the average 10,000 lbs., or 4·5 tons, per sq. in. for iron; 12,000 lbs., or 5·36 tons, per sq. in. for mild steel. The shearing resistance of the rivets may be taken as follows:

| | Pounds per sq. in. | Tons per sq. in. |
|---|---|---|
| Iron rivets in single shear | 8,500 | 3·80 |
| " " double " | 7,800 | 3·48 |
| Steel rivets in single shear | 9,500 | 4·24 |
| " " double " | 8,750 | 3·90 |

Hence the ratio $f_s/f_t$ of shearing to tearing resistance is—

| | Single shear $f_s/f_t$ = | Double shear $f_s/f_t$ = |
|---|---|---|
| Iron plates, iron rivets | ·85 | ·78 |
| Steel plates, steel rivets | ·79 | ·73 |

93. *Limits of working stress in tension joints in bridges and other structures.*—In iron and steel structures the joints are usually butt joints and multiple riveted. It is desirable to reduce the shearing stress on the rivets somewhat because uniform distribution of the load to all the rivets in a group is not to be relied on. A circumstance which greatly modifies the limit of working stress in structures is the effect of constant alternations of stress in causing fatigue.

In the older wrought-iron bridges working stresses of 5 tons per sq. in. in tension and 4 tons per sq. in. in compression were generally allowed, but these limits require reduction in members exposed to a great range of stress. Modern structures are usually made of mild steel. For members not exposed to a great range of stress the greatest working stress may be taken at 7½ tons per sq. in. in tension and 6 tons per sq. in. in shear. In the booms of steel bridges the working stress is more commonly 6 to 6½ tons per sq. in.; in smaller members, such as the vertical and diagonal bracing, 5 tons per sq. in. In members exposed at times to tension and compression, the working tensile stress should not exceed 3 to 4½ tons per sq. in. The shearing resistance of the rivets is provided for in this way. The section of rivets in shear is made 1½ times the net section of the plates in tension. That is equivalent to taking $f_s/f_t = 2/3$.

94. *Joints in compression.*—Joints in compression are almost

always butt joints and are usually multiple riveted. The conditions of strength are essentially different from those in tension joints. The rivet holes do not weaken the plate in the same way, for if the rivets fit the holes the thrust is transmitted as in a solid plate. If the ends of the plates are well butted, the riveting is only required to prevent displacement of the plates. In early wrought-iron bridges girders were made with single-riveted joints in the top boom. But then, unless the plates were very well fitted—and this was not sufficiently attended to in early bridges—some crushing of the plate edges and consequent shearing of the rivets was inevitable. This being observed, bridges were designed with joints so arranged that the rivets would carry the whole thrust independently of any abutting of the plates. That is, they were designed like tension joints, except that the rivet shearing area was made about $\frac{8}{9}$ths of the gross section of a plate. If the ends of the plates are planed and well fitted, it certainly seems unnecessary to proportion the rivets to take the whole thrust at the joint. Nor is so much shearing area now usual. Thus, in the compression joints of the Forth Bridge, the shearing area of the rivets on one side of a joint was made half the gross section of a plate.

## DETAILED CALCULATIONS OF RIVETED JOINTS IN TENSION
### SINGLE RIVETED JOINTS IN MILD STEEL

95. *Single riveted joints in single shear.*—Taking $f_t = 5{\cdot}36$ tons per sq. in. and $f_s = 4{\cdot}24$ tons per sq. in., $f_s/f_t = 0{\cdot}79$. To each rivet section of diameter $d$ corresponds a net plate section $(p-d)t$, where $p$ is the pitch and $t$ the thickness of plate. When the joint has the best proportions for strength the shearing and tearing resistance are equal, and then

$$(p-d)\,t f_t = \pi d^2 f_s/4$$

$$p = 0{\cdot}785\,\frac{d^2 f_s}{t f_t} + d$$

$$= (0{\cdot}62\,d^2)/t + d \qquad (3)$$

and when the joint has these proportions the ratio of strength of joint to strength of solid plate, which will be termed the *efficiency* of the joint, is

$$\eta = (p-d)/p$$

With ordinary proportions of rivets and the thicker plates, this rule gives the pitch less than $2\,d$, so that the joint could not

be constructed. It is necessary, then, to increase the pitch. For various reasons it may be necessary to vary the pitch from the theoretical best value. For instance, it may be desirable that the pitch should be a multiple of $\frac{1}{16}$ of an inch. If the pitch is reduced the shearing area is in excess and the efficiency is lowered, but the expression for it remains the same. On the other hand, if the pitch is increased, the tearing area is in excess, and the efficiency is

$$\eta = \left(\frac{\pi}{4} d^2 f_s\right) / (p\, t\, f_t) = (0{\cdot}62\, d^2)/pt$$

*Single-riveted joint in double shear* (fig. 65).—One rivet shears through two sections, for each pitch length of joint.

$$(p - d)\, t\, f_t = \pi\, d^2 f_s/2$$

Taking $f_t = 5{\cdot}36$ and $f_s = 3{\cdot}9$ tons per sq. in.,

$$f_s/f_t = 0{\cdot}73,$$

$$p = 1{\cdot}15 \frac{d^2}{t} + d \quad . \quad . \quad . \quad (4)$$

$$\eta = (p - d)/p.$$

In this case the rule gives values to the pitch which are possible. But if the pitch is increased the efficiency becomes

$$\eta = \left(\frac{\pi}{2} d^2 f_s\right) p\, t\, f_t = (1{\cdot}15\, d^2)/p\, t$$

If a larger factor of safety is desired, that is a lower value of $f_t$, the proportions of the joint need not be altered. If $f$ is the selected working tensile stress for the material, the riveted joint will carry a working stress $\eta f$, reckoned on the gross cross section of the plate.

*Single Riveted Steel Joints*

| Thickness of plates = $t$ M.S. | Diameter of rivets = $d$ M.S. | Single shear joints | | Double shear joints | |
|---|---|---|---|---|---|
| | | Pitch = $p$ M.S. | Efficiency = $\eta$ | Pitch = $p$ M.S. | Efficiency = $\eta$ |
| $\frac{5}{16}$ | $\frac{11}{16}$ | 1·63 | ·58 | 2·44 | ·72 |
| $\frac{3}{8}$ | $\frac{3}{4}$ | 1·68 | ·55 | 2·48 | ·70 |
| $\frac{7}{16}$ | $\frac{13}{16}$ | 1·75 | ·54 | 2·55 | ·68 |
| $\frac{1}{2}$ | $\frac{7}{8}$ | 1·82 | ·52 | 2·64 | ·67 |
| $\frac{5}{8}$ | 1 | 2·25[1] | ·44 | 2·84 | ·65 |
| $\frac{3}{4}$ | $1\frac{1}{8}$ | 2·50[1] | ·42 | 3·07 | ·63 |
| $\frac{7}{8}$ | $1\frac{3}{16}$ | 2·69[1] | ·37 | 3·04 | ·61 |

[1] In these cases the pitch has been increased.

## Double-riveted Joints—Mild Steel

96. *Double-riveted joints in single shear* (fig. 66).—Two rivet sections correspond to the plate section between two rivet holes—

$$(p - d)\, t\, f_t = \pi\, d^2 f_s / 2$$

$$p = 1{\cdot}571 \frac{d^2}{t} \frac{f_s}{f_t} + d$$

Taking $f_s / f_t = 0{\cdot}79$,

$$p = 1{\cdot}24 \frac{d^2}{t} + d \quad . \quad . \quad . \quad . \quad (5)$$

$$\text{Efficiency} = \eta = (p - d) / p.$$

If the pitch is increased above the value calculated for the rivet diameter the efficiency becomes

$$\eta = \left(\frac{\pi}{2} d^2 f_s\right) / (p\, t\, f_t) = (1{\cdot}24\, d^2) / (p\, t)$$

*Double-riveted joint in double shear.*—Four rivet sections correspond to the plate section between two rivet holes.

$$(p - d)\, t\, f_t = \pi\, d^2 f_s$$

Taking $f_s / f_t = 0{\cdot}73$,

$$p = 2{\cdot}29 \frac{d^2}{t} + d \quad . \quad . \quad . \quad . \quad (6)$$

$$\eta = (p - d) / p.$$

If the pitch is increased the efficiency is

$$\eta = (\pi\, d^2 f_s) / (p\, t\, f_t) = (2{\cdot}29\, d^2) / (p\, t)$$

*Double Riveted Steel Joints*

| Thickness of plate $= t$ M.S. | Diameter of rivet $= d$ M.S. | Single shear joints | | Double shear joints | |
|---|---|---|---|---|---|
| | | Pitch $= p$ M.S. | Efficiency $= \eta$ | Pitch $= p$ M.S | Efficiency $= \eta$ |
| $\frac{5}{16}$ | $\frac{11}{16}$ | 2·57 | ·73 | 4·17 | ·84 |
| $\frac{3}{8}$ | $\frac{3}{4}$ | 2·61 | ·71 | 4·19 | ·82 |
| $\frac{7}{16}$ | $\frac{13}{16}$ | 2·68 | ·70 | 4·27 | ·81 |
| $\frac{1}{2}$ | $\frac{7}{8}$ | 2·78 | ·68 | 4·38 | ·80 |
| $\frac{5}{8}$ | 1 | 2·98 | ·67 | 4·66 | ·79 |
| $\frac{3}{4}$ | $1\frac{1}{16}$ | 2·92 | ·64 | 4·50 | ·76 |
| $\frac{7}{8}$ | $1\frac{1}{8}$ | 2·93 | ·62 | 4·45 | ·75 |
| 1 | $1\frac{1}{4}$ | 3·18 | ·61 | 4·82 | ·74 |
| $1\frac{1}{8}$ | $1\frac{9}{32}$ | 3·10 | ·59 | 4·65 | ·73 |
| $1\frac{1}{4}$ | $1\frac{11}{32}$ | 3·13 | ·57 | 4·64 | ·71 |

### TREBLE-RIVETED JOINTS—MILD STEEL

97. *Treble-riveted joints. Rivets in single shear* (fig. 71).—Three rivet sections correspond to the plate section between two rivet holes.

$$(p - d)\, t f_t = \frac{3}{4}\pi d^2 f_s ; \qquad p = 2{\cdot}356\, \frac{d^2}{t} \cdot \frac{f_s}{f_t} + d ;$$

When

$$f_s / f_t = 0{\cdot}79 ; \quad p = 1{\cdot}86\, \frac{d^2}{t} + d ; \quad \eta = (p - d) / p$$

If the pitch is increased

$$\eta = (\tfrac{3}{4}\pi d^2 f_s) / (p t f_t) = (1{\cdot}86\, d^2) / (p\, t)$$

*Treble-riveted joint. Rivets in double shear.*

$$p = 4{\cdot}712\, \frac{d^2 f_s}{t f_t} + d$$

When

$$f_s / f_t = 0{\cdot}73 ; \quad p = 3{\cdot}34\, \frac{d^2}{t} + d ; \quad \eta = (p - d) / p.$$

If the pitch is increased

$$\eta = (1{\cdot}5\, \pi d^2 f_s) / (p t f_t) = (3{\cdot}34\, d^2) / (p\, t)$$

*Treble Riveted Steel Joints*

| Thickness of plate = $t$ M.S. | Diameter of rivets = $d$ M.S. | Single shear joints. | | Double shear joints | |
|---|---|---|---|---|---|
| | | Pitch = $p$ M.S. | Efficiency = $\eta$ | Pitch = $p$ M.S. | Efficiency = $\eta$ |
| $\frac{3}{8}$ | $\frac{3}{4}$ | 3·17 | ·88 | 5·37 | ·93 |
| $\frac{7}{16}$ | $\frac{13}{16}$ | 3·24 | ·87 | 5·47 | ·92 |
| $\frac{1}{2}$ | $\frac{7}{8}$ | 3·33 | ·85 | 5·56 | 91 |
| $\frac{5}{8}$ | 1 | 3·62 | ·83 | 5·99 | ·90 |
| $\frac{3}{4}$ | $1\frac{1}{16}$ | 3·54 | ·79 | 5·74 | ·87 |
| $\frac{7}{8}$ | $1\frac{3}{16}$ | 3·86 | ·78 | 6·23 | ·86 |
| 1 | $1\frac{1}{4}$ | 3·90 | ·74 | 6·19 | ·84 |
| $1\frac{1}{8}$ | $1\frac{5}{16}$ | 3·98 | ·72 | 6·22 | ·82 |
| $1\frac{1}{4}$ | $1\frac{3}{8}$ | 4·08 | ·69 | 6·31 | ·80 |

### FRICTIONAL RESISTANCE OF RIVETED JOINTS

98. In England riveted joints have been designed with reference to the ultimate tearing and shearing resistance, as stated above. On the Continent, a different view has been adopted, especially in such cases as joints for boilers which have to be staunch against leakage. The rivets contract as they cool and draw the plates together with considerable force. At the same time,

if they filled the holes when hot, they cannot, in consequence of lateral contraction, perfectly fill them when cold. Hence the resistance of the rivets to shearing cannot act without some slight slipping of the plates, which is resisted by the friction of the surfaces in contact. But any slipping will practically destroy a joint intended to be staunch, and the safety of the joint as regards this kind of failure depends on the resistance to slipping. The proportions of the joints arrived at on this view are not very different from those given above, and within proper limits the Continental theory appears to be justified.

Let $R_s$ (fig. 76) be the resistance to slipping at a pair of faces in contact, and $Q$ the tension in the rivet. Then—

$$R_s = \mu Q;$$

where $\mu$ is a kind of coefficient of friction depending on the condition of the surfaces, and $Q$ will vary directly as the cross

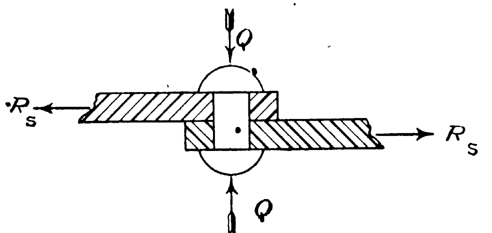



Fig. 76

section of the rivet, and depend to some extent on the temperature of the rivet when closed. The object in designing joints on the Continental theory must be to make $R_s$ as large as possible, consistently with not reducing more than necessary the strength of the plate. The shearing resistance of the rivet is entirely neglected.

Experiments by Bach showed that the frictional resistance when slipping of a joint begins amounts to 14,000 to 30,000 lbs. per sq. in. of rivet section at each pair of surfaces in contact. The resistance increases a little as the rivets are longer, and in multiple riveting does not increase quite in proportion to the number of rows of rivets. In butt riveting with double cover plates there is rather less resistance at each pair of faces than in lap riveting. Caulking the joint sensibly increases the resistance

to slipping. On these general results the following rules have been based by Bach.

Let $t$ be the thickness of the plates, $d$ the diameter of the rivets, R the resistance to slipping per sq. in. of rivet section.

MILD STEEL BOILER JOINTS, DESIGNED FOR FRICTIONAL RESISTANCE

99. (1) *Single-riveted lap joints or butt joints with single cover plate.*

Diameter of rivet $= d = 1{\cdot}4 \sqrt{t} - 0{\cdot}16$.
Pitch of rivets $= p = 2\,d + 0{\cdot}32$.
Width of overlap $= 3\,d$.

The working resistance to slipping may be taken at $R = 8{,}500$ to 10,000 lbs. per sq. in. of the rivet section. Hence the working tensile stress in the plates, reckoned on the gross section, without deducting rivet holes, is

$$f = \tfrac{1}{4}\frac{\pi\, d^2}{p\,t}\,R \text{ lbs. per sq. in.}$$

$$= 6{,}670 \text{ to } 7{,}850\frac{d^2}{p\,t}$$

The stress on the net section of the plates is

$$f_t = (fp)\,/\,(p - d)$$

and this must be not greater than the allowable working stress in tension.

*Example.*—Let $t = \frac{3}{4}$. Then the rivet diameter should be $d = 1{\cdot}052$, or, say, $1\frac{1}{16}$ inch. Pitch $= 2 \times 1\frac{1}{16} + 0{\cdot}32 = 2{\cdot}445$, say, $2\frac{7}{16}$ inch. The tension on the gross section of the plate is then $f = 4{,}118$ to $4{,}848$ lbs. per sq. in. The stress on the net section of the plate is $f_t = 7{,}302$ to $8{,}594$ lbs. per sq. in.

(2) *Double-riveted lap joints or double-riveted butt joints with one cover plate.*

Diameter of rivets $= d = 1{\cdot}4 \sqrt{t} - 0{\cdot}16$.

If the rivets are arranged zigzag

$$p = 2{\cdot}6\,d + 0{\cdot}6.$$

If arranged as chain riveting

$$p = 2{\cdot}6\,d + 0{\cdot}4.$$

The distance between two rows of rivets may be $0{\cdot}6\,p$ for zigzag, and $0{\cdot}8\,p$ for chain riveting. The width of overlap may be

$4{\cdot}6\,d + 0{\cdot}9$ for zigzag and $5{\cdot}1\,d + 0{\cdot}8$ for chain riveting. The working resistance to slipping may be taken at $R = 7{,}800$ to $9{,}200$ lbs. per sq. in. of rivet section. Hence the working tensile stress, reckoned on the gross section of the plates, is

$$f = \tfrac{1}{2}\,\frac{\pi d^2}{p\,t}\,R \text{ lbs. per sq. in.}$$

$$= 12{,}240 \text{ to } 14{,}400\,\frac{d^2}{p\,t}$$

The stress on the net section is

$$f_t = (f\,p)\,/\,(p - d).$$

(3) *Triple-riveted lap joints or triple-riveted butt joints with single cover plate.*

$$\text{Diameter} = d = 1{\cdot}4\sqrt{t} - 0{\cdot}16$$
$$\text{Pitch} = p = 3\,d + 0{\cdot}88.$$

The working resistance to slipping may be taken at $R = 7{,}000$ to $8{,}500$ lbs. per sq. in. of rivet section. Hence the working tensile stress in the plates, reckoned on the gross section, is

$$f = \tfrac{3}{4}\,\frac{\pi d^2}{p\,t}\,R \text{ lbs. per sq. in.}$$

$$= 16{,}500 \text{ to } 20{,}000\,\frac{d^2}{p\,t}.$$

The distance between the rows of rivets may be $0{\cdot}5\,p$, if placed zigzag. The stress on the net section is as before—

$$f_t = (f\,p)\,/\,(p - d).$$

(4) *Single-riveted butt joints with double cover plates.*—There are two pairs of faces in contact at the joint. The thickness of the cover plates may be $\frac{5}{8}\,t$ to $\frac{3}{8}\,t$.

$$\text{Diameter} = d = 1{\cdot}4\,\sqrt{t} - 0{\cdot}2$$
$$\text{Pitch} = p = 2{\cdot}6d + 0{\cdot}4.$$

The working resistance to slipping may be taken at $R = 14{,}200$ to $17{,}000$ lbs. per sq. in. of rivet section. Hence the tension in the plates, reckoned on the gross section, is

$$f = \tfrac{1}{4}\,\frac{\pi d^2}{p\,t}\,R \text{ lbs. per sq. in.}$$

$$= 11{,}100 \text{ to } 13{,}400\,\frac{d^2}{p\,t}$$

(5) *Double-riveted butt joints with double cover plates.*

$$\text{Diameter} = d = 1{\cdot}4\sqrt{t} - 0{\cdot}24$$
$$\text{Pitch} = p = 3{\cdot}5\,d + 0{\cdot}6.$$

The working resistance to slipping may be taken at R=13,500 to 16,500 lbs. per sq. in. of rivet section. Hence the tension in the plates, reckoned on the gross section, is

$$f = \tfrac{1}{2}\frac{\pi d^2}{p t}\,\text{R lbs. per sq. in.}$$
$$= 21{,}200 \text{ to } 25{,}900\,\frac{d^2}{p t}$$

(6) *Triple-riveted butt joints with double cover plates.*

$$\text{Diameter} = d = 1{\cdot}4\sqrt{t} - 0{\cdot}28$$
$$\text{Pitch} = p = 6\,d + 0{\cdot}8$$

The resistance to slipping=R=13,000 to 14,000. Hence the tension in the plates, reckoned on the gross section, is

$$f = \tfrac{3}{4}\frac{\pi d^2}{p t}\,\text{R lbs. per sq. in}$$
$$= 30{,}600 \text{ to } 33{,}000\,\frac{d^2}{p t}$$

100. *Graphic method of designing joints.*—Schwedler has introduced a mode of designing joints which is particularly useful for irregular or complicated joints. The width $w$ of a strip of plate of a strength equivalent to that of one rivet in single shear is given by the equation

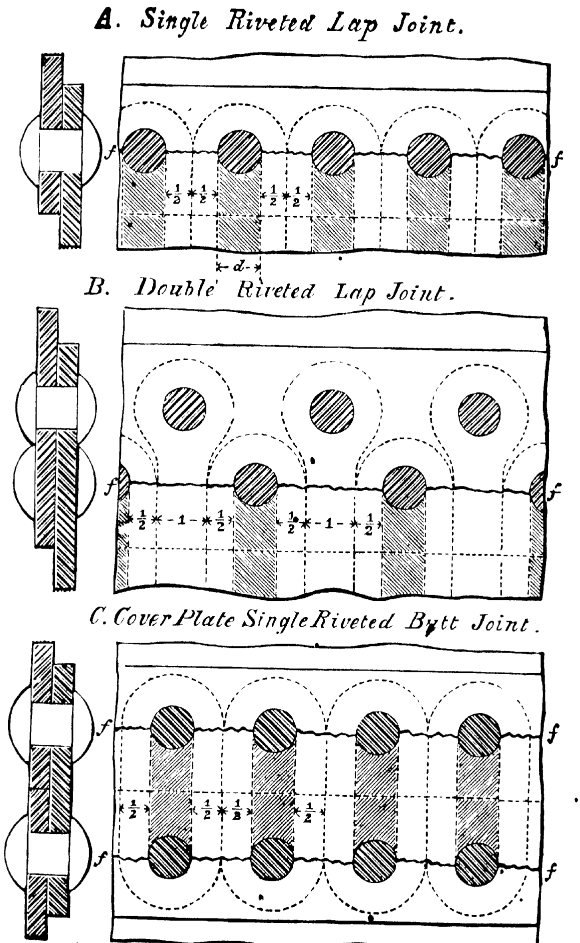
$$w\,t f_t = \frac{\pi}{4} d^2 f_s$$

Then $$w = 0{\cdot}785\,\frac{d^2}{t}\cdot\frac{f_s}{f_t} \qquad (7)$$

Suppose $d$ and $t$ given, and the value of $f_t/f_s$ also given or taken from the table above § 92. Then $w$ can be calculated. If round each rivet a circle is drawn of diameter $d + w$, and from these circles lines are drawn cutting up the plate into strips of width $w$, a portion of plate of sufficient strength will have been allotted to each rivet, and any redundant portions will indicate useless material in the joint. If the rivets are in double shear then $w$ has double the value given above.

The following figures show the application of this method to some forms of joint, the shaded portions being parts of the plate which do not add to the strength of the joint.

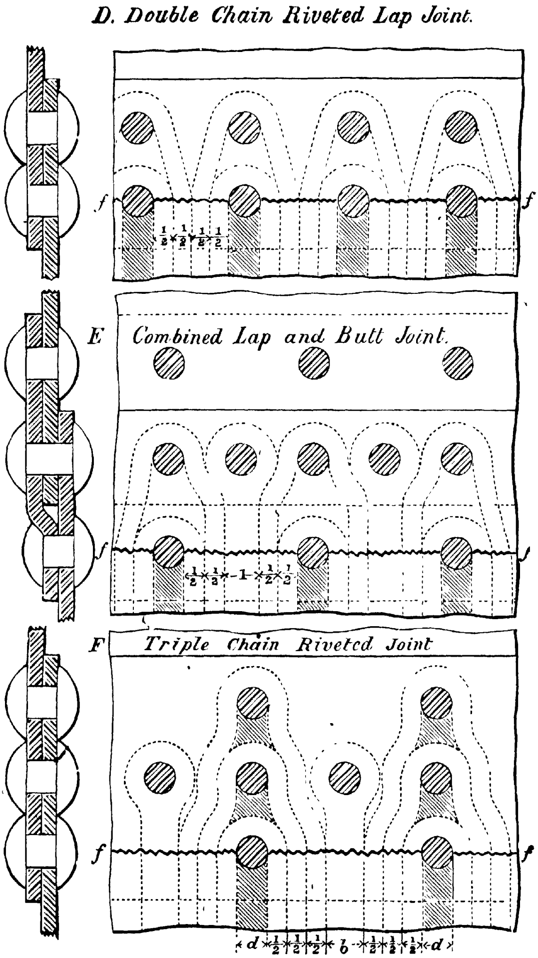
In fig. 77, A is a single-riveted and B a double-riveted joint so arranged that the shearing and tearing resistances are equal. The shaded portions represent metal unavoidably wasted,




Unit = *w*. *ff*, Line of fracture.

Fig. 77

because the joint will give way at the line of fracture $ff$. c is the cover strip of a butt joint similarly designed. Fig. 78 shows joints of a more complicated construction. Fig. 79 shows the




Unit = *w*., *ff*, Line of fracture.

Fig. 78

same method applied in designing a joint in a tie bar. As to the distance apart of the rows of rivets, no specific rule can be given, but it is rational to suppose that the strips carrying the load due to each rivet should not be too sharply bent.

101. *Cold riveting for thin plates.*—For gasometers and tanks thin plates are used, and questions of strength are relatively unimportant. The thickness is fixed with reference to deterioration by corrosion. In such cases the rivets are hammered up cold. With plates $\frac{1}{8}$ inch to $\frac{3}{16}$ inch, the rivets are $\frac{1}{4}$ inch to $\frac{5}{16}$ inch

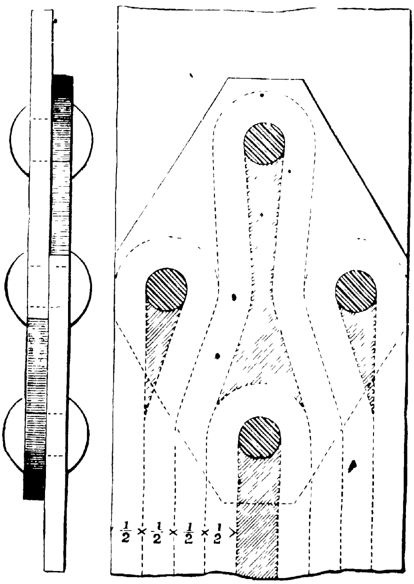



Fig. 79 Unit = $w$

diameter and $\frac{7}{8}$ inch pitch. The half width of overlap is about $\frac{1}{2}$ inch, and the staunchness of the joint is secured by laying between the plates, in a zigzag direction round the rivets, a string smeared with red lead.

102. *Junctions of three plates.*—In boiler work where the riveted seams must be watertight, a difficulty arises where the cross joints and longitudinal joints meet, because there three plates overlap. At such places one or more plates are thinned out by forging so that the joint may be solid throughout.

Fig. 80 shows three plates, *a*, *b*, *c*, overlapping and single riveted. Fig. 81 a similar joint where the longitudinal seam is double riveted and the cross seam single riveted.

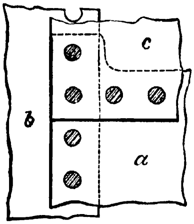



Fig. 80

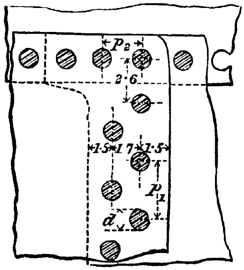



Fig. 81

Fig. 84 shows a junction of three plates with butt straps, the longitudinal butt strap being planed down and tucked under the cross butt strap.

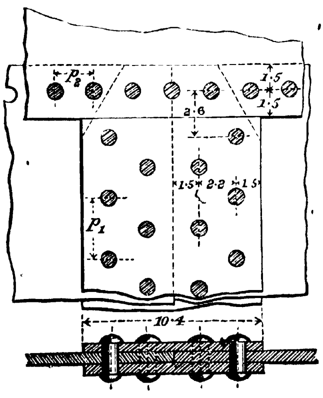



Fig. 82

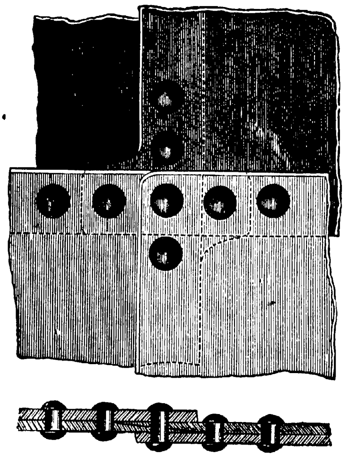


Fig. 83

In fig. 82 the cross seam is overlapped and single riveted, but the longitudinal seam has double butt straps and is double riveted. The upper butt strap is planed at the end so that it can be better caulked where it abuts against the plate. The proportional unit is the rivet diameter.

*Junctions of four plates.*—Figs. 83, 85 show four overlapping plates, single riveted. Each of the two interior plates is thinned out at the junction. It will be seen that the forged part is lengthened out so as to be gripped by an additional rivet in the thin part.

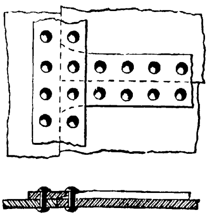


Fig. 84

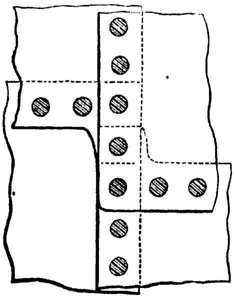


Fig. 85

103. *Connection of plates not in one plane.*—This is commonly effected by the use of a kind of angular joint strip, called an *angle iron.* These angle irons are rolled of a great variety of sizes, and are of very great service in all descriptions of wrought-iron work. Fig. 86 shows an angle-iron joint. No very definite rule can be given for the size of angle-iron to be used, but generally

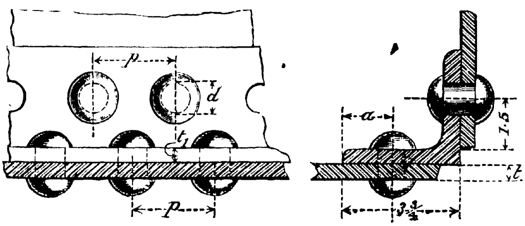



Fig. 86

the mean thickness of the angle iron is about equal to, or a little greater than, that of the plates to be connected. If the mean thickness of the angle iron is $t_1 = t + \frac{1}{16}$, then the width $w$ of each limb of the angle iron may be $3\frac{3}{4}t$; the diameter $d$ of the rivets $= 2t$: $a = \frac{1}{2}(w - t_1)$. The angle iron usually tapers so that it is rather thicker at the root than at the point. In bridge work the angle irons are often heavier. Care must be taken in

arranging the rivets that the heads in one row do not prevent the proper riveting of those in the other row.

Fig. 87 shows a T iron joint, the object being to stiffen the plates against flexure.

Fig. 88 shows methods of connecting plates by flanging the plates themselves, instead of using angle irons. This is more expensive and is impracticable when the plates are not of good quality. The curvature should not be too sharp. The

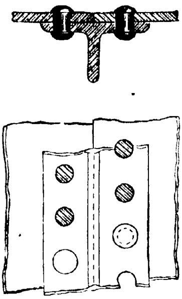


Fig. 87

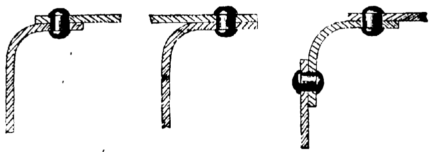


Fig. 88

inside radius may be at least four times the thickness of the plates. The width of overlap must be, at least, three times the diameter of the rivet.

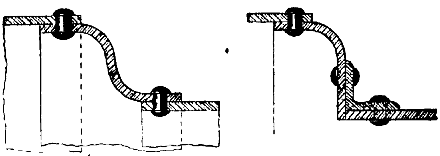


Fig. 89

Fig. 89 shows joints used at the junction of the cylindrical barrel of locomotive boilers with the external fire-box.

Fig. 90 shows ordinary proportions for flanged plates.

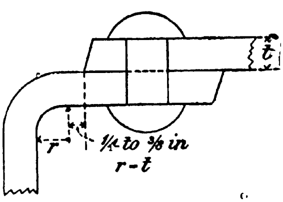



Fig. 90

104. *Connection of parallel plates.*—A case which frequently occurs is where two plates, near together, require to be connected. For instance, at the bottom of the fire-box of locomotives, a connection has to be made between the inner and outer fire-box. The following sketches show how this may be effected.

In fig. 91 *a* there are two angle irons. This is rather complicated, and there are inside joints, which cannot be caulked.

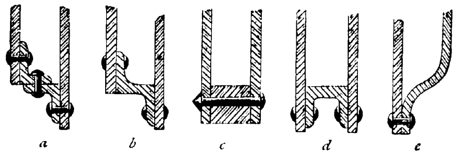



Fig. 91

Fig. 91 *b* is simpler, but has an inside joint, which cannot be caulked. Fig. 91 *d* is an admirable joint, and is formed by what is termed a channel iron. But it is difficult to bend the channel

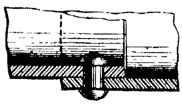


Fig. 92

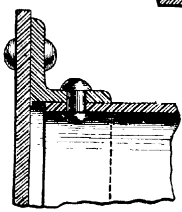


Fig. 93

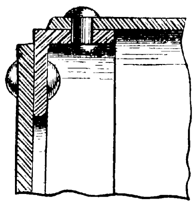


Fig 94

iron round the corners of the fire-box. Fig. 91 *e* is simple, but forms a corner for the lodgment of sediment. Fig. 91 *c* is the form most commonly used.

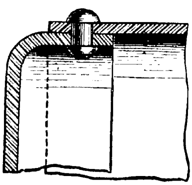


Fig. 95

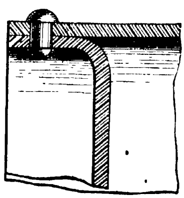


Fig 96

*Connections in cylindrical boilers.*—Fig. 92 shows the ordinary way of overlapping the cross joints in shells and flues. Figs. 93 to 96 show arrangements used for connecting the boiler shell and

end plates. Figs. 97, 98 show modes of connecting the furnace flues and end plates. When the joint is made as in fig. 98, a half ring cover is sometimes used like that shown dotted, to protect the upper half of the joint from grooving.

105. *Corner connections.*—Figs. 99, 100 show the connection of three plates in different planes by means of angle irons. The proportions of the angle irons may be the same as in fig. 86.

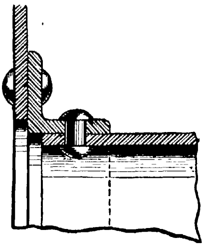


Fig. 97

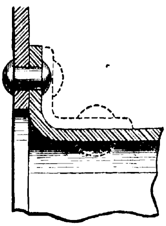


Fig 98

*Elliptical rivets.*—Since the efficiency of the joint is the ratio $(p-d)/p$ of the distance between the holes to the pitch, we may increase the efficiency by using rivets of elliptical section. With such rivets, placed with their least breadth in the line of fracture of the plates, the quantity $p-d$ would be greater, while the shearing section remained the same. By adopting the

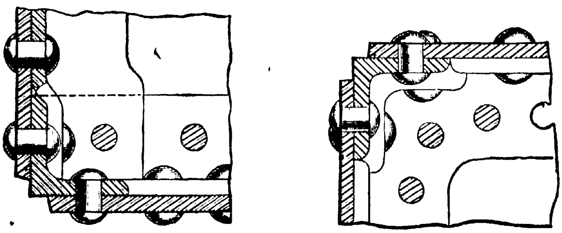


Fig. 99

elliptical form, two variables, the axes of the ellipse, take the place of the single variable $d$, in the equations.

106. *Position of rivets in tie bars and struts.*—When a bar, subjected to a longitudinal straining force, is attached at each end by a single rivet or pin, these should be placed on the centre line of the bar. It is a fair assumption, and must be nearly true, that the straining force acts through the centre of the rivet.

Hence, if the rivets are in the centre line of the bar, the resultant straining force passes through the axis of the bar, and the stress on each transverse section is uniform. If the rivets are not so placed, one side of the bar is more strained than the other, and gives way before the other has fully exerted its powers of resistance. When there are several rivets at each end of a bar, they should, for the same reason, be placed symmetrically on either side of the axis, and as uniformly distributed as possible over the area in which they are placed. If they cannot be placed

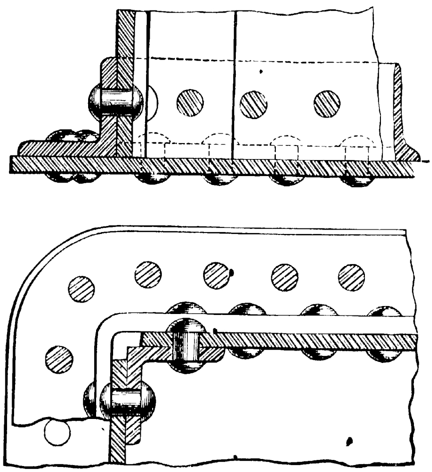


Fig. 100

symmetrically, an approximation is made to the best conditions by arranging them so that their common centre of gravity falls on the axis of the bar. In that case, if each rivet supports the same fraction of the load, the resultant force will still pass through the axis of the bar.

107. *Cylindrical riveted structures.*—A cylindrical vessel made of numerous plates may be formed of a series of cylindrical rings alternately larger and smaller, so that each alternate ring can be slipped inside the others. Then if D is the inside diameter of the smaller rings, $D+2t$ is that of the larger ones. A second plan is to make the rings of equal diameter, and to use a butt strap over the joints. A third plan, common in boiler flues, is to make the rings conical, the diameters being D at one end

and $D+2t$ at the other. The rings are then slipped into each other, and the joints should be so placed that the flame does not directly strike the edges of the plate.

*Strength of cylindrical vessels subjected to internal pressure. Boiler shells.*—The general equation for the stress in a cylindrical shell of diameter $d$ and thickness $t$, in inches, for a working internal pressure $p$ lbs. per sq. in., is

$$f = p\, d \,/\, 2t \text{ lbs. per sq. in.} \qquad (8)$$

If the shell is composed of plates with riveted joints having an efficiency $\eta$,

$$f = p\, d \,/\, 2\eta t \qquad (8a)$$

where $\eta$ is the efficiency of the joint, for which rules are given above. Sometimes the efficiency has been taken at roughly $\eta = 0{\cdot}56$ for single riveted, $0{\cdot}70$ for double riveted, and $0{\cdot}75$ for treble riveted joints. But the efficiency is not always quite so great as this, especially for single riveted joints. The safe internal working pressure of the boiler is

$$p = 2\,\eta\, t f / d \qquad (9)$$

where, on the average, $f = 10{,}000$ lbs. per sq. in. for iron and 12,000 lbs. per sq. in. for steel. The factor of safety in boiler work is generally not less than 4·5.

*Safe working stress in practice.*—The following values of the working stress $f$ give values of $p$ identical with those prescribed by Lloyd's rules for marine boilers.

*Working Stress $f$ in Cylindrical Boiler Shells*

| | Plate thickness in inches | | | |
|---|---|---|---|---|
| | $\frac{1}{2}$ and less | $\frac{1}{2}$ to $\frac{3}{4}$ | over $\frac{3}{4}$ | |
| *Wrought iron.* | | | | |
| Lap joints, punched plates | 7,750 | 8,250 | 8,500 | |
| ,, drilled ,, | 8,500 | 9,000 | 9,500 | |
| Butt joints, punched plates | 8,500 | 9,000 | 9,500 | |
| ,, drilled ,, | 9,000 | 9,500 | 10,000 | |
| | $\frac{3}{8}$ and less | $\frac{3}{8}$ to $\frac{9}{16}$ | $\frac{9}{16}$ to $\frac{3}{4}$ | above $\frac{3}{4}$ |
| *Steel.* | | | | |
| Lap joints | 10,000 | 10,750 | 11,500 | 12,000 |
| Butt joints [1] | 10,750 | 11,500 | 12,500 | 13,000 |

[1] Double butt straps.

## BOILER STAYS

108. Boiler stays are fastenings which support flat surfaces, and, although they are sometimes more complex than simple fixed fastenings, they are conveniently treated here.

*Locomotive or water space stays.*—The flat surfaces of locomotive fire-boxes are supported by stays screwed into both the inner and outer fire-box plates and riveted over. Sometimes

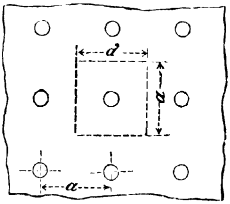



Fig. 101

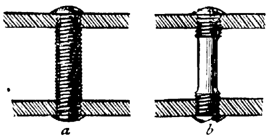



Fig. 102

the stays are screwed over the whole length (*a*, fig. 102). Sometimes the middle part is turned down to the diameter at the bottom of the screw thread (*b*, fig. 102), and there is then less liability to fracture. Occasionally, especially in stays near the level of the fire-bars, a small hole is drilled into the stay (fig. 103), the leakage from which gives warning if fracture has occurred. The screw threads of stays are usually of finer pitch than Whitworth threads: 6 or 8 threads per inch for large nutted stays, and 10 or 11 threads per inch for screw stays screwed into the plate are usual.

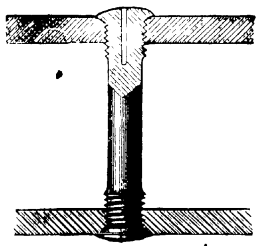


Fig. 103

Stays are almost always of copper in copper fire-boxes, and of iron in iron or steel fire-boxes. Steel stays have also been used. In marine boilers the stays are put in with a nut at each end or a nut at one end and a head at the other. In other cases the stays are double nutted at each plate. Where the nuts are not exposed to excessive heat, so that they burn away, they are preferable to riveting and support the plates better. Copper stays are more easily riveted cold than iron or steel stays. The stays vary in diameter from $\frac{3}{4}$ inch upwards, and are pitched uniformly at a distance *a* so that each stay

supports the pressure on an area of $a \times a$ square inches (see fig. 101). The working stress may be 4,000 to 5,000 lbs. per sq. in. for copper, 6,000 to 7,500 lbs. for iron, and 8,000 to 9,000 for steel.

The following rules give ordinary proportions for stays: Let $a$ be the pitch of the stays, $d$ their gross diameter, $d_1$ the net diameter at bottom of screw thread, $t$ = thickness of plate, $p$ = excess of internal over external pressure in lbs. per sq. in. The thickness of the plates is determined by their resistance to bending, and rules are given in § 79, Case III. Usually for stays screwed and riveted

$$t = 0{\cdot}011\ a \sqrt{p} \text{ for copper plates ;}$$
$$= 0{\cdot}009\ a \sqrt{p} \text{ for iron or steel plates.}$$
$$a = 95\ t / \sqrt{p} \text{ for copper plates ;}$$
$$= 120\ t / \sqrt{p} \text{ for iron or steel plates.}$$

If $a = c\,t/\sqrt{p}$, then, according to Lloyd's rules, $c$ would have approximately the following values, the stays being made without welds:

| Plates | Stays | Values of $c$ | |
|---|---|---|---|
| | | Plates not over $\frac{7}{16}$ in. thick | Plates over $\frac{7}{16}$ in. thick |
| Iron or steel . | Screw stays riveted over . | 151 | 160 |
| Iron or steel . | " and nuts . . | 168 | 175 |
| Iron . . . | " and double nuts | 189 | 189 |
| Steel . . . | " " " | 212 | 212 |

For the limits of stress given above we get—

$$d_1 = k_1\, a \sqrt{p}$$
$$d = k\, a \sqrt{p} + 0{\cdot}055$$

where the constants have the following values:

| — | $k_1 =$ | $k =$ |
|---|---|---|
| Copper stays . . . . | 0·0178 to 0·0160 | 0·0196 to 0·0176 |
| Iron stays . . . . | 0·0146 to 0·0130 | 0·0160 to 0·0143 |
| Steel stays . . . . | 0·0126 to 0·0119 | 0·0139 to 0·0131 |

M. Le Chatellier has investigated the causes which lead to the troublesome fractures of locomotive fire-box stays. He

finds that on lighting the fire, the fire-box expands more than the outer casing, and this bends the stays. The deflection in the length of a stay may amount to 0·04 to 0·07 inch. When the steam pressure rises, the bending diminishes, and may even be reversed in direction. This bending fatigues the stays, and they break as if sheared, usually at the point of encastrement in the steel casing.

109. *Diagonal stays.*—The simplest form of diagonal stay is shown in fig. 104. Here it is connected by pins to angle irons riveted to the boiler shell. There is then no tendency to produce bending stresses in the stay. Very often such stays are forged out into palms at the ends and these are riveted to the boiler shell, but there will then be a bending couple acting on the stay of indeterminate amount. Now let A be the area of the end plate supported by the stay, and $p$ the pressure on it. Then the

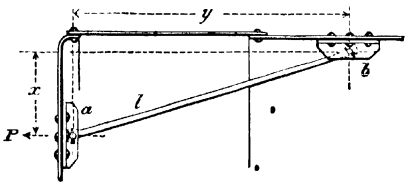



Fig 104

force P, acting at the end of the stay, which is the horizontal component of the tension in the stay, is $p$ A. Consequently, the tension in the stay is $p$ A $l/y$. The safe limit of stress for such stays is about $f =$ 5,000 lbs. per sq. in. for welded iron, 7,000 lbs. per sq. in. for unwelded iron, 8,000 lbs. per sq. in. for steel. Hence, the area of section $a$ of the stay is

$$a = (p \text{ A } l) / f y.$$

The horizontal shear on the rivets at $b$ is $p$ A; the vertical shear on the rivets at $a$ is $p$ A $x / y$; the horizontal pull on the rivet heads at $a$ is $p$ A; and the vertical pull on the rivet heads at $b$ is $p$ A $x / y$.

*Gusset stays.*—Fig. 105 shows the arrangement commonly adopted for staying the ends of Cornish and Lancashire boilers. The dotted lines in the end view show the areas which may be taken to be supported by each stay. The distance $x$ between the end rivets of the stays and the rivets of the flues is ordinarily

9 inches, and the unsupported space round the flues of this breadth is called the breathing space. If the gusset stays are brought too near the flues, so that the boiler ends are too rigid and cannot spring, considerable stresses are induced by the

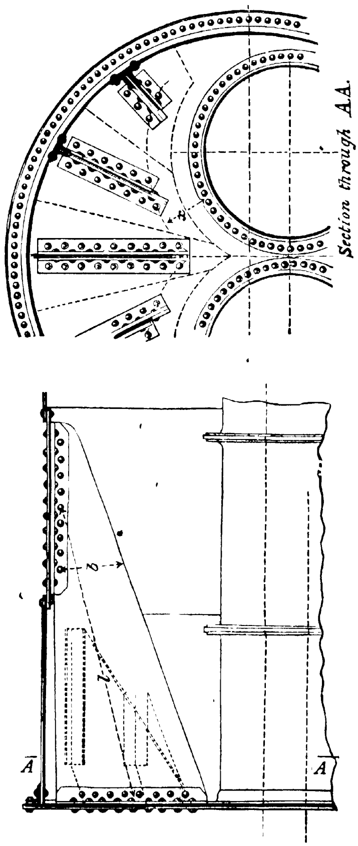



Fig. 105

expansion and contraction of the flue tubes. There is then danger of leakage at the joints of the flues or grooving round the edges of the flue angle irons. Below the flues the gusset stays must be brought nearer to the flues, say about 7 inches

instead of 9 inches. The least section of the gusset stays may be calculated as if they were simple diagonal stays by the rule above. The length of the stay may be taken as the distance $l$ between the centres of the end attachments, and its breadth $b$ may be measured at the first rivet at the narrow end.

110. *Bridge or girder staying.*—In many cases it is inconvenient to support the roofs of fire-boxes by ordinary stays. Then wrought-iron bars of considerable strength are placed over the plate to be supported, and rest at the ends on the side plates. To these bars the stays are attached. Fig. 106 shows the ordinary arrangement of bridge and stays. The bridge is sometimes a solid bar, as shown at $a$ and in plan at $b$. A cheaper form is shown at $c$, the bridge consisting of two flat plates connected by rivets with spacing ferrules and having room between the plates for the stays. The water space between girder and roof

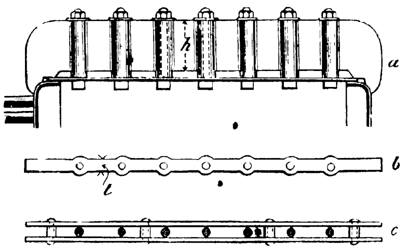



Fig. 106

of fire-box should be as open as possible and at least $1\frac{1}{2}$ inch in depth. The girders should have bearing enough at the ends to prevent crushing of the plates on which they rest. The pitch of the stays, when girders are used, is rarely less than 4 inches.

The girder or bridge is subjected to simple bending with an approximately uniform distributed load. Let $a$ be the area supported by one bolt, $p$ the steam pressure, then the load on the bolt is

$$P = p\,a.$$

The bolts may be designed for a stress of 7,000 lbs. per sq. in. or 9,000 lbs. if of steel, on the core section at the bottom of the thread. From a plan of the surface supported, the load on each bolt in the bridge can be found and the bending moment computed exactly. But if $l$ is the span of the bridge, $d$ the

ance between the bridges, centre to centre, $n$ the number of s in a bridge, then approximately

$$P = (p\,l\,d)\,/\,n$$

, also approximately, the bending moment at the centre ıe bridge is

$$M = (p\,d\,l^2)\,/\,8.$$

$h$ be the height of the bridge, $t$ its thickness. (If there are plates in each bridge, $t$ is the sum of their thicknesses.)

$$M = \tfrac{1}{6} f\,t\,h^2 = \tfrac{1}{8}\,p\,d\,l^2.$$

is assumed

$$h = 0{\cdot}87\sqrt{\frac{p\,d\,l^2}{f\,t}}$$

steel or steel castings, $f = 12{,}000$ to $13{,}000$ lbs. per sq. in.
n modern locomotive practice it is difficult to make simple ɛr stays strong enough for the high steam pressures now

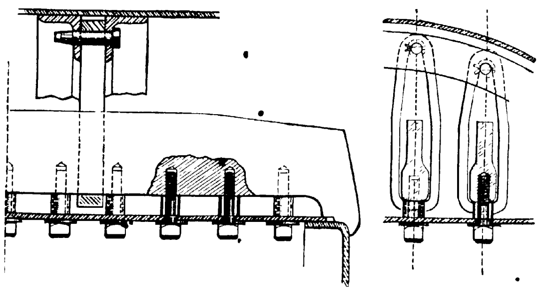


Fig 107

ted. Hence, the girder is hung by slings from the outer of the boiler. It thus becomes a continuous beam on ·al supports. The straining action is in this case really erminate, for it is impossible with any certainty to ascertain upporting forces at the ends and at the slings. If we could ne the girder supported by points at the same level exactly, with two slings dividing the girder into three equal spans ;reatest bending moment would be only $\frac{1}{90}\,p\,d\,l^2$ and with slings $\frac{1}{160}\,p\,d\,l^2$. But as minute differences of level very great differences in the supporting forces and ng moments, it would be unsafe to expect the bending

moments to be as small as this. Perhaps a fair approximate assumption is to take the supporting forces equal. Then the supporting forces and bending moments are as follows :

| | Reactions at ends and slings | Greatest bending moment |
|---|---|---|
| Two slings . . . . . . | $\frac{1}{4}\, p\, d\, l$ | $\frac{1}{24}\, p\, d\, l^2$ |
| Three slings . . . . | $\frac{1}{5}\, p\, d\, l$ | $\frac{1}{40}\, p\, d\, l^2$ |

Two slings : $$t\, h^2 = p\, d\, l^2 / 4 f$$

Three slings : $$t\, h^2 = 3\, p\, d\, l^2 / 20 f$$

An examination of some examples seems to show that the bridges would be safe designed for this estimate of the bending moment and the stress limits given above.

Fig. 107 shows girders with slings designed by Mr. Webb of Crewe. The stays are tapped into the bridge with spacing ferrules to prevent the straining of the fire-box plate.

# CHAPTER V

## ON FASTENINGS

### Screw Threads and Bolts

111. A *screw* is a cylindrical bar on which has been formed a helical projection or *thread*. The screw fits accurately into a hollow corresponding form, termed its *nut*. Pairs of elements thus formed are used in machinery (*a*) as fastenings, in which case they are commonly termed bolts; (*b*) for adjusting the relative position of two pieces; (*c*) for transmitting energy. It is chiefly as fastenings that they will be treated in the present chapter.

The helical thread may be right- or left-handed, and in practice is ordinarily right-handed, unless there is some special reason for using a left-handed screw. Multiple threaded screws are occasionally used.

Bolts or fastening screws are chiefly used to resist straining forces which act parallel to the axis of the bolt and normal to the surfaces connected together. The bolt is then in tension. When the straining force acts perpendicularly to the axis of the bolt and parallel to the surfaces connected, the bolt is in shear and is then equivalent to a rivet, and may be proportioned by the same rules. A bolt differs from a rivet in this, that it permits the connected pieces to be easily disconnected again when necessary.

### Forms and Proportions of Screw Threads

For manufacturing reasons it is of great importance that standard dimensions and forms of screw threads should be adopted. Screws of the same nominal dimensions should be interchangeable, otherwise great trouble and cost is involved in making and repairing machines. To secure this interchangeability a very high degree of precision is necessary in the taps, dies, and chasing tools used in cutting the screw threads. Inac-

curacy in these is often due to the irregular contraction of the steel in the process of hardening them. In a system of *standard screws*, a limited number of diameters must be selected; for each diameter the number of threads per inch must be specified, and the threads for each size must be of an accurately defined form. Screws varying in diameter, number of threads or form of threads from the standard proportions are *bastard* screws.

112. *Conditions of interchangeability.*—In a screw (fig. 108), $d$ is the full diameter, and $d_1$ the core diameter. The distance $d_e$ between the slopes of the threads is termed the effective diameter; $p$ is the pitch (reciprocal of the number of threads

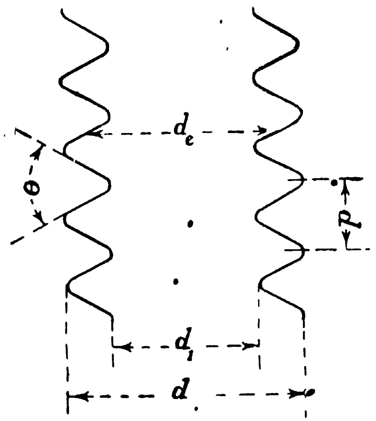



Fig. 108

per inch). In securing interchangeability, the most important conditions are uniformity of effective diameter and pitch. Next to these, uniformity of full and core diameters and thread angle. In most cases it is desirable that there should be a small clearance between the crest of the bolt and root of the nut thread and *vice versa*. The British Standards Committee have published tables giving the permissible 'tolerance' or error in pitch and diameters in screw threads of different types.[1]

113. *Whitworth system of screw threads.*—Sir J. Whitworth

[1] British Standard Systems for Screw Threads. London: Crosby Lockwood. 1907

first proposed a standard system of screw threads in 1841, and modified this a little in 1857 and 1861. Since then the Sellers system has been introduced and largely used in the United States, and other standard systems have been proposed in France and Germany. In selecting the form of screw thread the governing condition is that it should be capable of exact reproduction.

The Whitworth thread has an angle of 55°, and the top and bottom of the thread are rounded off. The objections to an exactly triangular thread are that the sharp outer angle is liable to injury and the sharp re-entrant angle weakens the bolt.

*Whitworth thread.*—The Whitworth thread is of the form shown in fig. 109; it has an angle of 55°, and one-sixth of the depth it would have, if exactly triangular, is rounded off at top and bottom. The radius of the rounding is 0·137 of the pitch. The pitch is fixed by practical experience so as to be suitable for either wrought or cast iron. The pitch and number of threads per inch are given in the following table for all standard sizes. The following approximate formulæ give values nearly the same as those in the table, and are convenient in calculations.

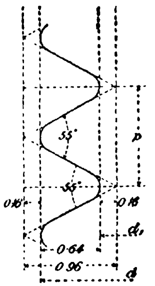



Fig. 109

Diameter at crest of thread $= d$.

Pitch $= p = 0{\cdot}08\, d + 0{\cdot}04$ nearly.

Number of threads per inch $= n = 1 / p$.

Core diameter or diameter at root of thread $= d_1 = 0{\cdot}9\, d - 0{\cdot}05$ nearly.

*Table of Whitworth Triangular Screw Threads*

| $d$ Diameter in ins. | | $n$ No. of threads per in. | $p$ Pitch in ins. | $d_1$ Diameter at bottom of thread in ins. | Area of section at bottom of thread in sq. ins. | Ratio $\frac{d_1}{d}$ |
|---|---|---|---|---|---|---|
| $\frac{1}{4}$ | 0·25 | 20 | 0·05 | ·186 | ·027 | ·75 |
| $\frac{5}{16}$ | ·313 | 18 | ·0555 | ·241 | ·046 | ·77 |
| $\frac{3}{8}$ | ·375 | 16 | ·0625 | ·295 | ·068 | ·79 |
| $\frac{7}{16}$ | ·438 | 14 | ·0714 | ·346 | ·094 | ·79 |
| $\frac{1}{2}$ | ·500 | 12 | ·0833 | ·393 | ·121 | ·79 |
| $\frac{5}{8}$ | ·625 | 11 | ·0909 | ·509 | ·204 | ·81 |
| $\frac{3}{4}$ | ·750 | 10 | ·100 | ·622 | ·304 | ·83 |
| $\frac{7}{8}$ | ·875 | 9 | ·111 | ·733 | ·422 | ·84 |
| 1 | 1·0 | 8 | ·125 | ·840 | ·554 | ·84 |
| $1\frac{1}{8}$ | 1·125 | 7 | ·143 | ·942 | ·697 | ·84 |

*Table of Whitworth Triangular Screw Threads*—(continued)

| $d$ Diameter in ins. | | $n$ No. of threads per in. | $p$ Pitch in ins. | $d_1$ Diameter at bottom of thread in ins. | Area of section at bottom of thread in sq ins. | Ratio $\frac{d_1}{d}$ |
|---|---|---|---|---|---|---|
| $1\frac{1}{4}$ | 1·25 | 7 | ·143 | 1·067 | ·894 | ·85 |
| $1\frac{3}{8}$ | 1·375 | 6 | ·167 | 1·161 | 1·060 | ·85 |
| $1\frac{1}{2}$ | 1·500 | 6 | ·167 | 1·286 | 1·300 | ·86 |
| $1\frac{5}{8}$ | 1·625 | 5 | ·200 | 1·369 | 1·472 | ·84 |
| $1\frac{3}{4}$ | 1·750 | 5 | ·200 | 1·494 | 1·753 | ·85 |
| $1\frac{7}{8}$ | 1·875 | $4\frac{1}{2}$ | ·222 | 1·590 | 1·986 | ·85 |
| 2 | 2·0 | $4\frac{1}{2}$ | ·222 | 1·715 | 2·311 | ·86 |
| $2\frac{1}{8}$ | 2·125 | $4\frac{1}{2}$ | ·222 | 1·840 | 2·659 | ·86 |
| $2\frac{1}{4}$ | 2·25 | 4 | ·250 | 1·930 | 2·926 | ·86 |
| $2\frac{1}{2}$ | 2·50 | 4 | ·250 | 2·180 | 3·733 | ·87 |
| $2\frac{3}{4}$ | 2·75 | $3\frac{1}{2}$ | ·286 | 2·384 | 4·464 | ·87 |
| 3 | 3·0 | $3\frac{1}{2}$ | ·286 | 2·634 | 5·450 | ·88 |
| $3\frac{1}{4}$ | 3·25 | $3\frac{1}{4}$ | ·308 | 2·855 | 6·402 | ·88 |
| $3\frac{1}{2}$ | 3·50 | $3\frac{1}{4}$ | ·308 | 3·105 | 7·563 | ·89 |
| $3\frac{3}{4}$ | 3·75 | 3 | ·333 | 3·323 | 8·673 | ·88 |
| 4 | 4·0 | 3 | ·333 | 3·573 | 10·027 | ·89 |
| $4\frac{1}{4}$ | 4·25 | $2\frac{7}{8}$ | ·348 | 3·804 | 11·370 | ·89 |
| $4\frac{1}{2}$ | 4·50 | $2\frac{7}{8}$ | ·348 | 4·054 | 12·910 | ·90 |
| $4\frac{3}{4}$ | 4·75 | $2\frac{3}{4}$ | ·364 | 4·284 | 14·414 | ·90 |
| 5 | 5·0 | $2\frac{3}{4}$ | ·364 | 4·534 | 16·146 | ·90 |
| $5\frac{1}{4}$ | 5·25 | $2\frac{5}{8}$ | ·381 | 4·762 | 17·810 | ·91 |
| $5\frac{1}{2}$ | 5·5 | $2\frac{5}{8}$ | ·381 | 5·012 | 19·73 | ·91 |
| $5\frac{3}{4}$ | 5·75 | $2\frac{1}{2}$ | ·400 | 5·239 | 21·55 | ·91 |
| 6 | 6·0 | $2\frac{1}{2}$ | ·400 | 5·489 | 23·65 | ·91 |

114. *British standard fine thread.*—For various purposes a thread of finer pitch than the Whitworth thread is required, and several systems of fine screw threads have been used by different manufacturers. The British Standards Committee have recommended the following system, which no doubt will be generally adopted. The pitch is based on the following rules :

For screws up to one inch in diameter,

$$p = \tfrac{1}{10} \sqrt[3]{d^2}.$$

For screws over one inch in diameter,

$$p = \tfrac{1}{10} \sqrt[8]{d^5}.$$

The form of thread is the same as that of the Whitworth

*screw, the angle being 55°, and the depth of thread, diminished by rounding, is 0·64 of the pitch. Approximately,*

$$d_1 = 0{\cdot}95\,d - 0.07.$$

*British Standard Fine Threads*

| $d$ Diameter in ins. | $n$ No. of threads per inch | $p$ Pitch in ins. | $d_1$ Diameter at bottom of thread in ins. | Area of section at bottom of thread in sq. in. | Ratio $\frac{d_1}{d}$ |
|---|---|---|---|---|---|
| $\frac{1}{4}$ | 25 | ·0400 | ·199 | ·0310 | ·80 |
| $\frac{5}{16}$ | 22 | ·0455 | ·254 | ·0508 | ·81 |
| $\frac{3}{8}$ | 20 | ·0500 | ·311 | ·0760 | ·83 |
| $\frac{7}{16}$ | 18 | ·0556 | ·366 | ·1054 | ·84 |
| $\frac{1}{2}$ | 16 | ·0625 | ·420 | ·1385 | ·84 |
| $\frac{9}{16}$ | 16 | ·0625 | ·483 | ·1828 | ·86 |
| $\frac{5}{8}$ | 14 | ·0714 | ·534 | ·2235 | ·86 |
| $\frac{3}{4}$ | 12 | ·0833 | ·643 | ·3250 | ·86 |
| $\frac{7}{8}$ | 11 | ·0909 | ·759 | ·4520 | ·87 |
| 1 | 10 | ·1000 | ·872 | ·5971 | ·87 |
| $1\frac{1}{8}$ | 9 | ·111 | ·983 | ·7585 | ·87 |
| $1\frac{1}{4}$ | 9 | ·111 | 1·108 | ·9637 | ·89 |
| $1\frac{3}{8}$ | 8 | ·125 | 1·215 | 1·1593 | ·88 |
| $1\frac{1}{2}$ | 8 | ·125 | 1·340 | 1·410 | ·89 |
| $1\frac{5}{8}$ | 8 | ·125 | 1·465 | 1·685 | ·90 |
| $1\frac{3}{4}$ | 7 | ·143 | 1·567 | 1·928 | ·90 |
| 2 | 7 | ·143 | 1·817 | 2·593 | ·91 |
| $2\frac{1}{4}$ | 6 | ·167 | 2·037 | 3·257 | ·91 |
| $2\frac{1}{2}$ | 6 | ·167 | 2·287 | 4·106 | ·92 |
| $2\frac{3}{4}$ | 6 | ·167 | 2·537 | 5·053 | ·92 |
| 3 | 5 | ·200 | 2·744 | 5·913 | ·92 |
| $3\frac{1}{4}$ | 5 | ·200 | 2·994 | 7·040 | ·92 |
| $3\frac{1}{2}$ | $4\frac{1}{2}$ | ·222 | 3·215 | 8·120 | ·92 |
| $3\frac{3}{4}$ | $4\frac{1}{2}$ | ·222 | 3·465 | 9·432 | ·92 |
| 4 | $4\frac{1}{2}$ | ·222 | 3·715 | 10·842 | ·93 |
| $4\frac{1}{2}$ | 4 | ·250 | 4·180 | 13·721 | ·93 |
| 5 | 4 | ·250 | 4·680 | 17·20 | ·94 |
| $5\frac{1}{2}$ | $3\frac{1}{2}$ | ·286 | 5·134 | 20·70 | ·93 |
| 6 | $3\frac{1}{2}$ | ·286 | 5·634 | 24·93 | ·94 |

*British Association standard screw threads.*—For small screws such as are used by opticians and electrical instrument makers another system has been devised by a committee of the British Association, recommended by the Standards Committee, and has been generally adopted. The angle of the thread is $47\frac{1}{2}°$; the threads are rounded equally at crest and root to a radius of nearly $\frac{2}{11}$ths of the pitch, and the depth of thread is 0·6 of the pitch. For such small screws metric dimensions are commonly used. The following table gives the metric dimensions and the equivalent in inches :

## *British Association Screw Threads*

| Designating No. | Diameter in mm. | Diameter in ins. | Pitch in mm. | Pitch in ins. | Depth of thread in m.m. | Diameter at bottom of thread in m m. | Diameter at bottom of thread in ins. |
|---|---|---|---|---|---|---|---|
| 0 | 6·0 | ·236 | 1·0 | ·039 | ·6 | 4·8 | ·189 |
| 1 | 5·3 | ·209 | 0·9 | ·035 | ·54 | 4·22 | ·166 |
| 2 | 4·7 | ·185 | ·81 | ·032 | ·485 | 3·73 | ·147 |
| 3 | 4·1 | ·161 | ·73 | ·029 | ·44 | 3·22 | ·127 |
| 4 | 3·6 | ·142 | ·66 | ·026 | ·395 | 2·81 | ·111 |
| 5 | 3·2 | ·126 | ·59 | ·023 | ·355 | 2·49 | ·098 |
| 6 | 2·8 | ·110 | ·53 | ·021 | ·32 | 2·16 | ·085 |
| 7 | 2·5 | ·098 | ·48 | ·0189 | ·29 | 1·92 | ·076 |
| 8 | 2·2 | ·087 | ·43 | ·0169 | ·26 | 1·68 | ·066 |
| 9 | 1·9 | ·075 | ·39 | ·0154 | ·235 | 1·43 | ·056 |
| 10 | 1·7 | ·067 | ·35 | ·0138 | ·21 | 1·28 | ·050 |
| 11 | 1·5 | ·059 | 31 | ·0122 | ·185 | 1·13 | ·044 |
| 12 | 1·3 | ·051 | ·28 | ·0110 | ·17 | ·96 | ·038 |
| 13 | 1·2 | ·047 | ·25 | ·0098 | ·15 | ·9 | ·0354 |
| 14 | 1·0 | ·039 | ·23 | ·0091 | ·14 | ·72 | ·0283 |
| 15 | 0·9 | ·035 | ·21 | ·0083 | ·125 | ·65 | ·0256 |
| 16 | ·79 | ·031 | ·19 | ·0075 | ·115 | ·56 | ·0220 |
| 17 | ·70 | ·028 | ·17 | ·0067 | ·10 | ·50 | ·0197 |
| 18 | ·62 | ·024 | ·15 | ·0059 | ·09 | ·44 | ·0173 |
| 19 | ·54 | ·021 | ·14 | ·0055 | ·085 | ·37 | ·0146 |
| 20 | ·48 | ·019 | ·12 | ·0047 | ·07 | ·34 | ·0134 |
| 21 | ·42 | ·017 | ·11 | ·0043 | ·065 | ·29 | ·0114 |
| 22 | ·37 | ·015 | ·10 | ·0039 | ·06 | ·25 | ·0098 |
| 23 | ·33 | ·013 | ·09 | ·0035 | ·055 | ·22 | ·0087 |
| 24 | ·29 | ·011 | ·08 | ·0031 | ·05 | ·19 | ·0075 |
| 25 | ·25 | ·010 | ·07 | ·0028 | ·04 | ·17 | ·0067 |

*British standard pipe threads.*—For the wrought-iron and steel tubes used largely in conveying gas, and for many other purposes, a form of thread has long been used known commonly as the gas thread. This thread has been recently revised by the Standards Committee. The form of thread is the same as the Whitworth thread, the angle being 55°, and the depth of thread approximately 0·64 of the pitch.

115. *Sellers' screw thread system.*—In the American or Sellers system of screw thread, which is practically identical with that shown in fig. 110, the angle of the thread is 60°, and the threads are truncated at top and bottom. This form can perhaps be produced more easily than the Whitworth thread, but as the taps and dies wear the threads become somewhat rounded.

There has been an impression that the Sellers thread is more

rational in form than the Whitworth. There is little ground for this, and the sharp corners are elements of weakness. The Standards Committee made some shock tests on both forms. For very mild steel there was not much difference in strength, but the Whitworth screw had a small advantage. With 40- or 50-ton steel, such as is now often used, the Whitworth screw carried a larger number of blows before breaking than the Sellers thread.

*British Standard Pipe Threads*

| Nominal bore of pipe in ins. | Outside diameter of thread in ins. $d$ | Depth of thread in ins. | Diameter at bottom of thread in ins. $d_1$ | No. of threads per in. $n$ |
|---|---|---|---|---|
| ¼ | ·383 | ·023 | ·337 | 28 |
| ⅜ | ·656 | ·034 | ·589 | 19 |
| ½ | ·825 | ·046 | ·734 | 14 |
| ⅝ | ·902 | ·046 | ·811 | 14 |
| ¾ | 1·041 | ·046 | ·950 | 14 |
| 1 | 1·309 | ·058 | 1·193 | 11 |
| 1¼ | 1·650 | ·058 | 1·534 | 11 |
| 1½ | 1·882 | ·058 | 1·766 | 11 |
| 1¾ | 2·116 | ·058 | 2.000 | 11 |
| 2 | 2·347 | ·058 | 2·231 | 11 |
| 2¼ | 2·587 | ·058 | 2·471 | 11 |
| 2½ | 2·96 | ·058 | 2·844 | 11 |
| 3 | 3·46 | ·058 | 3·344 | 11 |
| 4 | 4·45 | ·058 | 4·334 | 11 |
| 5 | 5·45 | ·058 | 5·334 | 11 |
| 6 | 6·45 | ·058 | 6·334 | 11 |
| 8 | 8·45 | ·064 | 8·322 | 10 |
| 10 | 10·45 | ·064 | 10·32 | 10 |
| 12 | 12·45 | ·080 | 12·29 | 8 |
| 15 | 15·68 | ·080 | 15·52 | 8 |
| 18 | 18·68 | ·080 | 18·52 | 8 |

The number of threads per inch for different diameters of bolt is given in the table below. This thread system is termed the U.S. standard thread, but it is not in universal use in the United States. The following equations give very approximately the pitch and diameter at bottom of thread in the Sellers system.

$$p = 0{\cdot}24\sqrt{d + 0{\cdot}625} - 0{\cdot}175$$

$$n = 1/p$$

$$d_1 = 0{\cdot}91d - 0{\cdot}08 \text{ nearly.}$$

The width of flat at top and bottom of thread is ⅛th of the pitch. The tap for cutting nuts has a diameter at the top of the

thread slightly greater than the screw to allow a small clearance. The clearance is from 0·004 for ¼-inch to 0·01 for 2-inch.

*Table of Sellers' U.S. Standard Threads*

(From 'Standards of Length' by G. M. Bond, M. E. Pratt & Whitney Company.)

| $d$ Diameter in ins. | Number of threads per in. | $p$ Pitch of thread in ins. | $d_1$ Diameter at bottom of thread in ins. | Area at bottom of thread in sq. ins. | $\frac{d_1}{d}$ |
|---|---|---|---|---|---|
| ¼ | 20 | 0·05 | 0·1850 | 0·027 | ·74 |
| 5/16 | 18 | ·0555 | ·2403 | ·045 | ·77 |
| ⅜ | 16 | ·0625 | ·2938 | ·068 | ·79 |
| 7/16 | 14 | ·0714 | ·3447 | ·093 | ·79 |
| ½ | 13 | ·0769 | ·4001 | ·126 | ·80 |
| 9/16 | 12 | ·0833 | ·4542 | ·162 | ·80 |
| ⅝ | 11 | ·0909 | ·5069 | ·202 | ·81 |
| ¾ | 10 | ·1000 | ·6201 | ·302 | ·83 |
| ⅞ | 9 | ·1111 | ·7307 | ·420 | ·83 |
| 1 | 8 | ·1250 | ·8376 | ·550 | ·84 |
| 1⅛ | 7 | ·1429 | ·9394 | ·694 | ·83 |
| 1¼ | 7 | ·1429 | 1·0644 | ·893 | ·85 |
| 1⅜ | 6 | ·1667 | 1·1585 | 1·057 | ·84 |
| 1½ | 6 | ·1667 | 1·2835 | 1·295 | ·86 |
| 1⅝ | 5½ | ·1818 | 1·3888 | 1·515 | ·85 |
| 1¾ | 5 | ·2000 | 1·4902 | 1·746 | ·85 |
| 1⅞ | 5 | ·2000 | 1·6152 | 2·051 | ·86 |
| 2 | 4½ | ·2222 | 1·7113 | 2·302 | ·86 |
| 2¼ | 4½ | ·2222 | 1·9613 | 3·023 | ·87 |
| 2½ | 4 | ·2500 | 2·1752 | 3·719 | ·87 |
| 2¾ | 4 | ·2500 | 2·4252 | 4·620 | ·88 |
| 3 | 3½ | ·2857 | 2·6288 | 5·428 | ·88 |
| 3¼ | 3½ | ·2857 | 2·8788 | 6·510 | ·88 |
| 3½ | 3¼ | ·3077 | 3·1003 | 7·548 | ·88 |
| 3¾ | 3 | ·3333 | 3·3170 | 8·641 | ·88 |
| 4 | 3 | ·3333 | 3·5670 | 9·963 | ·89 |
| 4¼ | 2⅞ | ·3478 | 3·7982 | 11·329 | ·89 |
| 4½ | 2¾ | ·3636 | 4·0276 | 12·753 | ·90 |
| 4¾ | 2⅝ | ·3810 | 4·2551 | 14·226 | ·90 |
| 5 | 2½ | ·4000 | 4·4804 | 15·763 | ·90 |
| 5¼ | 2½ | ·4000 | 4·7304 | 17·572 | ·90 |
| 5½ | 2⅜ | ·4211 | 4·9530 | 19·267 | ·90 |
| 5¾ | 2⅜ | ·4211 | 5·2030 | 21·262 | ·90 |
| 6 | 2¼ | ·4444 | 5·4226 | 23·098 | ·90 |

*Metric screw thread system.*—The Whitworth system of screw threads has been and is still largely used on the Continent, where the metric system of measurement prevails. But it has the obvious inconvenience of being based on the inch unit. Hence various metric systems have been proposed. Fig. 110 shows the 'International System,' agreed to by a conference of

Continental engineers at Zurich in 1898. The angle of thread is 60°. The total depth of thread before removing the points of the triangle is 0·866 $p$. One-eighth of this depth is cut off the points of the threads, and $\frac{1}{16}$th at the bottom of the threads, so that there is a clearance top and bottom.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diameter | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 14 | 16 | 18 | 20 mm. |
| Pitch . | 1 | 1 | $1\frac{1}{4}$ | $1\frac{1}{4}$ | $1\frac{1}{2}$ | $1\frac{1}{2}$ | $1\frac{3}{4}$ | 2 | 2 | $2\frac{1}{2}$ | $2\frac{1}{2}$ ,, |
| Diameter | 22 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 | 48 | ,, |
| Pitch . | $2\frac{1}{2}$ | 3 | 3 | $3\frac{1}{2}$ | $3\frac{1}{2}$ | 4 | 4 | $4\frac{1}{2}$ | $4\frac{1}{2}$ | 5 | ,, |

116. *Screw threads for special purposes.*—When a single screw for a special purpose is required it is not important to adhere to

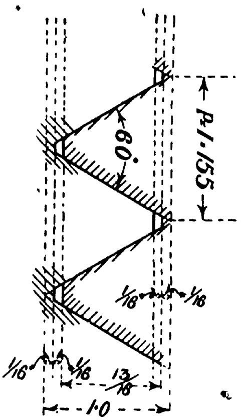



Fig. 110

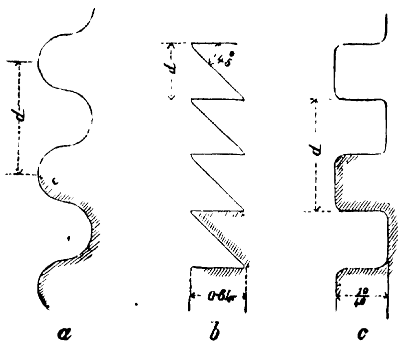



Fig. 111

standard proportions, and screw threads of various special forms, chased in the lathe, are used. Such screws may be single or multiple threaded, and right- or left-handed as is most convenient. The square thread, fig. 111 $c$, is used for the leading screw of lathes and in other cases where the screw drives a moving piece. With this thread there is no oblique pressure tending to burst the nut, and the bearing surface may be made large. Square threads are more costly to cut than triangular threads. Sometimes the thread is slightly trapezoidal to facilitate the engagement of the half nuts which drive the lathe saddle. The thickness of thread and width of space are generally each $p/2$, but sometimes the thickness of thread is $\frac{5}{8}\,p$ and the width of space $\frac{3}{8}\,p$,

to reduce the amount of metal removed in chasing. The following rules give ordinary proportions of square-threaded screws :

$$\text{Pitch} = p = 0{\cdot}16\,d + 0{\cdot}08$$

$$\text{Threads per inch} = n = 1/p$$

$$\text{Diameter at bottom of thread}$$

$$= d_1 = d - \frac{3{\cdot}8}{n} = 0{\cdot}85\,d - 0{\cdot}075$$

$$\text{Depth of thread, } 19\,p/40$$

Fig. 111 *b* is a section of a buttress thread which in some respects combines the advantages of a triangular and a square thread. The acting surface is normal to the axial thrust, as in a square thread, but in other respects it is similar to a triangular thread. The breech screw of large guns is of this form. The pitch and depth of thread may be the same as for a Whitworth

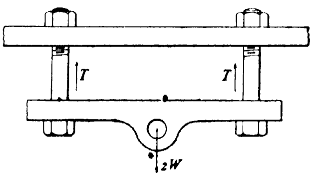



Fig. 112

or Sellers thread, or may be chosen arbitrarily. Fig. 111 *a* is a modified square thread, sometimes termed a knuckle thread, which is convenient for screws subjected to rough usage.

## General Considerations on the Strength and Friction of Screws

117. *Straining action on bolts.*—Suppose a plate suspended by two bolts and a load 2 W hung midway from it (fig. 112). Then each bolt is subjected to a simple axial tension

$$T = W \qquad \ldots \ldots \quad (1)$$

and the case presents no ambiguity. Commonly, however, bolts are used to connect pieces to some extent elastic, which are screwed tightly together, with more or less compressible packing between them, and there is an initial tension on each bolt before the load 2 W is applied. This tension is balanced by the elastic

reaction of the pieces fastened together. The case may be diagrammatically represented by imagining that, while the plates are rigid, elastic springs are interposed (fig. 113). Then, the arrangement being symmetrical, if $C_1$ is the initial thrust of each spring it is also the initial tension in each bolt due to screwing up. If the extension of the bolts is $\alpha$ and the compression of the springs $\beta$ per unit of load, the initial compression of the springs

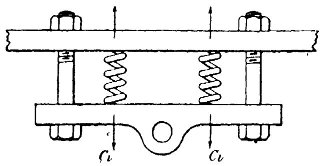



Fig. 113

due to screwing up is $\beta C_1$, and the corresponding extension of the bolts is $\alpha C_1$.

Now, after screwing up, let a load $2W$ be applied (fig. 114). In general, the tension of the bolts will increase to a value $T$ between $W$ and $W + C_1$, and the additional extension of the bolts will diminish the compression of the springs. If $2W$ is increased till the additional extension of the bolts is equal to, or exceeds, the initial compression of the springs, the thrust of the springs

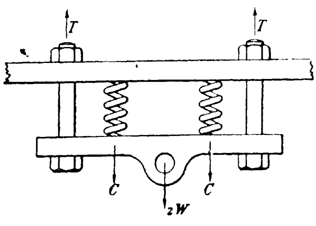



Fig. 114

vanishes, and the tension in the bolts is simply that due to the load, as in Eq. (1). In most practical cases the bolts must be screwed up initially so as to prevent this, or there will be a sensible separation of the connected pieces; and if there is a joint there would be leakage.

When the load $2W$ is added, the additional extension of the bolts is $\alpha(T - C_1)$, and the compression of the springs diminishes

to $\beta c_1 - \alpha(T - c_1)$. If c is the thrust of the springs in this condition—

$$\beta c = \beta c_1 - \alpha (T - c_1)$$

$$c = c_1 - \frac{\alpha}{\beta} (T - c_1)$$

$$T = W + c = W + c_1 - \frac{\alpha}{\beta} (T - c_1)$$

$$= \frac{\beta}{\beta + \alpha} W + c_1 \quad . \quad . \quad : \quad (2)$$

If the springs are very compressible, so that $\beta$ is large compared with $\alpha$, the tension when the load is applied is approximately $W + c_1$. If the compressibility of the springs and the extensibility of the bolts are equal, so that $\beta = \alpha$, the tension in each bolt is $\frac{1}{2} W + c_1$. Lastly, if the pieces connected are very rigid, so that $\beta$ is very small compared with $\alpha$, the tension is hardly increased by the application of the load and the tension in the bolts is $c_1$ nearly.

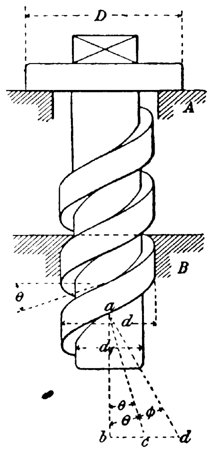


Fig. 115

118. *Friction in tightening up a square-threaded screw.*—Suppose that the square-threaded screw (fig. 115) is tightened by a force F applied at a leverage, $l$—for instance, by a spanner applied to the bolt head. The bolt connects two pieces, A and B, and there is a resistance to their relative approach either because B supports a load W, or because the elasticity of A and B, or of the packing between them, resists compression. This resistance is symmetrically disposed, and its resultant is an axial force T, which is the tension in the body of the bolt.

Let $d$ be the full and $d_1$ the core diameter of the bolt, D the diameter of the collar. Let $\mu = \tan \phi$ be the coefficient of friction between the bolt and B, and $\mu' = \tan \phi'$, the coefficient of friction between the collar and A. If $r$ is the mean frictional radius of the thread and R the mean frictional radius of the collar, then, to a sufficient approximation, $r = \frac{1}{4} (d + d_1)$ and $R = \frac{1}{4} (d + D)$.

Let $p$ be the pitch; then the inclination of the thread at radius $r$ to the normal to the axis is given by the relation

$\tan\theta = p/2\pi r$. If the thread were frictionless the pressure on it would be normal to the thread, or along $a\,c$, which makes an angle $\theta$ with the axis of the screw. In consequence of friction the pressure makes an angle $\phi$ with the normal to the thread, or acts along $a\,d$ at an angle $\theta + \phi$ with the axis of the screw. Then the screw is in equilibrium under the action of an axial force T along $a\,b$, a horizontal force H at radius $r$ along $b\,d$ due to the force applied to the spanner, and a pressure Q between the threads of bolt and nut along $a\,d$, making an angle $\theta + \phi$ with the axis. Hence

$$\mathrm{T} : \mathrm{H} : \mathrm{Q} :: a\,b : b\,d : a\,d,$$

$$\mathrm{H} = a\,b \tan(\theta + \phi) = \mathrm{T}\tan(\theta + \phi).$$

The moment of H about the axis of the screw is H $r$. Hence

$$\begin{aligned}\mathrm{H}\,r &= \mathrm{T}\,r\tan(\theta + \phi)\\ &= \mathrm{T}\,r\,\frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi},\end{aligned}$$

or, inserting the values of $\tan\theta$ and $\tan\phi$ above,

$$\mathrm{H}\,r = \mathrm{T}\,r\,\frac{p + 2\mu\pi r}{2\pi r - \mu p} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The collar friction is $\mu'$ T, and it acts at radius R. The moment of collar friction is

$$\mathrm{H}'\,\mathrm{R} = \mu'\,\mathrm{T}\,\mathrm{R}$$

The effort F at the spanner, acting at a leverage $l$, balances the frictional resistances of the screw.

$$\mathrm{F}\,l = \mathrm{H}\,r + \mathrm{H}'\,\mathrm{R} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Taking $\mu = \mu' = 0{\cdot}125$, and $\mathrm{R} = 1{\cdot}4\,r$,

$$\mathrm{F}\,l = \mathrm{T}\,r\left\{\frac{{\cdot}785\,r + p}{6{\cdot}28\,r - 0{\cdot}125\,p} + 0{\cdot}175\right\}$$

The useful work in tightening the screw or driving against a resistance T is T $p$ per revolution, and the work expended at the spanner is $2\pi\,\mathrm{F}\,l$. Hence the efficiency is,

$$\eta = (\mathrm{T}\,p)\,/\,(2\pi\,\mathrm{F}\,l) \quad . \quad . \quad . \quad . \quad (5)$$

The following are ordinary proportions of square-threaded

screws, and from these the turning moments and efficiency are calculated.

| $d$ | $d_1$ | $p$ | $r$ | $\frac{F\,l}{1\,r}$ | $\eta$ |
|---|---|---|---|---|---|
| 1 | 0·68 | ·24 | ·44 | ·215 | ·40 |
| 2 | 1·63 | ·40 | ·90 | ·197 | ·36 |
| 4 | 3·33 | ·72 | 1·83 | ·203 | ·31 |

The larger part of the inefficiency is due to the friction of the nut or collar.

119. *Co-efficient of friction for screws.*—Experiments have been made by Kingsbury,[1] on the friction between square-threaded screws and nuts of cast iron, steel, wrought iron and brass. The screws were 1·426 full diameter and 1·278 core diameter, and 0·33 inch pitch. The total thread bearing area was approximately one square inch. The axial thrust on the screw varied up to 14,000 lbs., producing an axial stress of 11,000 lbs. per square inch on the core section, or 14,000 lbs. per square inch of projected bearing surface of thread. The nuts fitted the screw loosely, and the threads had been worn to good condition. Each screw was turned about one revolution in two minutes. The screw and nut were amply lubricated at the beginning of a set of four or eight tests, and not lubricated again. No marked difference of friction was noted at different pressures or with different metals. The following are the results with different lubricants.

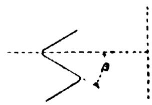



Fig. 116

*Coefficients of Friction*

| | Coefficient $\mu$ = | | |
|---|---|---|---|
| | Minimum | Maximum | Mean |
| Lard oil | ·09 | ·25 | ·11 |
| Heavy mineral machinery oil | ·11 | ·19 | ·14 |
| Heavy oil and graphite | ·03 | ·15 | ·07 |

120. *Friction of an angular threaded screw.*—Let $2\beta$ (fig. 116) be the angle between the sides of an angular screw thread, so

[1] *Trans. Am. Soc. Mechanical Engineers*, xvii. 96.

that an axial section of the thread makes an angle $\beta$ with the normal to the axis. Then the pressure between the thread of screw and nut is increased, because it is oblique to the axial tension. The moment required to tighten a bolt against the tension T, including the friction of the thread and collar or nut, is

$$\mathrm{F}\,l = \mathrm{T}\left\{\frac{\tan\theta + \mu\cos\theta\sqrt{(1+\tan^2\theta+\tan^2\beta)}}{1-\mu\sin\theta\sqrt{(1+\tan^2\theta+\tan^2\beta)}}\,r + \mu'\,\mathrm{R}\right\}$$

Since $\theta$ is small compared with $\beta$, $\tan^2\theta$ may be neglected. Then

$$\mathrm{F}\,l = \mathrm{T}\left\{\frac{\tan\theta + \mu\cos\theta\sec\beta}{1-\mu\sin\theta\sec\beta}\,r + \mu'\,\mathrm{R}\right\}$$

But $\tan\theta = p/2\pi r$, and as $\theta$ is small $\sin\theta = p/2\pi r$ nearly and $\cos\theta = 1$ nearly.

$$\mathrm{F}\,l = \mathrm{T}\left\{\frac{p + 2\pi\mu r\sec\beta}{2\pi r - \mu p\sec\beta}\,r + \mu'\mathrm{R}\right\}^{1}$$

Let $$\frac{p + 2\pi\mu r\sec\beta}{2\pi r - \mu p\sec\beta} = x,$$

and let $$\mu'\mathrm{R}/r = y.$$

Then, $$\mathrm{F}\,l = \mathrm{T}\,r\,(x+y) \quad . \quad . \quad . \quad . \quad (6)$$

For a Whitworth screw, $\beta = 27\frac{1}{2}°$; for a nut, $\mathrm{R} = 1{\cdot}4\,r$ nearly, and then, if $\mu = \mu' = 0{\cdot}125$, values corresponding to fairly good lubrication,

$$x = \frac{p + 0{\cdot}885\,r}{6{\cdot}28\,r}; \quad y = 0{\cdot}175$$

If $\mu = \mu' = 0{\cdot}25$ corresponding to bad or slight lubrication,

$$x = \frac{p + 1{\cdot}77\,r}{6{\cdot}28\,r}; \quad y = 0{\cdot}35$$

The work done against the axial resistance T is T$p$ per revolution,

[1] If the screw is slackened instead of tightened, then the pressure between the screw threads acts at $\theta - \phi$ with the axis of the screw, and then

If $\phi > \theta$ $$\mathrm{F}\,l = \mathrm{T}\left\{\frac{2\mu\pi r - p}{2\pi r + \mu p}\,r + \mathrm{R}\,\mu'\right\}$$

If $\phi < \theta$ $$\mathrm{F}\,l = \mathrm{T}\left\{\mathrm{R}\,\mu' - \frac{p - 2\mu\pi r}{2\pi r + \mu p}\,r\right\}$$

and the work expended at the spanner is $2\pi F l$. Hence the efficiency of the screw is

$$\eta = (T p)/(2\pi F l) \quad . \quad . \quad . \quad (7)$$

$$= p/\{2\pi r(x+y)\} \quad . \quad . \quad . \quad (7a)$$

The following table gives the calculated efficiency of some Whitworth screws.

| Diameter of screw $d$ in ins. | Diameter of core $d_1$ in ins. | Mean radius of thread $r$ | Pitch $p$ | $\frac{H r}{T r} = x$ | $\frac{H' r}{T r} = y$ | $\frac{F l}{T r}$ | Efficiency $\eta$ |
|---|---|---|---|---|---|---|---|
| | | | $\mu = \mu' = 0{\cdot}125$ | | | | |
| $\frac{5}{8}$ | ·509 | ·284 | ·091 | ·192 | ·175 | ·367 | ·140 |
| 1 | ·840 | ·460 | ·125 | ·184 | ·175 | ·359 | ·121 |
| 2 | 1·715 | ·929 | ·222 | ·179 | ·175 | ·354 | ·107 |
| 4 | 3·573 | 1·893 | ·333 | ·204 | ·175 | ·379 | ·074 |
| | | | $\mu = \mu' = 0{\cdot}25$ | | | | |
| $\frac{5}{8}$ | ·509 | ·284 | ·091 | ·333 | ·35 | ·683 | ·075 |
| 1 | ·840 | ·460 | ·125 | ·325 | ·35 | ·675 | ·065 |
| 2 | 1·715 | ·929 | ·222 | ·320 | ·35 | ·670 | ·057 |
| 4 | 3·573 | 1·893 | ·333 | ·309 | ·35 | ·659 | ·043 |

It is clear in this table that the quantity $F l/T r$ does not vary much with different sized bolts. If $r = 0{\cdot}45\, d$, where $d$ is the outside diameter of the bolt, then approximately,

| | $F l =$ |
|---|---|
| $\mu = \mu' = 0{\cdot}125$ | $\cdot 36\, T r = 0{\cdot}16\, d\, T$ |
| $= 0{\cdot}25$ | $\cdot 67\, T r = 0{\cdot}30\, d\, T$ |

*Tensile stress on bolts due to screwing up.*—If the bolt is unloaded so that the resistance is entirely due to the spring of the flanges and packing which are being compressed, then the total tension $T$ in the bolt depends only on the spanner moment $F l$ which the workman happens to apply. Necessarily this is a very uncertain quantity, but (1) the effective spanner length is usually proportional to the bolt diameter, and (2) the effort of the workman is almost certainly greater as the bolt diameter is greater.

Suppose the spanner length is $15\, d$ and the workman's effort is taken, merely as a provisional hypothesis, at $40\, d$ lbs. Then, in tightening up, the spanner moment is $F l = 600\, d^2$ inch pounds.

Equating this to the value found above for the smaller coefficient of friction

$$600\, d^2 = 0{\cdot}16\, d\, \text{T}$$
$$\text{T} = 3750\, d$$

The tensile stress on the core section of the bolt, taking diameter at the bottom of the thread to be $d_1 = 0{\cdot}8\, d$, is [1]

$$f_t = \text{T} \Big/ \left(\frac{\pi}{4} d_1^{\,2}\right) = 2\,\text{T} / d^2$$
$$= 7500 / d \text{ lbs. per sq. in.}$$

| | | | | |
|---|---|---|---|---|
| Diameter of bolt $d$ = | $\frac{5}{8}$ | 1 | 2 | 4 ins. |
| Total tension T | = 2,343 | 3,750 | 7,500 | 15,000 lbs. |
| Tensile stress $f_t$ | = 12,000 | 7,500 | 3,750 | 1,875 lbs. per sq. in. |

As the load is mainly a steady or permanent load, the bolts will carry a working stress of 15,000 lbs. per square inch. But clearly the tensile stress in small bolts in joints due merely to screwing up may be very serious, though it becomes negligibly small in large bolts. This is consistent with the experience that bolts less than $\frac{5}{8}$ inch should not be used in flanges, as they may be overstrained or broken if the workman making the joint is careless.

121. *Additional stress due to torsion.*—The twisting moment on the body of the screw is the spanner moment less the moment of the nut or collar friction. That is, the twisting moment is $x\, \text{T}\, r$. This produces a shearing stress

$$f_s = \frac{x\, \text{T}\, r}{0{\cdot}196\, d_1^{\,3}} = \frac{4{\cdot}5\, \text{T}\, x}{d^2} \text{ nearly.}$$

The stress due to the tension is

$$f_t = 2\,\text{T}/d^2 \text{ nearly}$$
$$f_s / f_t = 2{\cdot}25\, x \text{ nearly}$$
$$\mu = \mu' = 0{\cdot}125, \quad x = 0{\cdot}19 \text{ nearly}, \quad f_s / f_t = 0{\cdot}43 \text{ nearly.}$$

Combining the tensile and shearing stress, the resultant stress is $1{\cdot}16 f_t$, or the torsion increases the stress by 16 per cent. As the calculation of the strength of flange bolts is at best a very rough one, the effect of torsion may be neglected.

[1] Values of the ratio $d_1 / d$ for bolts of different sizes are given in the tables in §§ 113, 114, 115.

122. *Experimental determination of the stress in bolts due to screwing up.*—Experiments have been made at Sibley College on the tension in bolts of $\frac{1}{2}$ to $1\frac{1}{4}$ inch in diameter, with faced and rough nuts and washers, when screwed up by experienced workmen. The axial tension was measured by a testing machine, and the workmen were directed to use such force as in their judgment would ordinarily be necessary in making a leakage tight joint. The effective spanner length was 15 $d$. The results showed that the workmen graduated the effort on the spanner, so that it was, on the average, proportional to the diameter of the bolt. The average bolt tension was about

$$T = 16,000\, d \text{ lbs.} \qquad (8)$$

but in some cases this initial tension was considerably greater. Using this value, the following are the initial stresses due to screwing up on bolts of various sizes with Sellers threads. For Whitworth threads the results would be practically the same.

| | | | | | |
|---|---|---|---|---|---|
| Diameter of bolts $d$ | $\frac{5}{8}$ | 1 | 2 | 4 | ins. |
| Total tension T | 10,000 | 16,000 | 32,000 | 64,000 | lbs. |
| Stress per sq in. of core section | 49,500 | 29,100 | 14,000 | 6,430 | lbs. per sq. in. |

These are much larger values than those calculated above, and in the case of the $\frac{5}{8}$-bolt the breaking stress is nearly reached. Even for the 1-inch bolt the yield point is nearly or quite reached, especially if the additional torsional stress is considered. It is hardly possible to think that these stresses are ordinarily reached in making leakage-tight joints; but the experiments show that in such cases the initial stress may be very serious indeed in the smaller sizes of bolt, and that probably in almost all cases the stress due to screwing up is much larger than any stress due to the steam or water pressure. Probably to make a tight joint the initial tension of the bolts must be a multiple of the steam or water pressure, and only in that sense is it reasonable to calculate the bolts with reference to the load due to the fluid pressure in the vessel.

123. *Bearing pressure on screw threads. Fastening screws.*—Let $d$ be the diameter of the screw, $d_1$ the diameter at root of thread, $p$ the pitch, all in inches; also $n$ the number of threads per inch and H the height of the nut. Then the number of coils of the thread in bearing in the nut is $N = H/p = H\,n$. The

total projected area of threads in bearing (on a plane normal to the load) is $\frac{\pi}{4}(d^2 - d_1^2)$ N $= \frac{\pi}{2} d (d - d_1)$ N nearly. Let $k$ be the permissible pressure per sq. in. of projection of bearing surface and P the total load on the bolt in lbs. Then

$$\text{P} = \text{or} < k \frac{\pi}{4} (d^2 - d_1^2) \text{ N},$$

or the greatest permissible load is

$$\text{P} = \frac{\pi}{2} k\, d\, (d - d_1) \text{ N} \quad . \quad . \quad . \quad (9)$$

For wrought-iron fastening screws, on wrought iron or bronze, it is generally taken that $k$ should not exceed 2,100 lbs. per sq. in., and for steel screws on steel or bronze 2,700 lbs. per sq. in.; the nuts being of iron, steel, or gunmetal. Then the greatest load is

$$\text{P} = 3{,}300 \text{ to } 4{,}240\ d\ (d - d_1) \text{ N},$$

or, introducing the height of the nut,

$$\text{P} = 3{,}300 \text{ to } 4{,}240\ d\ (d - d_1) \text{ H } n.$$

If, as is common, the nut has a height H $= d$, then

$$\text{P} = 3{,}300 \text{ to } 4{,}240\ d^2\ (d - d_1)\ n,$$

or approximately, for Whitworth screws,

$$\text{P} = 4{,}100\ d^2 \text{ to } 5{,}300\ d^2.$$

The load calculated from this equation is less than, but not much less than, the greatest load consistent with the safe working tensile strength of the bolt. The bearing pressure on the threads can be reduced either by increasing the bolt diameter or by increasing the height of the nut.

*Bearing pressure. Screws transmitting motion.*—If a screw transmits motion (as the leading screw of a lathe driving the saddle) the bearing pressure should be less. At low rubbing

velocities below 50 ft. per minute, the bearing pressure should not exceed the values given above. For higher velocities

$$k = 300{,}000 \,/\, v^{1\cdot 2} \qquad . \quad . \quad . \quad (10)$$

where $v$ is in feet per minute.

## Practical Calculations on the Strength of Bolts

124. Bolts are usually of wrought iron or mild steel, which would carry a safe working stress in tension of 15,000 lbs. per sq. in. for a statical or dead load, and 10,000 lbs. per sq. in. for a varying tensile load. But for various reasons the ordinary working stress is less than this. At a section through the bottom of a screw thread the material is strengthened for purely statical loads by the diminution of contraction consequent on the presence of less strained material near the plane of fracture. Martens has shown that with a gradually applied load, the breaking stress per square inch at screw threads is 10 to 19 per cent. greater than at a section through the body of the bolt. Under varying stress, however, the screw-thread groove is a source of weakness from the unequal distribution of stress across the section at the bottom of the thread. The unstrained metal in the projecting thread hinders the extension of the neighbouring metal in the body of the bolt, and so increases the stress at the bottom of the re-entrant angle formed by the thread.

When screwed rods are exposed to alternating stresses of opposite sign, the screw thread is found to be a frequent source of accident. Thus slide-valve rods frequently break at a screw thread. Mr. Longridge points out that the thread in such cases should be a fine pitched thread, or, better still, a knuckle or rounded thread.

*Bolts not tightened up before the load is applied.*—In many cases—eye-bolts, for instance—there is no initial stress when the load is applied. Let T be the axial load and $f$ the safe working stress, $d$ the full and $d_1$ the core diameter of the bolt.

$$T = \frac{\pi}{4} f d_1^2 = \frac{\pi}{4} f d^2 \left(\frac{d_1}{d}\right)^2$$

Values of $d_1/d$ are given in the tables above. If $d_1/d$ is taken at 0·8 for Whitworth threads,

$$\left.\begin{array}{l} \mathrm{T} = 0{\cdot}5 f d^2 \\ d = 1{\cdot}41 \sqrt{(\mathrm{T}/f)} \end{array}\right\} \quad . \quad . \quad . \quad (11)$$

For loads varying from 0 to T, $f$ may be taken at 8,000 lbs. per sq. in. for wrought iron; 10,000 lbs. per sq. in. for mild steel; 2,800 lbs. per sq. in. for gunmetal.

$$\begin{array}{ll} f = \phantom{0}8{,}000 & d = 0{\cdot}0158\sqrt{\mathrm{T}} \\ \phantom{f} = 10{,}000 & \phantom{d} = 0{\cdot}0141 \;\;,, \\ \phantom{f} = \phantom{0}2{,}800 & \phantom{d} = 0{\cdot}0267 \;\;,, \end{array}$$

For Sellers threads the constants would differ very little from those for Whitworth threads. It must be remembered, however, that often a larger margin of strength in bolts is allowed. Sometimes this larger margin is an apparent margin only. Thus a stress of 6,000 lbs. per sq. in. is sometimes allowed in important screwed parts in steam engines, such as the screwed end of the piston rod. But then the load is taken at simply the steam pressure on the piston, though from various causes at times the load is greater than the steam pressure. But also very often the cost of the bolts is comparatively unimportant, and the load not ascertainable with certainty.

125. *Bolts for steam or water-tight flanged joints.*—Let D be the diameter of a cylinder exposed to an internal pressure $p$ lbs. per sq. in. Let the cover be attached by $n$ bolts. Then the tension on each bolt calculated from the steam or water pressure is at least

$$\mathrm{T} = \left(\frac{\pi}{4}\mathrm{D}^2 p\right)/n \text{ lbs.}$$

and is probably more because the pressure extends some distance between the flanges. But such bolts are screwed up to an initial tension before the load due to the steam or water pressure is applied. It has been shown that this initial tension may be very serious. It must exceed the tension T due to the steam or water pressure, or the joint would open. The stress due to it is very great, especially in small bolts. Unfortunately it is a stress which cannot be calculated, as it depends on the judgment of the workman.

Generally in such cases the bolts have been calculated from

the tension T, but with a very low working stress to allow a margin for the initial tension. Thus,

$$\left.\begin{aligned} f &= 3{,}600 \text{ to } 4{,}800 \text{ lbs. per sq. in.} \\ T &= 1{,}800 \text{ to } 2{,}400 \quad d^2 \\ d &= 0{\cdot}024 \text{ to } 0{\cdot}020 \sqrt{T} \end{aligned}\right\} \quad . \quad . \quad . \quad . \qquad (12)$$

Sometimes a quarter of an inch is added to the calculated diameter as an additional margin of safety.

Looking to the fact that the stress due to the initial tension is much greater in small than in large bolts, it would seem rational that, if the bolts are calculated from the steam or water pressure, a working stress should be assumed, diminishing as the size of the bolt is smaller. In rougher joints let

$$f = 1{,}600 + 1{,}600\ d^2$$

*Whitworth Threads*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Diameter of bolt $d$ | $\frac{5}{8}$ | $\frac{3}{4}$ | $\frac{7}{8}$ | 1 | $1\frac{1}{8}$ | $1\frac{1}{4}$ | $1\frac{1}{2}$ |
| Working stress $f$ | 2,220 | 2,500 | 2,800 | 3,200 | 3,600 | 4,100 | 5,200 |
| Core area . . | ·204 | ·304 | ·422 | ·554 | ·697 | ·894 | 1·30 |
| Total safe load T | 452 | 760 | 1,180 | 1,770 | 2,510 | 3,670 | 6,760 |

In faced joints with thin packing let

$$f = 2{,}500 + 3{,}000\ d^2$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Diameter of bolt $d$ | $\frac{5}{8}$ | $\frac{3}{4}$ | $\frac{7}{8}$ | 1 | $1\frac{1}{8}$ | $1\frac{1}{4}$ | $1\frac{1}{2}$ |
| Working stress $f$ . | 3,670 | 4,189 | 4,798 | 5,500 | 6,300 | 7,180 | 9,250 |
| Core area . . | ·204 | ·304 | ·422 | ·554 | ·697 | ·894 | 1·30 |
| Total safe load T . | 748 | 1,273 | 2,025 | 3,047 | 4,391 | 6,418 | 12,025 |

126. *Examples from practice.*—Flange bolts are extremely important parts of machines, and it is unfortunate that the straining action on them is essentially indeterminate. Hence only experience of what has proved satisfactory can be taken as a guide in designing. Now it happens that the flange joints of water and steam pipes have been standardised, and the following table contains a selection of cases, the strain on the bolts having been calculated. There is a difference among engineers as to how the load due to the steam or water pressure should be calculated. Most commonly, if D is the internal diameter of the pipe, the total load is taken to be the pressure acting on an area of diameter D. But some engineers assume that the pressure extends between the flanges. If $D_1$ is the diameter of a circle touching the inside of the bolts, they take the total load as

the pressure acting on a circle of diameter $D_1$. In the table the loads and stresses on both assumptions are given.

| Diameter of bolts $d$ M.S. | Number of bolts $n$ | Internal diameter of pipe D M.S. | Diameter inside bolts $D_1$ M.S. | Tension on each bolt: From D in lbs. | Tension on each bolt: From $D_1$ in lbs. | Stress on core section: Calculated from D in lbs. per sq. in. | Stress on core section: Calculated from $D_1$ in lbs. per sq. in. |
|---|---|---|---|---|---|---|---|
| | *British Standard. Working Pressure 55 lbs. per sq. in.* | | | | | | |
| $\frac{5}{8}$ | 4 | 4 | $6\frac{3}{8}$ | 173 | 437 | 848 | 2150 |
| $\frac{5}{8}$ | 8 | 8 | $10\frac{7}{8}$ | 346 | 638 | 1700 | 3130 |
| $\frac{7}{8}$ | 12 | 16 | $19\frac{5}{8}$ | 922 | 1384 | 2190 | 3220 |
| 1 | 16 | 24 | $28\frac{3}{4}$ | 1554 | 2231 | 2810 | 4010 |
| | *British Standard. Working Pressure 325 lbs. per sq. in.* | | | | | | |
| $\frac{3}{4}$ | 8 | 4 | $6\frac{3}{4}$ | 511 | 1454 | 1680 | 4790 |
| $\frac{7}{8}$ | 12 | 8 | $11\frac{7}{8}$ | 1362 | 3000 | 3230 | 7130 |
| $1\frac{1}{8}$ | 20 | 16 | $20\frac{5}{8}$ | 3265 | 5425 | 4690 | 7810 |
| $1\frac{3}{8}$ | 24 | 24 | $29\frac{3}{8}$ | 6120 | 9190 | 5780 | 8690 |
| | *German Standard. Cast Iron Water Pipes. Working Pressure 140 lbs. per sq. in.* | | | | | | |
| $\frac{5}{8}$ | 4 | $3\frac{1}{2}$ | 6 | 159 | 465 | 1800 | 5010 |
| $\frac{3}{4}$ | 8 | $11\frac{7}{8}$ | 15 | 882 | 1425 | 6680 | 10300 |
| 1 | 20 | $29\frac{5}{8}$ | $33\frac{3}{4}$ | 2209 | 2871 | 9180 | 11400 |
| | *German Standard. Steam Pipes. Working Pressure 284 lbs. per sq. in.* | | | | | | |
| $\frac{5}{8}$ | 6 | $2\frac{3}{4}$ | $5\frac{1}{16}$ | 282 | 961 | 1380 | 4710 |
| $\frac{3}{4}$ | 8 | 5 | 8 | 675 | 1742 | 2220 | 5730 |
| 1 | 16 | $11\frac{7}{8}$ | 16 | 1949 | 3542 | 3520 | 6390 |
| $1\frac{1}{8}$ | 20 | $15\frac{7}{8}$ | $20\frac{5}{8}$ | 2772 | 4708 | 3980 | 6750 |
| | *American Standard Cast Iron Water Pipes. Working Pressure 200 lbs. per sq. in.* | | | | | | |
| $\frac{5}{8}$ | 4 | $3\frac{1}{2}$ | $6\frac{3}{8}$ | 480 | 1595 | 2350 | 7820 |
| $\frac{3}{4}$ | 12 | 9 | $12\frac{1}{2}$ | 1060 | 2045 | 3490 | 6730 |
| $\frac{7}{8}$ | 12 | 12 | $16\frac{1}{8}$ | 1881 | 3433 | 4460 | 8140 |
| 1 | 16 | 16 | $20\frac{1}{4}$ | 2512 | 4025 | 4540 | 7270 |
| $1\frac{1}{8}$ | 20 | 20 | $23\frac{7}{8}$ | 3140 | 4470 | 4500 | 6420 |
| $1\frac{1}{2}$ | 44 | 48 | $54\frac{1}{2}$ | 8223 | 10500 | 6330 | 8080 |

127. *Proportions of bolts and nuts.*—Fig. 117 shows an ordinary type of bolt, nut, and washer. This bolt has a square head, and a square neck, to prevent the rotation of the bolt while the nut is being screwed up. The nut is hexagonal, and the washer circular.

Standard proportions for nuts and bolt heads have now been

fixed by the Standards Committee. These cannot be expressed exactly by formula, but are given very closely by the following rules.

*Hexagon Bolt Heads and Nuts*

Black nuts—width across flats, maximum $= D = 1{\cdot}5\,d + 0{\cdot}16$
,, ,, minimum $= 1{\cdot}5\,d + 0{\cdot}14$
Bright nuts—width across flats, maximum $= D = 1{\cdot}5\,d + 0{\cdot}16$
,, ,, minimum $= 1{\cdot}5\,d + 0{\cdot}15$

Above 2 inches in diameter the standard sizes are a little irregular. Thus, for a 4-inch bolt the width across flats is 5·95,

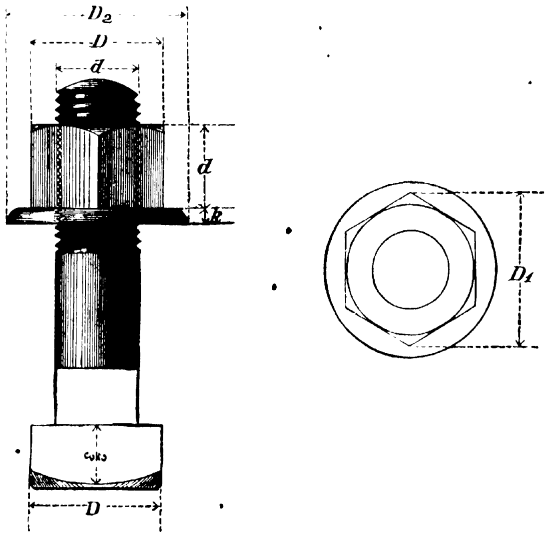


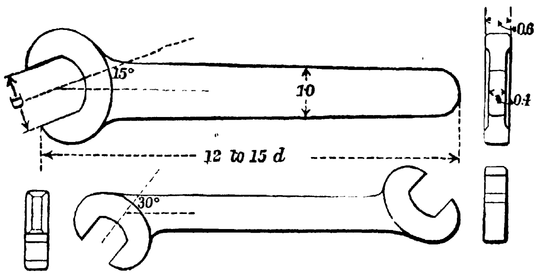

Fig. 117

for a 6-inch it is 10 inches. Square nuts have generally the same width across the flats as hexagon nuts for the same diameter of bolt.

Width of hexagon nuts across angles $= D_1 = 1{\cdot}155\,D$
,, square ,, ,, ,, $= 1{\cdot}414\,D$
Height or thickness of nut $= d$
,, ,, lock nut $= \frac{2}{3}\,d$
,, ,, bolt head $= 0{\cdot}875\,d$.

Nuts of weaker material than the bolt should have a greater height.

*Spanners* for tightening nuts are shown in fig. 118. They have on the average a length of about $9\,d + 6$. The thickness of the head is $0{\cdot}6\,d + 0{\cdot}7$; the width of jaw $1{\cdot}5\,d + 0{\cdot}17$; thickness of shank, $0{\cdot}4\,d + 0{\cdot}05$; width of shank, $1\frac{1}{8}\,d + \frac{1}{8}$ at one end to $d + \frac{1}{10}$ at the other. Standard dimensions have been fixed, but




Unit = *d*.
Fig. 118

they do not differ much from these proportions. The angle between the axis of head and shank varies according to the conditions in which the spanner is used. It is generally 15°, 30° or 45°. Spanners for hexagon nuts are available for square nuts, if the width across flats is the same.

*Washers* (fig. 119).—These are used under nuts (*a*) when the seating is rough and the washer provides a smooth surface on which the nut turns; (*b*) when the seating is of weak material—for instance, wood—and the washer distributes the pressure over a large surface.

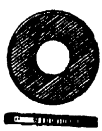


Fig. 119

The thinnest washers are 0·08 inch in thickness. Washers under nuts have a diameter about $2{\cdot}2\,d$ and a thickness $\frac{1}{4}\,d$ in small and $\frac{1}{5}\,d$ in large bolts. Washers for wood may be $3\,d$ in diameter or more, and $0{\cdot}3\,d$ in thickness. Large cast-iron washers are used to distribute the pressure on brick or masonry. The pressure on wood should not exceed 600 lbs. per sq. in.

128. *Different forms of nuts.*—Ordinary nuts are chamfered off at an angle of 30° to 45°, as shown at *a*, fig. 120; or they are finished with a spherical bevil, struck with a radius of about $2\,d$, as shown at *b*. *Flange nuts*, *c*, are used when the hole, in which the bolt is placed, is considerably larger than the bolt itself.

The flange covers and hides the hole. *Cap nuts, d,* are used where leakage along the screw thread is feared. In the figure, a thin, soft copper washer is shown, which prevents leakage under the nut. *Circular nuts, e,* are occasionally used. They have

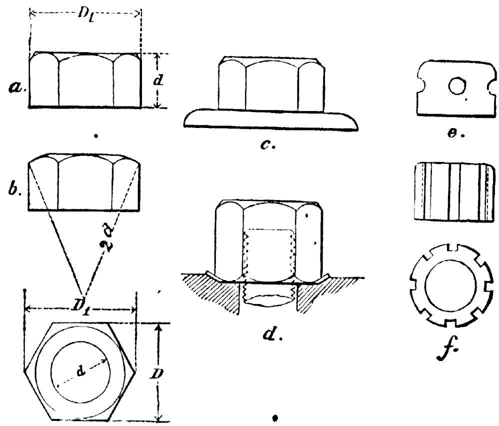


Fig. 120

holes, in which a bar, termed a 'Tommy,' is placed, for screwing them up. Sometimes grooves are cut as shown at *f.* Steel nuts may be used if great durability is required. Various forms of heads for screws are shown in fig. 121.

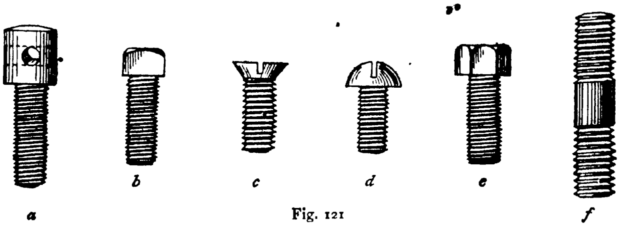


Fig. 121

Fig. 122 shows a turned bolt with a round head, washer, and split pin.

$$D = 1{\cdot}2\,d + \tfrac{3}{8}$$
$$h = 0{\cdot}2\,d + \tfrac{1}{4}$$
$$D_1 = D$$
$$h_1 = \tfrac{1}{8}\,d + \tfrac{1}{8}$$

Fig. 123 shows a similar bolt with a loose collar and split pin.

$$D_2 = 1{\cdot}2\,d + \tfrac{3}{8}$$

$$h_2 = \tfrac{3}{8}\,d + \tfrac{1}{8}$$

The diameter of the split pin may be $\frac{1}{8}\,d + \frac{1}{16}$.

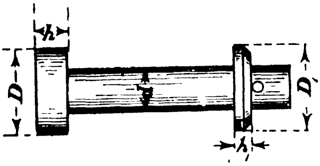



Fig. 122

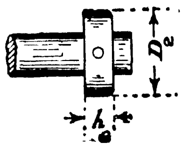



Fig. 123

128*a*. *Different forms of bolt heads.*—In fig. 124, *a* is a cup-shaped, *b* a countersunk, and *c* a square bolt-head. Rotation of the bolt is prevented in *a* by a square neck, in *b* by a set screw, in *c* by a snug forged on the bolt. Fig. 124, *d*, shows a T-headed bolt in front and side elevation. Fig 124, *e*, shows an

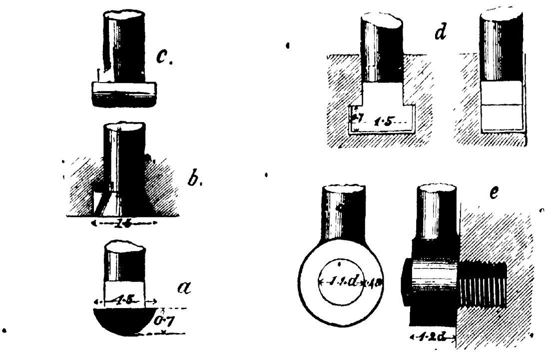



Fig. 124

eye-bolt. Fig 125 shows a spherical-headed bolt with a square neck, used sometimes for railway fastenings. The spherical head allows the bolt to take a fair bearing on the rail. The other figure shows a cup-head, with a snug forged on the bolt, to prevent rotation when the bolt is screwed up. Proportional unit = *d*, in all these figures.

Fig. 126 is a *hook bolt*, which is used when one piece is too small to have a bolt hole through it, or when it is objectionable to weaken the piece by a bolt hole. Fig. 127 is a *stud*, which is screwed into one of the connected pieces, and remains in position when the nut is removed. Fig. 128 is a *set screw*, or bolt not requiring a nut. Fig. 129 shows, at *a*, a nut-headed bolt, or

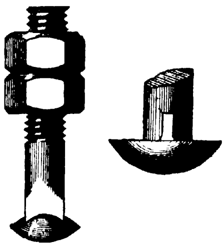


Fig 125

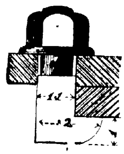


Fig. 126

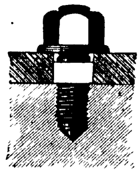


Fig. 127

bolt having two loose nuts, instead of a nut and head; at *b* and *c* similar bolts, with an intermediate head or flange. These bolts remain in place when the top nut is removed.

Fig. 130 is a bolt leaded into stone work. The tail of the bolt is rectangular, with jagged edges.

Fig. 131 is a fang bolt used for attaching ironwork to wood, and especially for attaching rails to sleepers. The fangs of the

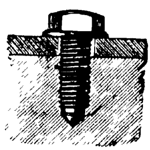
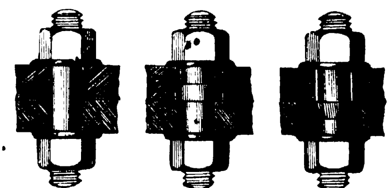


Fig. 128



*a* *b* *c*

Fig 129

broad triangular plate, which forms the nut, bite into the wood, while the bolt is rotated by the head, which bears on the ironwork. The large area of the nut prevents crushing of the wood.

129. *Locking arrangements for nuts* are intended to prevent the gradual unscrewing of nuts, subjected to vibration and frequent changes of load. No nut accurately fits its bolt; a certain amount of play, however minute, always exists. When

a nut, having play, is subjected to vibration, it gradually slacks back. This is, to a great extent, prevented by double nuts, shown in fig. 132. One of the nuts is termed a lock nut, and is

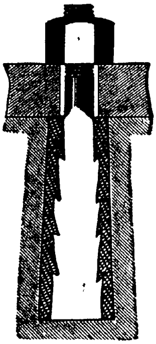


Fig. 130

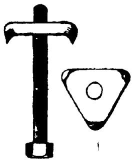


Fig. 131

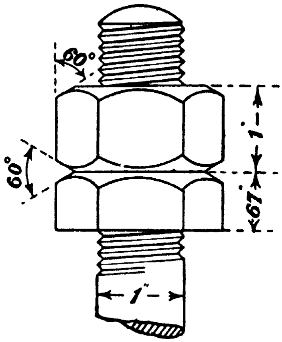



Fig. 132

usually two-thirds as thick as the ordinary nut. When there are two nuts, the whole load may be thrown on the outer nut. The outer nut ought, therefore, to be the thicker nut. It is common

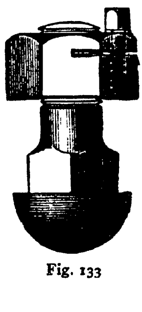
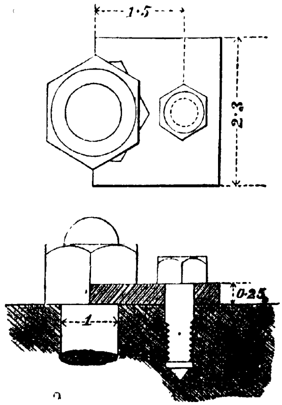


Fig. 133




Unit = a

Fig. 134

in practice to put the thinner nut outside, the reason being, that ordinary spanners are sometimes too thick to hold the thin nut, when screwed home first.

Fig. 133 shows the form of Wiles' lock nut, which is now often used on quick-running machine parts subjected to a good deal of vibration. It is very simple and can be locked on any part of the bolt. The nut is half cut through by a saw cut, and a small set screw is used to close slightly the jaws thus formed, after the nut is screwed home. The nut then grips the screw

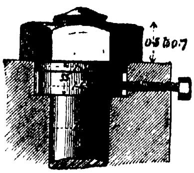


Fig. 135

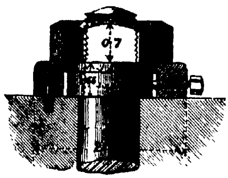


Fig. 136

thread tightly. For nuts under one inch in diameter the set screw is omitted, and the jaws of the saw cut are slightly closed by a hammer blow, before the nut is put in place.

Another plan is to drill a hole through the top of the bolt above the nut, and drive a split pin or cotter through. The nut must always be in the same place when screwed up. A better plan is shown in fig. 134, a stop plate being used, fixed on one

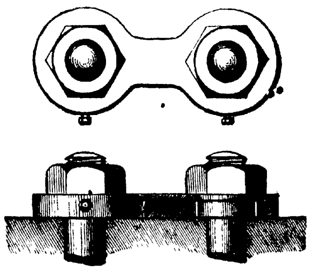


Fig. 137

side of the nut. The set screw in the stop plate may have its diameter $= \frac{1}{4} d + \frac{1}{8}$.

A very neat arrangement is shown in fig. 135; the lower part of the nut is turned circular, and fits in a recess in the piece connected by the bolt. A set screw is tapped through, and bears on the side of the nut. The diameter of the set screw may be $\frac{1}{4} d + \frac{1}{8}$. A stop ring is sometimes used (fig. 136), with a set

screw tapped through it. The stop ring is of brass, or wrought iron, and it is prevented from turning by a stop pin of the same size as the set screw.

Fig. 137 shows a neat way of applying this last mode of locking to a pair of nuts. The locking plate embraces the turned part of both nuts, and the stop pin is dispensed with.

Elastic washers have been used as substitutes for lock nuts. Fig. 138 shows Grover's spring steel washer. When the nut is

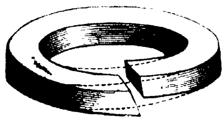


Fig. 138

tightened up, the washer becomes nearly, but not quite flat, and its elasticity neutralises the play of the nut on the bolt.

130. *Bolting of cast-iron plates.*—Cast-iron plates are united by bolts; flanges, to receive the bolts, are cast on the plates, and these may be external or internal. The flanges are a little thicker than the plates. Fig. 139 shows arrangements of bolts and flanges. *a* is an ordinary rust joint, the flanges fitting at a narrow chipping strip and the rest of the space being filled with

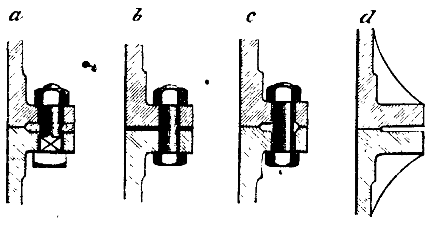



Fig. 139

iron cement. *b* is a faced joint made with a layer of asbestos millboard or other packing. In *c* there are two faced parts so that the tension of the bolt cannot bend the flange. In *d* the flange is strengthened with feathers or brackets between the bolts. Fig. 140 shows the section of a bolted flange on a larger scale.

If *d* is the diameter of the bolt and *t* the thickness of the cylinder or pipe, D its diameter, $t_1$ the thickness of flange, then,

on the average, $t_1 = 1{\cdot}3\,d$ for light and $= 1{\cdot}9\,d$ for heavy pressures. The pitch of the bolts is $6\,d$ for light and $3{\cdot}5\,d$ for heavy pressures, $w_1 = 1{\cdot}6\,d$ and $w_2 = 1{\cdot}2\,d$. The bolt hole may be $d + \frac{1}{8}$ in diameter. In pipes the number of bolts is usually a multiple of four, and the principal centre lines should bisect the interval between two bolts. The design of a flange is necessarily a process of trial and error. The number of bolts is assumed, the bolt diameter calculated, and then the pitch is examined to see if it is convenient.

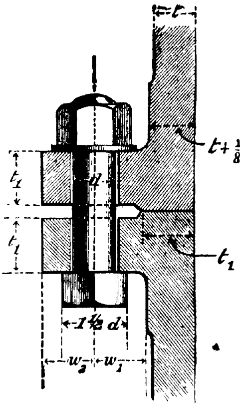


Fig. 140

For wrought-iron or steel pipes with flanges welded on, and for heavy steam pressures, $t_1 = 1{\cdot}6\,d$; $p = 3{\cdot}3\,d$; $w_1 = 1{\cdot}6\,d$; $w_2 = d$; on the average.

*Pipe flanges.*—Standard dimensions for pipe flanges for cast iron pipes and for steel pipes with welded flanges have been drawn up by the Standards Committee.[1] They are suitable for pressures of 55, 125, 225, and 325 lbs. per sq. in. In the United States standard dimensions for pipe flanges, given in the table on p. 208, have been adopted. The pipe thickness for a pressure $p$ lbs. per sq. in. and a pipe $d$ inches internal diameter is calculated by the empirical rule,

$$t = \frac{p + 100}{7200}\,d + 0{\cdot}333\left(1 - \frac{d}{100}\right)$$

Where one dimension only is given it is sufficient for 200 lbs. pressure per sq. in. Where two dimensions are given the smaller is suitable for pressures up to 100 lbs. per sq. in., the larger for pressures from 100 to 200 lbs. per sq. in.

131. *Stay bolts* for tanks. The flat surfaces of tanks are supported by stay bolts connecting opposite sides of the tank, and thus directly resisting the water pressure. Fig. 141 shows the form of such bolts for cast and wrought iron tanks. The end of the bolt is enlarged in fig. 141, *a*, so that the body of the bolt is of the same strength as the screw at the bottom of the thread. Then $d_1 = 0{\cdot}9\,d - 0{\cdot}05$. The cast-iron plate through

[1] Report No. 10. Crosby Lockwood & Son.

*Standard Pipe Flanges.* (*Committee of Am. Soc. Mech. Eng.*)

| Pipe size, ins. | Pipe thickness | Thickness, nearest fraction, ins. | Stress on pipe per square inch at 200 lbs. | Radius of fillet, ins. | Flange diameter, ins. | | Flange thickness at hub for iron pipe, ins. | Flange thickness at edge, ins. | | Width flange face, ins. | | Bolt circle diameter, ins. | | Number of bolts | Bolt size diameters, ins. | | Bolt length, ins. | Stress on each bolt, per square inch at bottom of thread at 200 lbs. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | ·409 | 7/16 | 460 | 1/8 | | 6 | 1 | | 5/8 | | 2 | | 4 3/4 | 4 | 1/2 | 5/8 | 2 | 825 |
| 2 1/2 | ·429 | 7/16 | 550 | 1/8 | | 7 | 1 1/8 | | 11/16 | | 2 1/4 | | 5 1/2 | 4 | 1/2 | 5/8 | 2 1/4 | 1,050 |
| 3 | ·448 | 7/16 | 690 | 1/8 | | 7 1/2 | 1 1/4 | | 3/4 | | 2 1/4 | | 6 | 4 | 1/2 | 5/8 | 2 1/2 | 1,330 |
| 3 1/2 | ·466 | 1/2 | 700 | 1/8 | | 8 1/2 | 1 1/4 | | 13/16 | | 2 1/2 | | 7 | 4 | 1/2 | 5/8 | 2 1/2 | 2,530 |
| 4 | ·486 | 1/2 | 800 | 1/8 | | 9 | 1 3/8 | | 15/16 | | 2 1/2 | | 7 1/2 | 4 | 5/8 | 3/4 | 2 3/4 | 2,100 |
| 4 1/2 | ·498 | 1/2 | 900 | 1/8 | | 9 1/4 | 1 3/8 | | 15/16 | | 2 3/8 | | 7 3/4 | 8 | 5/8 | 3/4 | 3 | 1,430 |
| 5 | ·525 | 1/2 | 1,000 | 1/8 | | 10 | 1 1/2 | | 15/16 | | 2 1/2 | | 8 1/2 | 8 | 5/8 | 3/4 | 3 | 1,630 |
| 6 | ·563 | 9/16 | 1,060 | 1/8 | | 11 | 1 1/2 | | 1 | | 2 1/2 | | 9 1/2 | 8 | 5/8 | 3/4 | 3 | 2,360 |
| 7 | ·60 | 5/8 | 1,120 | 1/8 | | 12 1/2 | 1 1/2 | | 1 1/16 | | 2 3/4 | | 10 3/4 | 8 | 5/8 | 3/4 | 3 1/4 | 3,200 |
| 8 | ·639 | 5/8 | 1,280 | 1/8 | | 13 1/2 | 1 3/4 | | 1 1/8 | | 2 3/4 | | 11 3/4 | 8 | 5/8 | 3/4 | 3 1/2 | 4,190 |
| 9 | ·678 | 11/16 | 1,310 | 3/16 | | 15 | 1 3/4 | | 1 1/8 | | 3 | | 13 1/4 | 12 | 5/8 | 3/4 | 3 1/2 | 3,610 |
| 10 | ·713 | 3/4 | 1,330 | 3/16 | | 16 | 2 | | 1 3/16 | | 3 | | 14 1/4 | 12 | 3/4 | 7/8 | 3 3/8 | 2,970 |
| 12 | ·79 | 13/16 | 1,470 | 3/16 | | 19 | 2 | | 1 1/4 | | 3 1/2 | | 17 | 12 | 3/4 | 7/8 | 3 3/4 | 4,280 |
| 14 | ·864 | 7/8 | 1,600 | 3/16 | | 21 | 2 | | 1 3/8 | | 3 1/2 | | 18 3/4 | 12 | 7/8 | 1 | 4 1/4 | 4,210 |
| 15 | ·904 | 15/16 | 1,600 | 3/16 | | 22 1/4 | 2 | | 1 3/8 | | 3 5/8 | | 20 | 16 | 7/8 | 1 | 4 1/4 | 3,660 |
| 16 | ·946 | 1 | 1,600 | 3/16 | | 23 1/2 | 2 1/4 | | 1 7/16 | | 3 3/4 | | 21 1/4 | 16 | 7/8 | 1 | 4 1/4 | 4,210 |
| 18 | 1·02 | 1 1/16 | 1,690 | 3/16 | | 25 | — | | 1 9/16 | | 3 1/2 | | 22 3/4 | 16 | 1 | 1 1/8 | 4 3/4 | 4,540 |
| 20 | 1·09 | 1 1/8 | 1,780 | 3/16 | | 27 1/2 | — | | 1 11/16 | | 3 3/4 | | 25 | 20 | 1 | 1 1/8 | 5 | 4,490 |
| 22 | 1·18 | 1 3/16 | 1,850 | 1/4 | | 29 1/2 | — | | 1 13/16 | | 3 3/4 | | 27 1/4 | 20 | 1 | 1 1/4 | 5 1/2 | 4,320 |
| 24 | 1·25 | 1 1/4 | 1,920 | 1/4 | 31 1/2 | 32 | — | 1 1/4 | 1 7/8 | 3 3/4 | 4 | 29 1/4 | 29 1/2 | 20 | 1 | 1 1/4 | 5 1/2 | 5,130 |
| 26 | 1·30 | 1 5/16 | 1,980 | 1/4 | 33 3/4 | 34 1/4 | — | 1 3/8 | 2 | 3 7/8 | 4 1/8 | 31 1/4 | 31 3/4 | 24 | 1 | 1 1/4 | 5 3/4 | 5,030 |
| 28 | 1·38 | 1 3/8 | 2,040 | 1/4 | 36 | 36 1/2 | — | 1 7/16 | 2 1/16 | 4 | 4 1/4 | 33 1/2 | 34 | 28 | 1 | 1 1/4 | 6 | 5,000 |
| 30 | 1·48 | 1 1/2 | 2,000 | 1/4 | 38 | 38 3/4 | — | 1 1/2 | 2 1/8 | 4 | 4 3/8 | 35 1/2 | 36 | 28 | 1 1/8 | 1 3/8 | 6 1/4 | 4,590 |
| 36 | 1·71 | 1 3/4 | 1,920 | 1/4 | 44 1/2 | 45 3/4 | — | 1 3/4 | 2 3/8 | 4 1/4 | 4 7/8 | 42 | 42 3/4 | 32 | 1 1/8 | 1 3/8 | 6 1/2 | 5,790 |
| 42 | 1·87 | 2 | 2,100 | 1/4 | 51 | 52 3/4 | — | 1 7/8 | 2 5/8 | 4 1/2 | 5 3/8 | 48 1/2 | 49 1/2 | 36 | 1 1/4 | 1 1/2 | 7 1/4 | 5,700 |
| 48 | 2·17 | 2 1/4 | 2,130 | 1/4 | 57 1/2 | 59 1/2 | — | 2 | 2 3/4 | 4 3/4 | 5 3/4 | 54 3/4 | 56 | 44 | 1 3/8 | 1 1/2 | 7 3/4 | 6,090 |

which the bolt passes is thickened to spread the pressure of the bolt. In fig. 141, *b*, the wrought-iron tank plate cannot easily be

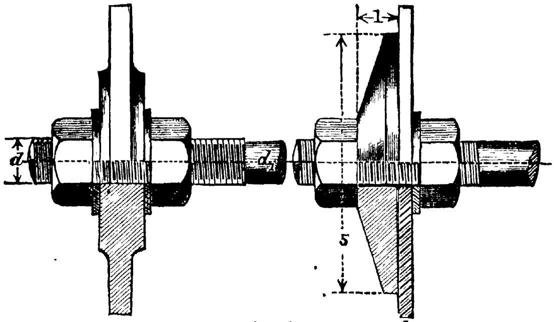
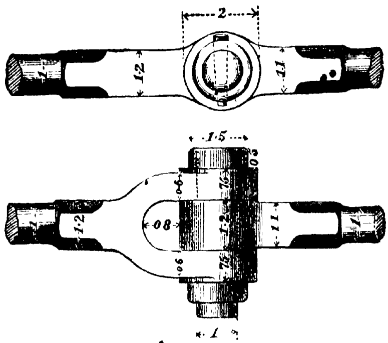



Fig. 141

thickened, but a large washer plate of cast iron is used to spread the bolt pressure.

### JOINT PINS. KNUCKLE JOINT

132. A joint pin is a kind of bolt, so placed as to be in shear. Fig. 142 shows an arrangement known as a knuckle joint. The




Unit = diam. of pin

Fig 142

proportions are empirical. If the joint pin were subjected to simple shear at two sections, it would be strong enough, when

its diameter was equal to 0·7 of the diameter of the rods. But the pin wears, and is then subjected to bending, as well as shearing. When there is much motion at the joint, the width of the eyes of the rods and the length of the pin may be increased.

*Straining action in a loosely fitting pin.*—Suppose a loosely-fitting pin (fig. 142A) in shear between plates of thickness $a, b$. If its strength is taken to depend simply on its resistance to shear, then the total section in shear is $\pi\, d^2 / 2$. The mean shearing stress is $f_{ms}$

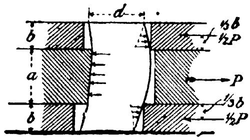



Fig. 142A

$= 2\,P / \pi\, d^2$, and allowing for the alteration of distribution of stress due to bending, the maximum shearing stress is $f_s = (8\,P) / (3\,\pi\, d^2) = 0{\cdot}85\ P/d^2$. If, however, there is sensible bending, the direct stress due to bending is greater usually than the shearing stress. The bending moment cannot be exactly calculated, because the distribution of pressure on the bearing surfaces is unknown. Suppose a uniform pressure is distributed along the middle part of the pin, and a uniformly varying pressure along the part in the side plates. The resultant pressures $\frac{1}{2}\,P$ in the side plates then act at $\frac{1}{3}\,b$ from the shearing plane. The bending moment at the centre of the pin is $\frac{1}{2}\,P\ (a/4 + b/3)$. The stress due to bending is

$$f = \left\{P\left(\frac{a}{8} + \frac{b}{6}\right)\right\} \Big/ \left\{0{\cdot}098\ d^3\right\}$$

$$= 5\ P\left(\frac{a}{4} + \frac{b}{3}\right) / \ d^3 \text{ nearly.}$$

Now let the safe working stress $f = 5{,}400$ lbs. per sq. in., which would be suitable for mild steel subjected to stresses reversing in sign, then,

$$d = 0{\cdot}097 \sqrt[3]{\left\{P\left(\frac{a}{4} + \frac{b}{3}\right)\right\}} \quad . \quad . \quad . \quad (13)$$

where P is in lbs. and the dimensions in ins

If $a = 1{\cdot}2\ d$ and $b = 0{\cdot}7\ d$, then,

$$d = 0{\cdot}0224\ \sqrt{P}$$

| | | | | | | |
|---|---|---|---|---|---|---|
| P = | 1,000 | 2,500 | 5,000 | 7,500 | 10,000 | 15,000 lbs. |
| $d$ = | 0·71 | 1·12 | 1·58 | 1·94 | 2·24 | 2·73 ins. |

Calculating for shear only, and taking $f_s$=4,000 for mild steel, then from formula above—

$$d = 0\cdot0146\sqrt{P}$$

| | | | | | | |
|---|---|---|---|---|---|---|
| P = | 1,000 | 2,500 | 5,000 | 7,500 | 10,000 | 15,000 lbs. |
| $d$ = | 0·46 | 0·73 | 1·03 | 1·26 | 1·46 | 1·78 ins. |

It will be seen that if bending is considered the pin must be larger than if shearing only is allowed for. For pins fitting loosely the calculation for bending should be used.

# CHAPTER VI

## Keys and Cotters

### Keys

133. Keys are small wedge-shaped pieces used to fix wheels, pulleys, cranks, and other pieces on shafts. The function of a key is to prevent the piece rotating relatively to the shaft; but, from the friction of the key in the keyways, it offers also some resistance to sliding along the shaft.

The following is a classification of keys, and a statement of the straining action to which they are subjected :—

| | Straining Action |
|---|---|
| (*a*) Fixed keys or feathers . . | Shearing. |
| (*b*) Keys on flats and saddle keys | Chiefly radial compression; partly friction. |
| (*c*) Sunk keys . . . . . | Partly shearing; partly radial compression |
| (*d*) Tangent keys . . . . . | Shearing on diagonal section. |

Fig. 143 shows an ordinary feather or fixed key, parallel both top and bottom and laterally.

Fig. 144 shows some ordinary forms of taper keys. The lower figure is a key with a head which is necessary for driving back the key, when the smaller end is inaccessible. Cross sections of keys are shown in fig. 145. A is a saddle key which prevents relative rotation almost entirely by friction. Such keys are used for fixing light pulleys and other pieces not transmitting great efforts. B is a key on a flat which has more holding power than a saddle key and is often used in fixing pulleys or wheels on shafts which cannot be conveniently dismounted. C is a sunk key which is the strongest and most effective form. Keyways are slotted in the shaft and hub and the key accurately

fitted to both. Opinions differ as to the way such a key should be fitted. Some engineers think that the lateral fit in the key-way is unimportant, and that the key resists in the same way as

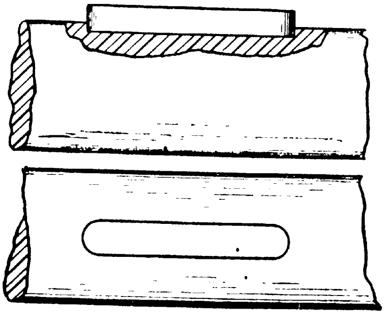


Fig. 143

a key on flat in consequence of the compression on the top and bottom faces. Most engineers think the lateral fit as important

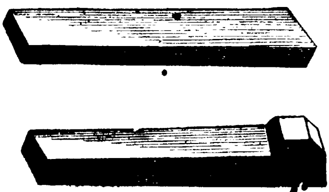


Fig. 144

as the top and bottom fit, so that no relative motion is possible without shearing the key.

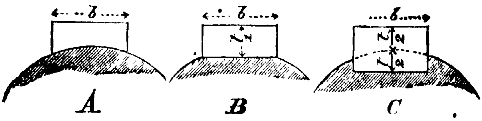



Fig. 145

The taper of keys is 1 in 48 to 1 in 96, or from $\frac{1}{4}$-inch per foot to $\frac{1}{8}$-inch per foot.

A sunk key with a saddle key placed at right angles to it, fig. 146, is a very good arrangement. If a wheel nave is acci-

dentally bored out slightly larger than the boss on which it is fitted, it rocks if held by a sunk key only. This rocking is entirely prevented by the additional saddle key, which insures a bearing between the eye and shaft boss at three points in the circumference.

*Staking on.*—In the case of large pieces four keys are sometimes used, fig. 147, or, when the shaft boss is square, eight

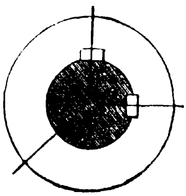


Fig. 146

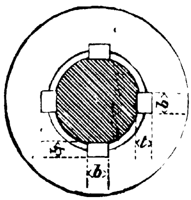


Fig. 147

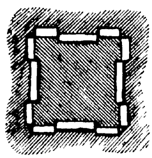


Fig. 148

keys, fig. 148. In the latter case the labour of planing the whole of the shaft boss and slotting out the square eye of the piece to be fixed to it is generally saved, only the key seats being faced. When four or eight keys are used, there is a limited power of adjusting the piece keyed to the shaft so as to be co-axial with the

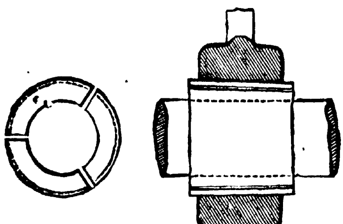


Fig. 149

shaft. Half the keys are usually driven from one side and half from the other. Large gear wheels should have four broad sunk keys, two driven from one side and two from the other.

*Cone keys.*—When a wheel has to be bored out, to pass over a shaft boss, cone keys may be used to fix it on the shaft, fig. 149. These are of cast iron and are cast in a single piece, with three parting plates, nearly but not quite dividing it into three pieces.

The casting is bored and turned, and afterwards split and the rough edges chipped away. There are thus obtained three cast-iron slightly tapering conical or saddle keys, of the thickness necessary to fill the space between the eye of the pulley and the shaft. They resist the tendency to rotation of the piece keyed on

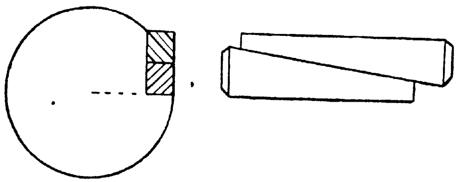


Fig. 150

by friction alone. The eye of the pulley must be bored slightly conical.

A very secure and rational form of key is the tangent key, fig. 150, made in two taper parts like a gib and cotter, and used for shafts over four inches in diameter. This key resists relative motion in one direction only; if the relative motion may occur in either direction, two such keys are used. The stress on the key is a simple shear along a diagonal plane.

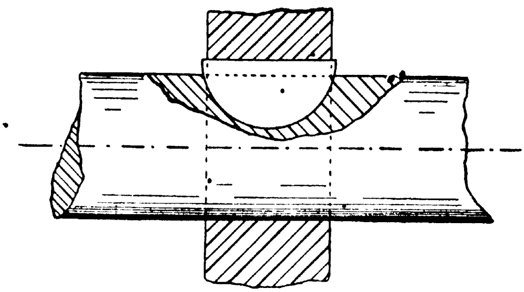


Fig. 151

Fig. 151 shows a Woodruff key used in America for small machines: The key is a slice of a semicircular bar. The keyway in the shaft is milled out by a circular cutter. The key adjusts itself to any inclination of the keyway in the hub, and is easily fitted. It rather weakens the shaft.

*Pins.*—A taper pin (fig. 152) is sometimes used in place of a key. It is sunk half in the shaft, half in the piece keyed on.

Where very fast connection between the pieces is required, as in the case of cranks keyed on crank shafts, the crank is usually bored slightly smaller than the crank shaft, expanded by heat, and shrunk on. A key is then fitted as an additional security.

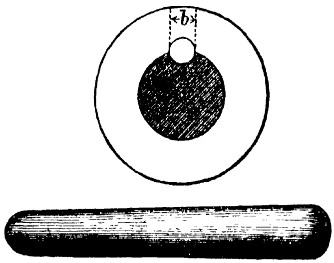



Fig. 152

134. *Stress on a key on flat.*—The normal thrust P between key and shaft cannot be taken to act at more than $b/6$ from the centre of the key or the pressure would not extend over the whole surface, and one edge of the key would lift out of contact. The resultant thrust R acts at an angle $\theta$ with P such that $\tan \theta = \mu$ the coefficient of friction. Now let $b = \frac{1}{4}d$, $t = \frac{1}{10}d$, $\mu = 0{\cdot}15$, then the leverage at which R acts, measured from the centre of the shaft, is found to be $0{\cdot}11\, d$ approximately. Hence the moment of resistance of the key to relative motion of shaft and hub is $0{\cdot}11\ \text{R}\ d = 0{\cdot}12\ \text{P}\, d$ nearly. Suppose the key is to be as strong as the shaft. If the torsion on the shaft produces a shearing stress $f_s = 9{,}000$ lbs. per sq. in., a usual working stress, then the twisting moment is $0{\cdot}2\ f_s\, d^3$. Hence

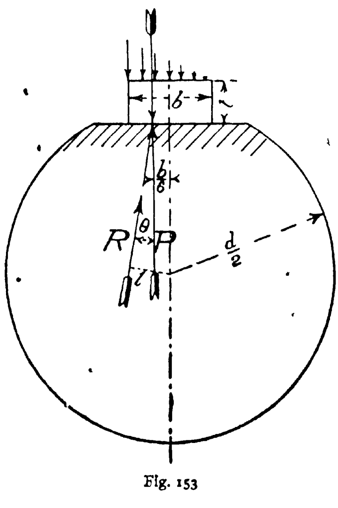



Fig. 153

$$0{\cdot}12\ \text{P}\, d = 0{\cdot}2\, f_s\, d^3$$

$$\text{P} = 1{,}500\, d^2 \quad . \quad . \quad . \quad (1)$$

Let the length of the key be $l = 1{\cdot}6\, d$. The area on which P acts

is $b l = 0{\cdot}4\ d^2$. The mean compressive stress on the key is $P/b\,l = 37{,}500$ lbs. per sq. in. and, as the stress is uniformly varying, the maximum at the edge is 75,000 lbs. per sq. in. The proportions selected are ordinary proportions and the compressive stress is very large, but not more probably than a good steel key will carry. Probably not more than the material of the shaft will carry, at any rate for some time, as near the key edge it is supported by comparatively unstrained material. Still the compressive stress is excessive and this is the reason why keys on flats subjected to a large twisting moment become slack and rock. Such keys should not be used in that case unless made very wide. Sunk keys fit partly on the sides, partly top and bottom. If the fit top and bottom is better than the fit at the sides, and this is probably usually the case, they are subjected to a similar compressive stress and are liable to become loose and rock. Hence heavy gear wheels should be fitted with more than one key.

135. *Keys subjected chiefly to shearing.*—A fixed or feather key is subjected chiefly to shearing. Let $b$ be the width and $l$ the length of the key. Let $f_s$ be the shearing resistance of the shaft and $f$ that of the key per sq. in. The total shearing resistance of the key is $f\,b\,l$. The moment of this about the shaft axis is $\frac{1}{2} f\,b\,l\,d$ nearly. Equating this to the twisting moment of the shaft, the shaft and key are of equal strength if—

$$\tfrac{1}{2}\, b\, l\, d\, f = 0{\cdot}2\, f_s\, d^3$$

$$b\, l = 0{\cdot}4 \frac{f_s}{f} d^2 \qquad . \quad . \quad . \quad . \qquad (2)$$

Let $l = 6\,b$, and as the key is usually of harder steel than the shaft, let $f_s/f = 0{\cdot}8$. Then

$$b = 0{\cdot}258\ d.$$

This is about a usual proportion for fixed and even for sunk keys.

Let $t$ be the mean thickness of the key. Then $t$ is generally about $0{\cdot}5\ b$. The side of the key has a bearing surface on the keyway of the shaft of about $0{\cdot}4\,t\,l = 0{\cdot}2\,b\,l$. Let $f_c$ be the crushing stress between key and keyway. Equating the crushing pressure to the shearing resistance of the key—

$$0{\cdot}2\, b\, l\, f_c = b\, l\, f$$

$$f_c / f = 5$$

*Hence with ordinary proportions the crushing stress on the key, when fully strained in shear, is five times the shearing stress. In the case of sunk keys no doubt much of the resistance is due to the radial compression, so that the shearing stress does not reach a high value and then the crushing stress between key and keyway is not excessive.*

Tangent keys resist mainly by shearing strength, along a diagonal plane from corner to corner. Also as the whole depth of key is both in shaft and hub the compression between key and keyway is only about half what it is in ordinary sunk keys.

### Common Proportions of Keys

136. Ordinarily keys are proportioned as if the whole energy transmitted by a shaft had to pass through the key. But of course, when only part of the energy transmitted is given off at a key, smaller dimensions suffice. The convenience of a general rule as to the proportions of keys is so great that except for light pulleys and gear wheels key dimensions are not generally reduced even though in many cases their strength is excessive.

Let $d$ be the shaft diameter ; $b$ the width, $t$ the mean thickness, $l$ the length of the key ; then the following empirical rules represent ordinary practice.

*Feather keys*

$$b = 0{\cdot}2\,d\,;\ t = 0{\cdot}2\,d$$

*Saddle key or key on flat*

$$b = 0{\cdot}25\,d + 0{\cdot}25$$

$$t = 0{\cdot}1\,d + 0{\cdot}125$$

*Sunk key*

$$b = 0{\cdot}25\,d + 0{\cdot}125$$

$$t = 0{\cdot}125\,d + 0{\cdot}0625$$

The depth of keyway in shaft is about $0{\cdot}4\,t$ and in hub $0{\cdot}6\,t$. The length of key is generally when possible not less than about $1{\cdot}6\,d$. Taper $\frac{1}{4}$-inch to $\frac{1}{2}$-inch in 12 inches.

***Tangent keys***

$$b = 0{\cdot}3\,d\,;\ t = 0{\cdot}1\,d$$

In settling the dimensions of keys, the *nearest sixteenth or* thirty-second of an inch is generally taken.

Fig. 154 gives a scale of proportions for keys. It is convenient to draw a diagram of this kind full size and to use it in place of formulæ or tables.

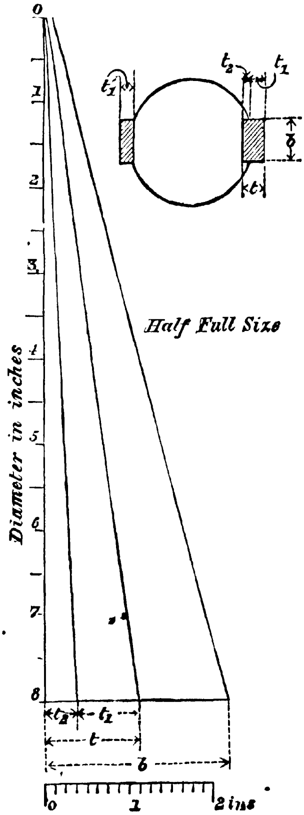


Fig 154

### Cotters

137. A cotter is a tapered bar driven through two pieces which are to be connected, and prevents their separation by resistance to shearing at two cross-sections. The cotter should be so designed as to diminish as little as possible the strength of the connected pieces. When the cotter is long it serves to adjust the length of the pieces connected. When driven home it shortens the total length, and *vice versa*.

The simplest forms of cotter are shown in fig. 155. In fig. 155, *a*, *b*, the fixing resists equally a thrust or tension. Fig. 155, *c*, is an arrangement for resisting tension only, often used for foundation bolts. The enlarged or gib ends of the cotter prevent its displacement.

Fig. 155, *e*, *f*, shows another arrangement for resisting only a tension, the cotter being divided into two parts termed respectively the gib and cotter. The cotter way is here exactly parallel. The rod is diminished beyond the connected pieces to a section equivalent to that through the cotter way. Fig. 155, *g*, shows an arrangement for both thrust and tension, used for pistons and piston rods. Often part of the rod is enlarged to receive the cotter, so that the area through the cotter hole is equal to the section

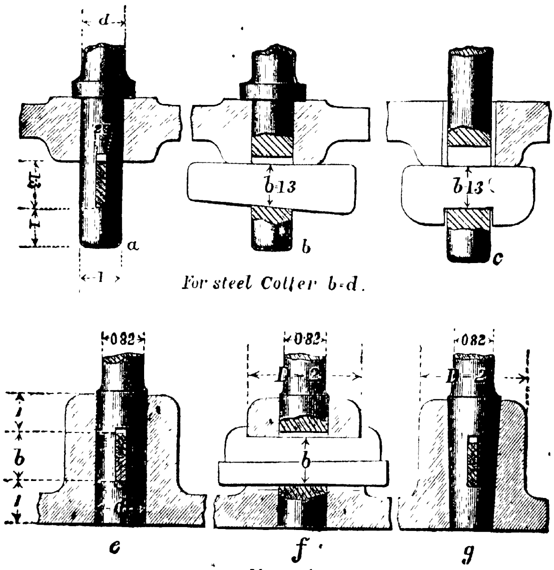
of the rest of the rod. If $d_1$ is diameter of the rod, $d$ diameter of the enlarged part, and $t$ thickness of the cotter—

$$\frac{\pi}{4} d_1^2 = \frac{\pi}{4} d^2 - d\,t$$

If $t = \frac{1}{4} d$, as is common—

$$d = 1{\cdot}21\ d_1, \text{ and } d_1 = 0{\cdot}82\ d.$$

The sides of the cotter slope at 1 in 30 to 1 in 60 with the axis of the rod, so that the total taper in simple cotters is 1 in 30 to




Unit = $d$

Fig 155

1 in 15. By driving the cotter the rod is drawn up to its seating and clearance is left to permit this.

Fig. 156 shows an ordinary foundation bolt and cast-iron washer. The body of the bolt when long may be reduced to $d_1 = 0{\cdot}9\ d - 0{\cdot}05$. It is then as strong as the screwed part at the bottom of the thread.

*Taper of cotter.*—It is easy to show that the horizontal force necessary to drive the cotter home is

$$Q_1' + Q_2' = P\,\{\tan(\theta + \alpha_1) + \tan(\theta + \alpha_2)\}$$

where $\theta$ is the angle of repose of the materials. Similarly the force necessary to drive back the cotter is

$$Q^{1''} + Q_2'' = P\,\{\tan(\theta - \alpha_1) + \tan(\ - \alpha_2)\}$$

and the cotter will slip back without any additional force if

$$Q_1'' + Q_2'' = 0, \text{ and then } \alpha_1 + \alpha_2 = 2\,\theta.$$

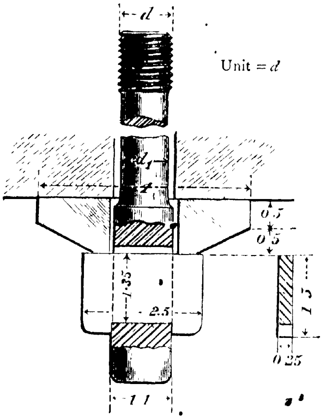



Fig. 156

Taking $\theta$ for slightly greasy metal at $4\frac{1}{2}°$,

$$\alpha_1 + \alpha_2 \text{ must not exceed } 9°$$

corresponding to a total taper of 1 in 7. The taper in practice is less than this for safety. For simple cotters a taper of 1 in 24 to 1 in 48 is usual. When a set screw or other means of preventing the slacking of the cotter is added, the taper may be 1 in 8 or 1 in 6.

138. *Strength and proportions of cotters and cottered rods.* (Figs. 157, 158.) Let P be the total tension transmitted, $d_1$ the smallest diameter of the rod; $d$ the diameter in the cottered part, D the diameter of the socket, the thickness of the socket $= \frac{1}{2}(D - d) = w$, $b$ the width of cotter (or gib and cotter),

$t$ its thickness, $f_s$, $f_t$, $f_b$, the working stresses in shear, tensio and bending. For equal strength of rod, and sections of ro and socket through the cotter hole—

$$P = \frac{\pi}{4} d_1^2 f_t \quad \ldots \quad (a)$$

$$= \left(\frac{\pi}{4} d^2 - d\,t\right) f_t \quad \ldots \quad (b)$$

$$= \left\{\frac{\pi}{4} (D^2 - d^2) - 2\,w\,t.\right\} f_t' \quad \ldots \quad (c)$$

If, as is common, $t = 0{\cdot}25\, d$, from $(a)$ and $(b)$—

$$d = 1{\cdot}22\, d_1 \text{ or } d_1 = 0{\cdot}82\, d.$$

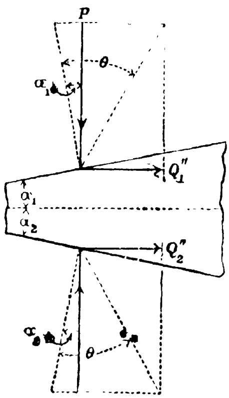



Fig. 157

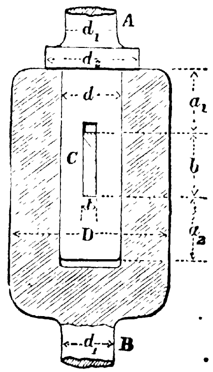



Fig. 158

The rod and socket are not necessarily of the same material. Let $f_t$ be the safe tearing resistance of the rod and $f_t'$ that of the socket in lbs. per sq. in. From $(b)$ and $(c)$, taking $t = 0{\cdot}25\, d$,

$$(\cdot 785\, D^2 - \cdot 25\, D\, d - \cdot 535\, d^2) f_t' = \cdot 535\, d^2 f_t$$

$$D = \left[0{\cdot}16 + \sqrt{\left\{\cdot 682\left(1 + \frac{f_t}{f_t'}\right) + \cdot 0256\right\}}\,\right] d.$$

If the socket and rod are of the same material, $f_t = f_t'$ and

$$D = 1{\cdot}34\, d.$$

If the rod is of mild steel and the socket of cast iron,

$$f_t / f_t' = 10000/2800 = 3{\cdot}57, \text{ and}$$
$$D = 1{\cdot}93\ d.$$

The crushing stress of the cotter on the rod and socket is

$$P = d\,t\,f_c = 2\,w\,t\,f_c' \qquad . \qquad . \qquad . \qquad (d)$$

Let the rod and socket be of steel, $t = 0{\cdot}25\ d$, and $D = 1{\cdot}34\ d$ so that $w = 0{\cdot}17\ d$.

Also insert the value of P in $(b)$—

$$0{\cdot}535\ f_t = 0{\cdot}25\ f_c = {\cdot}085\ f_c'$$

The crushing stress on the rod is

$$f_c = 2{\cdot}14\ f_t$$

and that on the socket is

$$f_c' = 6{\cdot}3\ f_t$$

This last is much greater than is desirable and hence the thickness of the socket must be greater than that necessary for tension. If $f_c'$ is taken equal to $f_c$, then from $(d)$—

$$w = 0{\cdot}5\ d, \text{ and } D = 2\ d.$$

This is a more usual proportion and with this value experience shows that the crushing action of the cotter is not too great. The unstrained material on each side increases the resistance of the seat of the cotter, and the cotter itself is usually of harder steel than the rod. If the socket is of cast iron the same diameter of the socket is sufficient.

The cotter itself is subjected to a bending action. Treating it in a similar way to the loose pin in § 132, the bending moment is $\frac{1}{2}$ P $(\frac{1}{4}\ d + \frac{1}{3}\ w)$. Equating this to the moment of resistance of a cotter of width $b$ and thickness $t$—

$$P\left(\frac{d}{8} + \frac{w}{6}\right) = \frac{b^2\,t}{6} f_b \qquad . \qquad . \qquad . \qquad (e)$$

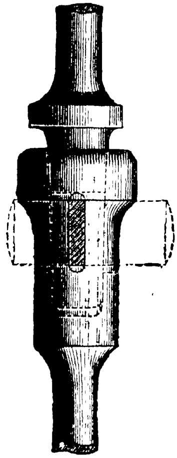
Let $w = 0{\cdot}5\ d$, $t = 0{\cdot}25\ d$, and inserting the value of P in $(b)$-

$$b = 1{\cdot}31\ d\sqrt{\frac{f_t}{f_b}}$$

If $\qquad f_b = f_t,$

then $\qquad b = 1{\cdot}31\ d.$

In practice the width of cotter is often rather larger and $b = 1{\cdot}5\ d$. The distances $a_1$, $a_2$ are generally taken each equal to $0{\cdot}75\ b$.



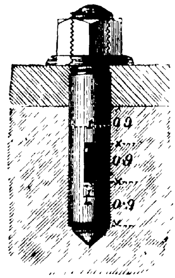


Fig. 160

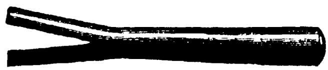


Fig. 159

Fig. 161

139. *Various arrangements of cotters.*—Fig. 159 shows the mode of cottering pump rods. The socket is conical. The collar is provided merely to facilitate the disengagement of the

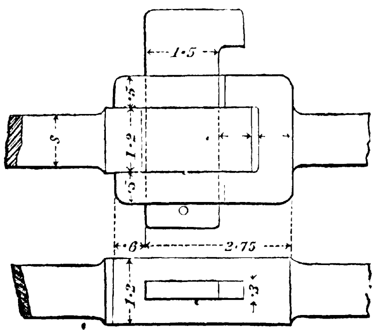



Fig. 162

parts after they have been for some time in use. The cotter having been driven out, wedges can be driven between the collar and socket so as to force them apart.

*Split pins* are virtually very small cotters, driven home like ordinary cotters and the split end opened out to prevent slacking back (fig. 161).

Fig. 160 shows a bolt cottered into a casting. The effective diameter of the bolt is the diameter at the bottom of the screw

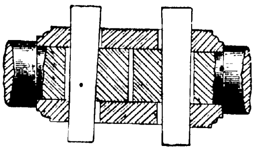


Fig. 163

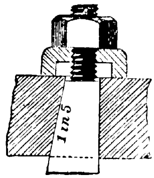



Fig 164

thread, and that is the value to be taken for $d$ in proportioning the cotter. The proportions marked on the figure have been so modified that the unit is the gross diameter $d$ of the bolt.

Fig. 162 shows a simple cottered joint for two bars.

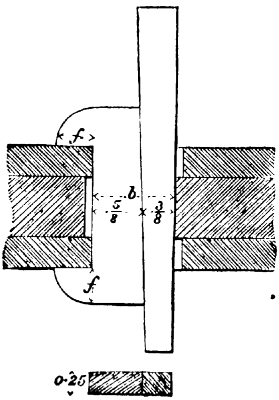



Fig 165

Unit = $b$

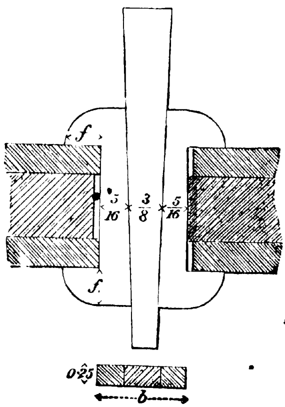



Fig 166

Fig. 163 is a cottered coupling suitable for rods transmitting a longitudinal thrust or pull.

If excessive taper must be given, to obtain sufficient draught, the end of the cotter is screwed, as shown in fig. 164, and a nut, bearing on a recessed washer or short tube, holds it in place.

## Gib and Cotter

140. When a cotter is used to connect strap-shaped parts to a more rigid rod, the cotter is divided into two parts, one acting as an ordinary cotter, the other having hooked ends, intended to prevent the spreading of the strap. It is convenient to make the outside of the gib parallel to the outside of the cotter, and to obtain the necessary draught by inclining the

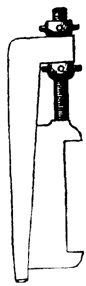


Fig 167

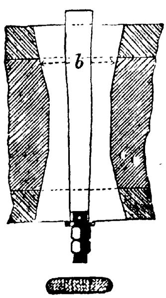



Fig. 168

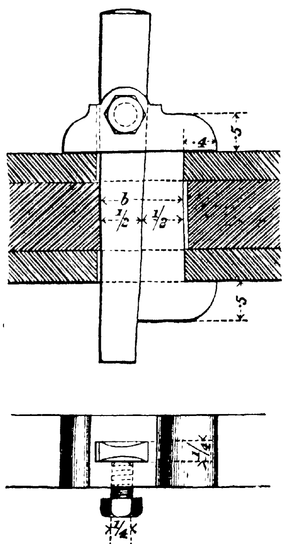



Fig. 169

division plane between them. The taper is usually 1 in 24 to 1 in 48 for simple cotters, and 1 in 8 to 1 in 16, when the slacking of the cotter is prevented by a screw. The total breadth $b$ and thickness $t$ of the gib and cotter are the same as for a simple cotter.

Figs. 165, 166 show ordinary proportions of gibs and cotters. The unit for the proportions is the breadth $b$.

The simplest way of securing a cotter is by a screwed pro-

longation of the gib, as shown in fig. 167. A set screw, passed through one of the connected pieces, is sometimes used.

Fig. 168 shows another arrangement of gibs and cotter. In this case the space is restricted, and the draught required is very small. The cotter is secured by a screwed end, nut and washer.

Fig. 169 shows another way of securing the cotter which is neat. The cotter passes through the head of the gib and is fixed by a set screw.

# CHAPTER VII

## JOURNALS, CRANK PINS, PIVOT AND COLLAR-BEARINGS

### JOURNALS

141. Journals are parts of rotating pieces supported in steps, brasses or bearings. The journal and its step form an *elementary pair* of elements, usually cylindrical, sometimes spherical or conical. Some journals run constantly, others support pieces which only move occasionally. In the latter case the strength of the journal is the chief consideration in designing. In the former durability and freedom from liability to heating are of equal importance, and these are directly dependent on reduction of friction. As to strength, some journals are subjected to straining forces acting in a plane through the axis only, and have to resist bending and shearing stresses. Others transmit power, are subjected to bending and torsion, and are calculated by the rules for combined stress.

Let $d$ = diameter of journal in inches.
$l$ = length " " "
N = revolutions per minute.
P = total load normal to the axis, in lbs.
$p$ = P$/d l$ = the bearing pressure in lbs. per square inch.
$v = (\pi d \mathrm{N})/(60 \times 12) = \cdot 00436\, d\mathrm{N}$ = surface velocity of journal in feet per second.

*Forms of journals.*—Some journals are fixed at one end only, or overhang, and are termed *end journals*; others are supported at both ends, and are termed *neck journals.*

An ordinary form of end journal is shown in fig. 170. The journal is cylindrical, and is terminated by shoulders or collars which limit the end play. Sometimes the length of the step or brass is only 0·9 of the length of the journal, so that there is a limited amount of end play, which ensures uniform wear. In other cases end play would interfere with the action of the

machine and is reduced to a very small amount. The collar height may be

$$e = 0{\cdot}08\, d + 0{\cdot}2 \text{ to } 0{\cdot}12\, d + 0{\cdot}2$$

The journal and its steps are usually of different metals, because then the liability to 'cutting' and 'seizing' is reduced. Cast-iron and wrought iron journals run well on cast-iron steps at slow speeds. At higher speeds the steps are made of brass, gun-metal, or phosphor bronze. In important bearings, the steps are lined with Babbitt metal or some similar soft alloy, which from its plasticity forms an accurate surface of contact with the journal very favourable to good lubrication and reduction of friction. The lubricant, oil, grease or graphite, which is supplied to journals to reduce friction and wear, forms a film between the journal and step. It should have such an amount of viscosity, greater as the bearing pressure is greater, that it is not squeezed out.

142. *Heating of journals.*—The work expended in friction, due to the rubbing of the journal on its step, produces an equivalent amount of heat which is conducted away and radiated. The rate of dissipation depends on the area of journal surface and the difference of temperature of the journal and surrounding space. The journal takes a temperature at which the rates of production and dissipation of heat balance. The heat produced by a journal of diameter $d$, making N revolutions per minute and carrying a load P, is

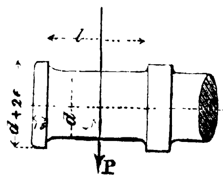


Fig. 170

$$\frac{\mu \text{P}}{\text{J}} \frac{\pi d \text{N}}{12} \text{ Th.U. per minute} \quad . \quad . \quad . \quad (1)$$

Where $\mu$ is the coefficient of journal friction and $\text{J} = 778$. The viscosity of the lubricant diminishes as the temperature is greater. This diminishes the rate of heat production, but also renders the lubricant more liable to be squeezed out. If from bad proportions or any other cause the temperature of the journal rises above a certain limit the lubricant is squeezed out, the friction and the rate of production of heat rapidly increase and the bearing 'seizes,' or in some cases the soft lining of the step is fused.

It is sometimes prescribed that the temperature of bearings when running under full load should not exceed 100° F. This is a good general rule, but Mr. Mattice ('Trans. Am. Soc. Mech. Eng.,' vol. xxvii.) has stated that large engine bearings lubricated with mineral oil have commonly a temperature of 135° F., and that in one case a bearing was working without giving trouble at a temperature of 180° F.

The tendency of a journal to get overheated depends on a number of conditions which cannot be quantitatively estimated, and overheating often occurs and gives great trouble in circumstances in which it is difficult to assign a cause. Whatever reduces the friction reduces the liability to overheating, and hence the journal and step should be very accurately fitted, the rubbing surfaces smooth, and the lubricant of suitable quality and properly distributed. The friction is least when there is a complete oil film between journal and step.

143. *Bearing pressure.*—The conditions just stated being complied with, the next most important condition of cool running is that the bearing pressure of the journal on its step should not exceed certain limits. Let P be the mean load on a journal running in ordinary full-load conditions, and $d$ and $l$ the diameter and length of the journal. The mean bearing pressure, reckoned on the projected area of the journal, is—

$$p = \mathrm{P}/d\,l \text{ lbs. per sq. in.}$$

The limiting safe-bearing pressure in different cases has been roughly ascertained by experience. It varies from 50 to 3 000 lbs. per sq. in. All that can be done at present to help a designer is to give values permitted in certain definite cases, and to indicate the conditions which most obviously determine it.

The allowable bearing pressure depends primarily on the materials of which journal and step are composed and on the accuracy of their fitting. It depends in some way on the velocity of rubbing of journal and step and on local conditions which determine the rate at which the heat produced is conducted away. It varies with the arrangements for lubrication and the kind of lubricant used, the most favourable conditions being those in which a continuous oil film is maintained between the rubbing surfaces. At low-rubbing velocities the oil is not so well carried in as at higher velocities and lower bearing pressures are necessary.

If a journal works intermittently, the amount of heat produced is comparatively small, and the time for cooling large. Hence high-bearing pressures are permissible. If the direction of the load on the journal reverses in each revolution, as in the case of an engine crank pin, so that the pressure comes alternately on two brasses or steps, it is found that rather high-bearing pressures can be sustained. Probably the alternate relief of pressure on each step facilitates lubrication, and the heating is divided over two steps through which it is conducted away. Crosshead pins on which the motion is a limited oscillation carry high-bearing pressures for similar reasons. Journals on which the load is constant in magnitude and direction, and which rest on a single step, must be proportioned for smaller bearing pressures.

In difficult cases where the bearing pressure cannot be sufficiently reduced, and trouble from heating is feared, forced lubrication may be adopted or the casing is made hollow and a water circulation to carry away the heat is provided.

So far as the materials of which journals and steps are constructed, the following are maximum limiting values of the bearing pressure in ordinary cases :—

| | Bearing Pressure $p = P/dl$ lbs. per sq. in. |
|---|---|
| Hardened steel on hardened steel | 2 000 |
| Unhardened steel on gunmetal | 800 |
| Mild steel on iron or bronze | 500 |
| Mild steel or wrought iron on cast iron or lignum vitæ | 350 |

## Friction of Journals

144. *General considerations on friction.*—The so-called ordinary laws of friction were ascertained by Morin chiefly by experiments on plane sliding surfaces with moderate velocities and pressures. They are approximately true over a considerable range of conditions for dry or *imperfectly lubricated* surfaces. According to Morin the frictional resistance to sliding is directly proportional to the total pressure between the surfaces, and is independent of the area of the surfaces and the velocity of sliding. Hence if R is the resistance to sliding, P the total pressure between the surfaces,

$$R = \mu P$$

where $\mu$ is a constant for given surfaces termed the *coefficient*

*of friction.* This coefficient Morin found to be about 0·25 for wood sliding on wood, dry; about 0·15 for metal on metal, dry; and about 0·07 to 0·08 for metal on metal fairly well lubricated.

At very low intensities of pressure, $p = P / a$, where $a$ is the area of surface in contact, $\mu$ increases. At very high intensities of pressure there is sensible abrasion and $\mu$ also increases. At high velocities, over 15 ft. per second with dry surfaces, $\mu$ diminishes, ranging from 0·1 to 0·2.

*Journal friction.*—Morin made some experiments on journal friction, with velocities and pressures such that the values of $p\,v = (P\,v) / (d\,l)$ did not exceed 100. In these conditions the relation $R = \mu\,P$ was approximately satisfied, $\mu$ having the value 0·18 for dry surfaces, 0·07 to 0·08 for intermittent lubrication, and 0·03 to 0·05 for continuous lubrication. It is probable that in these experiments the lubrication was always imperfect and that contact between journal and step was restricted to a line or narrow strip of surface.

If the diameter of the journal is $d$ inches, the revolutions N per minute, the rubbing velocity $v$ ft. per sec., and the load P lbs., then the work expended in friction is—

$$T = \mu\,P\,\frac{\pi\,d\,N}{12 \times 60} = 0{\cdot}00436\,\mu\,P\,d\,N = \mu\,P\,v \text{ foot lbs. per sec.} \quad (2)$$

The running conditions of ordinary journals in practice are different from those in Morin's experiments. The intensities of pressure and velocities are much greater, $p\,v$ reaching values of 10,000 or more. The lubrication also is more copious. In such conditions a more or less complete film of oil is formed between the journal and step, and the nature of the friction is completely changed. The resistance is due to the shearing of the oil film, and depends directly on the velocity of shearing and the viscosity of the oil, which again varies with the temperature of the oil film.

In some remarkable experiments by Hirn, in 1855, it was shown that for lubricated journals in ordinary normal conditions of running, the coefficient of friction or factor $\mu$ in the relation $R = \mu\,P$, so far from being constant, was more nearly proportional to the square root of the velocity of rubbing, $v$, and inversely proportional to the square root of the intensity of pressure, $p$. That is—

$$\mu = (c\sqrt{v})/\sqrt{p} \quad . \quad . \quad . \quad . \quad (3)$$

where $c$ is a constant.

In experiments by Beauchamp Tower,[1] to which fuller reference will be made presently, with still more complete lubrication, it was found that $\mu$ was independent of the total load on the journal, so that

$$\mu = (c\sqrt{v})/p \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $c$ is a constant.

If a bearing is imagined having a uniformly thick film of oil between journal and step, maintained at a constant temperature, the resistance would be proportional to $v$, and then

$$\mu = c\,v\,/\,p \quad . \quad . \quad . \quad . \quad . \quad (5)$$

145. *Beauchamp Tower's Experiments.*—In this research, carried out by the direction of the Institution of Mechanical Engineers, the journal was a horizontal steel journal 4 inches diameter and 6 inches long, with a gunmetal step embracing nearly the top semi-circumference. A steady vertical load was applied at the step. In these conditions it was found impossible to get regular and consistent values of the frictional resistance till an *oil bath* under the journal was adopted giving a continuous supply of lubricant. Then the coefficient of friction had very small values, and the friction was independent of the total pressure between the journal and step as it would be in a case of pure fluid friction.

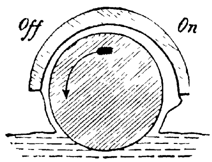


Fig 171

Further experiments by Mr. Tower, and an investigation by Professor Osborne Reynolds, proved that in the oil-bath experiments the condition of the journal and step was that shown in an exaggerated way in fig. 171. The step, either by wear or expansion, was of slightly greater radius than the journal, and the oil was carried in on the ' on ' side between the journal and step, forming a complete and continuous film, and carried out again at the ' off ' side. A pressure is generated in the fluid film which lifts and supports the whole load on the step, and the film is thinnest and the pressure greatest at a point on the ' off ' side of the centre of the step.

*Distribution of pressure in the oil film.*—By drilling holes $\frac{1}{16}$ inch in diameter, and connecting these by passages with

[1] *Proc. Inst. Mechanical Engineers*, 1883 and 1884.

pressure gauges, the pressure in the oil film at nine points distributed over one-half of the length of the journal was determined. The total load on the bearing was 8,008 lbs., or 667 lbs. per sq. in. The bearing was run at 150 revolutions per minute, or a rubbing speed of 2·6 feet per second. Calling the three planes parallel to the axes of the journal in which the pressures were observed, the on, centre, and off planes, and the transverse planes from the centre towards the ends of the journal the centre, No. 1 and No. 2 planes, the pressures observed in the oil film were as follows :—

*Oil Pressures at Different Points of a Bearing.*
*Oil Bath Lubrication. Mineral Oil.*

| Longitudinal Planes | Pressures in lbs per square inch On | Centre | Off |
|---|---|---|---|
| Transverse plane, centre | 370 | 625 | 500 |
| No. 1 | 355 | 615 | 485 |
| No. 2 | 310 | 565 | 430 |

The magnitude of these pressures is surprising. Curves drawn using the observed pressures as ordinates are shown in

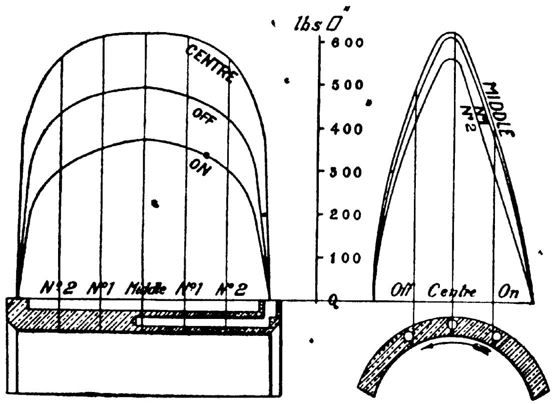



Fig. 172

fig. 172. It will be seen that the greatest pressure is on the 'off' side of the journal. The total upward pressure calculated from these figures is practically identical with the total load on the journal, so that the step was wholly supported by a continuous fluid film. What occurs with less perfect lubrication is not so clear, but probably, with moderately good lubrication, the step

is still fluid borne, the film extending over part only of the bearing surface. When the lubrication is very imperfect the film breaks down and there is contact between journal and step. The coefficient of friction then rises towards the value found by Morin.

To secure the best conditions for lubrication obviously the step must be symmetrically loaded. Any eccentricity of the loading relatively to the middle of the journal will create a tendency to drive out the oil at one end. This is the reason why steps which swivel so as to accommodate themselves to any inclination of the journal due to bending are found to be advantageous.

146. *Experiments with perfect or oil-bath lubrication—Beauchamp Tower's.*—The results of these experiments prove that for the range of pressures and velocities observed, the frictional resistance of the journal is directly proportional to the square root of the rubbing velocity $v$, independent of the total load P, and therefore inversely as the bearing pressure, $p$.

The journal takes a temperature such that the coefficient $\mu$ in the ordinary friction equation has the value

$$\mu = (c \sqrt{v}) / p \qquad . \quad . \quad . \qquad (6)$$

where $c$ is a constant for a given lubricant and a given type of bearing. The total frictional resistance of the journal is—

$$R = \mu P = c\,d\,l \sqrt{v} \qquad . \quad . \quad . \qquad (6a)$$

proportional to the surface area of the journal and to the square root of the rubbing velocity, and independent of the load on the journal.

The following tables contain a reduction of all Beauchamp Tower's experiments, with the exception of a few dealt with later. They give the values of $\mu$, the product $\mu p$, which is practically constant at each velocity, and the values of $c = \mu p / \sqrt{v}$ deduced from the mean values of $\mu p$ at each velocity. These values of $c$ are practically constant for each lubricant, notwithstanding the difficulty of ensuring constant conditions of the rubbing surfaces, and temperature in different tests. The journal was 4 inches diameter, 6 inches long, with a chord of contact of 3·92 inches, and the temperature was kept as constant at 90° as possible. The load was a steady load in one direction.

TABLE I.—*Olive Oil.*

| Load in lbs. per sq. inch of bearing surface $p$ | Rubbing speed $v$ in ft. per sec. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1·75 | 2·61 | 3·49 | 4·37 | 5·23 | 6·10 | 6·99 | 7·85 |
| | *Values of coefficient $\mu$* | | | | | | | |
| 520 | — | ·0008 | ·0010 | ·0012 | ·0013 | ·0014 | ·0015 | ·0017 |
| 468 | — | ·0011 | ·0013 | ·0014 | ·0015 | ·0017 | ·0018 | ·0020 |
| 363 | — | ·0013 | ·0016 | ·0017 | ·0019 | ·0020 | ·0022 | ·0025 |
| 310 | — | ·0015 | ·0017 | ·0019 | ·0021 | ·0022 | ·0024 | ·0027 |
| 258 | ·0014 | ·0017 | ·0020 | ·0023 | ·0025 | ·0026 | ·0029 | ·0031 |
| 205 | ·0018 | ·0021 | ·0025 | ·0028 | ·0030 | ·0033 | ·0036 | ·0040 |
| 153 | ·0023 | ·0030 | ·0035 | ·0040 | ·0044 | ·0047 | ·0050 | ·0057 |
| 100 | ·0036 | ·0045 | ·0055 | ·0063 | ·0069 | ·0077 | ·0082 | ·0089 |
| | *Values of $\mu p$* | | | | | | | |
| 520 | — | ·41 | ·52 | ·61 | ·66 | ·71 | ·77 | ·87 |
| 468 | — | ·52 | ·61 | ·66 | ·70 | ·80 | ·84 | ·94 |
| 363 | — | ·47 | ·58 | ·62 | ·69 | ·73 | ·80 | ·91 |
| 310 | — | ·47 | ·53 | ·59 | ·65 | ·68 | ·74 | ·84 |
| 258 | ·36 | ·44 | ·52 | ·59 | ·65 | ·67 | ·75 | ·80 |
| 205 | ·37 | ·43 | ·51 | ·57 | ·62 | ·68 | ·74 | ·82 |
| 153 | ·35 | ·46 | ·54 | ·61 | ·67 | ·72 | ·77 | ·87 |
| 100 | ·36 | ·45 | ·55 | ·63 | ·69 | ·77 | ·82 | ·89 |
| | *Values of $c = \mu p / \sqrt{v}$* | | | | | | | |
| — | ·27 | ·28 | ·29 | ·29 | ·29 | ·29 | ·30 | ·30 |

Here the values of $c$ are satisfactorily constant. The mean value of $c$ for olive oil with bath lubrication is $c = 0·290$.

TABLE II.—*Lard Oil.*

| $p$ lbs. per sq. in. | $v$ in ft. per sec. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1·75 | 2·61 | 3·49 | 4·37 | 5·23 | 6·10 | 6·99 | 7·85 |
| | *Values of $\mu$* | | | | | | | |
| 520 | — | ·0009 | ·0010 | ·0011 | ·0013 | ·0015 | ·0015 | ·0017 |
| 415 | — | ·0012 | ·0014 | ·0015 | ·0016 | ·0018 | ·0019 | ·0021 |
| 310 | — | ·0014 | ·0017 | ·0020 | ·0022 | ·0025 | ·0026 | ·0029 |
| 205 | ·0017 | ·0020 | ·0023 | ·0028 | ·0031 | ·0034 | ·0039 | ·0042 |
| 153 | ·0022 | ·0027 | ·0032 | ·0037 | ·0041 | ·0050 | ·0051 | ·0052 |
| 100 | ·0035 | ·0042 | ·0050 | ·0060 | ·0067 | ·0076 | ·0081 | ·0090 |
| | *Values of $\mu p$* | | | | | | | |
| 520 | — | ·46 | ·52 | ·56 | ·66 | ·77 | ·77 | ·87 |
| 415 | — | ·50 | ·58 | ·62 | ·66 | ·75 | ·79 | ·87 |
| 310 | — | ·43 | ·53 | ·62 | ·68 | ·78 | ·81 | ·90 |
| 205 | ·35 | ·41 | ·47 | ·57 | ·64 | ·70 | ·80 | ·86 |
| 153 | ·34 | ·41 | ·49 | ·57 | ·63 | ·77 | ·78 | ·80 |
| 100 | ·35 | ·42 | ·50 | ·60 | ·67 | ·76 | ·81 | ·90 |
| | *Values of $c = \mu p / \sqrt{v}$* | | | | | | | |
| — | ·26 | ·27 | ·28 | ·28 | ·29 | ·30 | ·30 | ·31 |

The mean value of $c = 0·286$.

TABLE III.—*Mineral Grease.*

| $p$ lbs. per sq. in | $v$ in ft. per sec. 1·75 | 2·61 | 3·49 | 4·37 | 5·23 | 6·10 | 6·99 | 7·85 |
|---|---|---|---|---|---|---|---|---|
| | *Values of μ* | | | | | | | |
| 625 | — | ·0010 | ·0012 | ·0014 | ·0014 | ·0016 | ·0018 | ·0020 |
| 520 | — | ·0014 | ·0016 | ·0018 | ·0019 | ·0020 | ·0021 | ·0022 |
| 415 | — | ·0016 | ·0019 | ·0021 | ·0023 | ·0025 | ·0026 | ·0027 |
| 310 | ·0020 | ·0022 | ·0026 | ·0029 | ·0032 | ·0035 | ·0038 | ·0040 |
| 205 | ·0026 | ·0034 | ·0040 | ·0047 | ·0053 | ·0058 | ·0062 | ·0066 |
| 153 | ·0028 | ·0038 | ·0048 | ·0057 | ·0065 | ·0071 | ·0077 | ·0083 |
| | *Values of μ p* | | | | | | | |
| 625 | — | ·63 | ·75 | ·88 | ·88 | 1·00 | 1·13 | 1·25 |
| 520 | — | ·73 | ·83 | ·94 | ·99 | 1·04 | 1·09 | 1·14 |
| 415 | — | ·66 | ·79 | ·87 | ·96 | 1·04 | 1·08 | 1·12 |
| 310 | ·62 | ·68 | ·81 | ·90 | ·99 | 1·09 | 1·18 | 1·24 |
| 205 | ·53 | ·70 | ·82 | ·96 | 1·09 | 1·19 | 1·27 | 1·35 |
| 153 | ·43 | ·58 | ·73 | ·87 | 1·00 | 1·09 | 1·18 | 1·27 |
| | *Values of $c = \mu p / \sqrt{v}$* | | | | | | | |
| — | ·40 | ·41 | ·42 | ·43 | ·43 | ·43 | ·44 | ·44 |

The mean value is $c = 0·426$.

TABLE IV.—*Sperm Oil.*

| $p$ lbs. per sq. in. | $v$ in ft. per sec. 1·75 | 2·61 | 3·49 | 4·37 | 5·23 | 6·10 | 6·99 | 7·85 |
|---|---|---|---|---|---|---|---|---|
| | *Values of coefficient μ* | | | | | | | |
| 310 | — | ·0011 | ·0012 | ·0014 | ·0016 | ·0017 | ·0018 | ·0019 |
| 205 | ·0013 | ·0016 | ·0018 | ·0021 | ·0023 | ·0024 | ·0025 | ·0027 |
| 153 | ·0016 | ·0019 | ·0023 | ·0028 | ·0030 | ·0033 | ·0035 | ·0037 |
| 100 | ·0025 | ·0030 | ·0038 | ·0044 | ·0051 | ·0057 | ·0061 | ·0064 |
| | *Values of μ p* | | | | | | | |
| 310 | — | ·34 | ·37 | ·43 | ·50 | ·53 | ·56 | ·59 |
| 205 | ·27 | ·33 | ·37 | ·43 | ·47 | ·49 | ·51 | ·55 |
| 153 | ·25 | ·29 | ·35 | ·43 | ·46 | ·51 | ·54 | ·57 |
| 100 | ·25 | ·30 | ·38 | ·44 | ·51 | ·57 | ·61 | ·64 |
| | *Values of $c = \mu p / \sqrt{v}$* | | | | | | | |
| — | ·19 | ·20 | ·20 | ·21 | ·21 | ·21 | ·21 | ·21 |

The mean value is $c = 0·204$.
This journal seized with a load of 520 lbs. per sq. in.

TABLE V.—*Rape Oil.*

| $p$ lbs. per sq. in. | $v$ in ft. per sec. 1·75 | 2·61 | 3·49 | 4·37 | 5·23 | 6·10 | 6·99 | 7·85 |
|---|---|---|---|---|---|---|---|---|
| | Values of coefficient $\mu$ | | | | | | | |
| 415 | — | ·0009 | ·0011 | ·0012 | ·0013 | ·0014 | ·0015 | ·0016 |
| 363 | — | ·0008 | ·0010 | ·0011 | ·0012 | ·0013 | ·0015 | ·0016 |
| 258 | ·0011 | ·0014 | ·0016 | ·0018 | ·0020 | ·0021 | ·0023 | ·0024 |
| 153 | ·0016 | ·0020 | ·0024 | ·0027 | ·0030 | ·0033 | ·0037 | ·0040 |
| 100 | ·0028 | ·0036 | ·0042 | ·0050 | — | — | — | — |
| | Values of $\mu p$ | | | | | | | |
| 415 | — | ·39 | ·44 | ·49 | ·54 | ·58 | ·62 | ·66 |
| 363 | — | ·30 | ·35 | ·40 | ·44 | ·49 | ·53 | ·56 |
| 258 | ·28 | ·36 | ·42 | ·46 | ·50 | ·55 | ·59 | ·63 |
| 153 | ·25 | ·30 | ·36 | ·41 | ·46 | ·51 | ·56 | ·60 |
| 100 | ·28 | ·36 | ·42 | ·50 | — | — | — | — |
| | Values of $c = \mu p / \sqrt{v}$ | | | | | | | |
| — | ·20 | ·21 | ·21 | ·22 | ·21 | ·22 | ·22 | ·22 |

The mean value is $c = 0·213$.

The bearing seized on reversing with a load of 573 lbs. per sq. in.

TABLE VI.—*Mineral Oil.*

| $p$ lbs. per sq. in. | $v$ in ft. per sec. 1·75 | 2·61 | 3·49 | 4·37 | 5·23 | 6·10 | 6·99 |
|---|---|---|---|---|---|---|---|
| | Values of coefficient $\mu$ | | | | | | |
| 310 | — | ·0014 | ·0016 | ·0018 | ·0021 | ·0023 | ·0024 |
| 205 | ·0018 | ·0021 | ·0024 | ·0027 | ·0030 | ·0033 | ·0035 |
| 100 | ·0033 | ·0042 | ·0049 | ·0056 | ·0062 | ·0068 | ·0073 |
| | Values of $\mu p$ | | | | | | |
| 310 | — | ·44 | ·50 | ·57 | ·64 | ·70 | ·75 |
| 205 | ·36 | ·42 | ·48 | ·55 | ·61 | ·67 | ·72 |
| 100 | ·33 | ·42 | ·49 | ·56 | ·62 | ·68 | ·73 |
| | Values of $c = \mu p / \sqrt{v}$ | | | | | | |
| — | ·27 | ·26 | ·26 | ·27 | ·27 | ·28 | ·28 |

The mean value is $c = 0·270$.

With greater loads the coefficient of friction increased.

The bearing carried 625 lbs. per sq. in., running both ways, but seized on the weight being increased.

It appears, therefore, that within the range of speeds and

pressures stated above $c$ is very approximately constant for a given lubricant. Hence the frictional resistance is—

$$R = \mu P = c P \sqrt{v/p} = c\, d\, l \sqrt{v} \text{ lbs.},$$

where—

| | |
|---|---|
| For Olive oil . . . . . | $c = 0\cdot290$ |
| Lard oil . . . . . . | $0\cdot286$ |
| Mineral grease . . . | $0\cdot426$ |
| Sperm oil. . . . . . | $0\cdot204$ |
| Rape oil . . . . . . | $0\cdot213$ |
| Mineral oil . . . . . | $0\cdot270$ |

147. *Coefficient for low bearing pressures.*—In one case, that of the mineral grease, the friction suddenly increased when the pressure was reduced from 153 to 100 lbs. per sq. in., and that at all velocities. At 153 lbs. per sq. in. and upwards $c = 0\cdot43$. But at 100 lbs. per sq. in. it was $0\cdot52$.

*Coefficient for excessive bearing pressures.*—In three of the series of tests, when the bearing pressure exceeded a certain value, and approached the load at which the journal seized, there was a gradual increase of friction at all velocities, probably due to squeezing out of the lubricant. The values of $c$ are nearly, but not quite, constant at different velocities, but they increased considerably as the load increased.

*Values of c, when the load approached the point of seizing.*

| Bearing pressure, $p$ | Velocity $v$: 2·61 | 3·49 | 4·37 | 5·23 | 6·10 | 6·99 |
|---|---|---|---|---|---|---|
| | Values of $c = \mu p / \sqrt{v}$ | | | | | |
| | *Sperm Oil.* | | | | | |
| 415 | ·385 | ·378 | ·357 | ·345 | ·336 | ·330 |
| | *Rape Oil.* | | | | | |
| 573 | ·354 | ·331 | ·323 | ·315 | ·306 | ·301 |
| 520 | ·306 | ·292 | ·286 | ·284 | ·280 | ·280 |
| | *Mineral Oil.* | | | | | |
| 625 | ·502 | ·464 | ·440 | ·428 | ·417 | — |
| 520 | ·414 | ·387 | ·373 | ·366 | ·358 | ·351 |
| 415 | ·315 | ·317 | ·318 | ·319 | ·320 | ·315 |

So far as these experiments go, the seizing pressure appears to be independent of the velocity.

148. *Coefficient of friction at low velocities.*—Beauchamp Tower made one series of tests with the same journal and oil-bath

lubrication at the very low speed of 20 revs. per min. or 0·35 feet of rubbing velocity per second. The results in no way agree with the results at the higher velocities. The friction is greater, and is not independent of the load on the journal. At some point the law that $\mu$ varies inversely as $\sqrt{v}$ must change. As there is only one series, nothing can be done, but to give the values of $\mu$ found in these tests.

*Values of $\mu$ at a Rubbing Speed of 0·35 ft. per sec.*

| | | | | |
|---|---|---|---|---|
| $p =$ | 443 | 333 | 211 | 89 |
| $\mu =$ | ·0013 | ·0017 | ·0025 | ·0044 |

149. *Coefficient with perfect lubrication at different temperatures.*—It has already been pointed out that a journal takes a definite temperature at which the rate of generation and dissipation of heat balance. In the tests above, means were taken to keep the temperature of the oil fed into the bearing from the oil-bath at 90° F. A special series of tests were made with the oil fed in at different temperatures. The rubbing speed varied from 1·75 to 7·85 ft. per second, the bearing pressure was in all cases 100 lbs. per sq. in., and the lubricant was lard oil, supplied from a bath. The results gave very different values of $\mu$, but the values of $c$ are satisfactorily constant at each temperature.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Temperature F. . | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| Value of $c$ . . | ·54 | ·44 | ·35 | ·29 | ·25 | ·21 | ·19 |

Mr. W. K. Jervis ('Am. Machinist,' 1907, 655) has made some interesting tests of a journal run at 1,020 revs. per min., or 6·17 feet of surface speed per sec., at different temperatures, and over a much greater range of temperature than those given above. The bearing pressure was 250 lbs. per sq. in. The values of $\mu$ were a minimum at 190° for machine oil, but continued to decrease for lard oil up to 280° F. Probably above 190° the machine oil disintegrates. The oil was supplied by an oil-bath, so that the lubrication was practically perfect.

| Temperature Fahr. | Machine Oil | | Lard Oil | |
|---|---|---|---|---|
| | $\mu$ | $c$ | $\mu$ | $c$ |
| 100 | ·00429 | ·433 | ·00432 | ·436 |
| 120 | ·00292 | ·294 | ·00306 | ·309 |
| 140 | ·00232 | ·234 | ·00242 | ·244 |
| 180 | ·00187 | ·188 | ·00188 | ·189 |
| 220 | ·00203 | ·205 | ·00170 | ·172 |
| 260 | ·00287 | ·289 | ·00164 | ·165 |

The values of $c$ are higher than those of Beauchamp Tower at corresponding temperatures. Possibly this may be due to a difference in the place at which the temperature was observed.

150. *Coefficient of friction when the lubrication is imperfect.*— Tests were made with a needle lubricator placed at the centre of the top of the step. With this arrangement the bearing would not run cool even with 100 lbs. per sq. in. The oil, instead of entering the oil-hole, was forced out there. Other attempts to introduce oil at the top of the brass equally failed. Mr. Tower remarks that if such an arrangement answers in the case of railway axles, it must be due to the fact that a railway axle has continual end play when running, which prevents the step becoming the perfect oil-tight fit which it was in the experimental apparatus.

In another series of tests oil was introduced from a siphon into grooves in the step, parallel to the axis and near the 'on' edge of the step. These succeeded better, though the results are irregular, and the bearing seized with a load of 380 lbs. per sq. in. The values of $\mu$ sometimes increased and sometimes decreased with the velocity, but are nearly proportional to the bearing pressure. Hence approximately in these conditions

$$\mu = c'p \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where $c'$ is constant.

*Siphon Lubrication with Rape Oil*

| Bearing pressure $p$ | Velocity of rubbing $v_o$ | | | | | |
|---|---|---|---|---|---|---|
| | 2·61 | 3·49 | 4·37 | 5·23 | 6·10 | 6·99 |
| | Values of $c' = \mu p$ | | | | | |
| 317 | 1·77 | 1·80 | 2·00 | 2·16 | — | — |
| 252 | 2·47 | 1·76 | 1·94 | 2·07 | 2·19 | — |
| 123 | 1·53 | 1·80 | 1·87 | 2·00 | 2·10 | 2·19 |
| Means . . . | 1·92 | 1·79 | 1·94 | 2·08 | 2·15 | 2·19 |

Next, a felt pad, soaked in rape oil, was placed under the journal. The results with this mode of lubrication were fairly regular and consistent, and the journal carried 550 lbs. per sq. in., but seized when the load was increased to 582 lbs. The law of variation of friction in this case was wholly different from that with oil-bath lubrication. The value of $\mu$ did not definitely vary either with velocity or pressure. For rubbing speeds of 1·75 to 7·0 feet per second, and bearing pressures from 582 to 178 lbs. per sq. in, $\mu$ varied in different trials from 0·015 to 0·008, and its mean value was $\mu = 0·01$. In these trials the temperature of the

journal rose regularly with the bearing pressure. It was 75° F. with 178 lbs. per sq. in., and 90° with 582 lbs. per sq. in.

*Average Values of the Coefficient $\mu$, deduced from Mr. Beauchamp Tower's Experiments*

| Pressure per sq in. of bearing surface $p$ | Surface velocity of journal in ft per sec. 2 | 4 | 6 | 8 | 12 |
|---|---|---|---|---|---|
| | Oil-bath lubrication, mineral oil, $\mu = 0{\cdot}27\sqrt{v}/p$ | | | | |
| 100 | ·0039 | ·0055 | ·0068 | ·0078 | ·0092 |
| 200 | ·0020 | ·0028 | ·0034 | ·0039 | ·0046 |
| 300 | ·0013 | ·0018 | ·0023 | ·0026 | ·0031 |
| 400 | ·0010 | ·0014 | ·0017 | ·0020 | ·0023 |
| | Siphon lubrication, rape oil, $\mu = 2{\cdot}0/p$ | | | | |
| 100 | — | ·0202 | at all the speeds | | |
| 200 | — | ·0101 | | | |
| 300 | — | ·0067 | | | |
| 400 | — | ·0051 | | | |
| 100 to 400 | Pad lubrication, rape oil, $\mu$ = ·01 at all speeds | | | | |

The following table contains the results of some experiments by Kuhn and Mickle on journals of wrought iron and steel, 8 ins. diameter and 1⅝ ins. long. Step of gunmetal of such a width that the bearing surface was 4 sq. ins. Imperfect lubrication with lard oil. Temperatures in different tests from 90° to 137° F.

*Values of $\mu$ for a Journal with Ordinary Lubrication*

| Pressure on bearing surface in lbs. per sq. in. $p$ | Surface velocity of journal 4 | 8 | 15 |
|---|---|---|---|
| 125 | ·0047 | ·0068 | ·0070 |
| 250 | ·0044 | ·0034 | — |
| 400 | ·0027 | — | — |

These results are rather irregular, but they agree fairly with Beauchamp Tower's results for oil-bath lubrication at the lowest bearing pressure, but are greater at higher bearing pressures.

In applying these results it must be remembered that Mr. Beauchamp Tower used a practically steady load acting always in one direction. Any condition tending to facilitate the inflow of oil between the journal and step and to distribute it lowers the friction towards the values for oil-bath or perfect lubrication.

Thus in the case of railway axle bearings, which have pad lubrication, the great vibration so facilitates lubrication that $\mu$ has values nearly as low as those given above for oil-bath lubrication. Still more, when the load on a journal changes direction at each revolution, as the thrust on a crank pin, even with imperfect methods of introducing oil, the lubrication is good. The crank pin presses alternately on the two brasses, and oil gets into each during the relief of pressure.

151. *Pressures at which journals seize.*—As to the limiting pressures which can be carried without seizing of the journal, they depend on the viscidity of the lubricant, the material of the journal and step, the temperature, and probably the velocity of rubbing. Probably, when lubrication for any reason becomes a little imperfect, the temperature rises, the oil becomes less viscid and is squeezed out, and then the friction suddenly rises and seizing occurs. The intensity of pressure at which seizing occurred in Mr. Tower's experiments was rather low, and higher pressures are carried without danger in many practical cases. In most of such cases the pressure is intermittent or reverses in direction, a condition favourable to the maintenance of an oil film in the bearing. Professor Goodman states that he has had a journal running for weeks with a surface velocity of 4 ft. per second under a load of two tons per sq. in., the journal being kept at a temperature of 110° F. by a stream of water forced through it. This is, of course, a very extreme case.

*Forced lubrication.*—Messrs. Bellis & Co. and others have adopted the method of continuously forcing oil into all the bearings of engines, running at high speeds of rotation, and have achieved very remarkable results in reducing friction and wear and in securing noiselessness, economy, and freedom from trouble in running.[1] An oscillating oil pump is provided, worked by an eccentric, and the oscillation effects the opening and closing of the admission and delivery passages without valves. The oil is delivered at a pressure of 15 lbs. per sq. in. and upwards, into pipes leading to every bearing.

In large hydraulic installations a complete system of circulation of oil under pressure to every bearing is adopted. The oil after passing through the bearings is collected, filtered and circulated again. The economy of oil with this arrangement

[1] 'High Speed, Self-lubricating Steam Engines.' By Alfred Morcom, *Proc. Inst. Mech. Engineers*, 1897, p. 316.

is very great. Not only is the system adapted to secure a perfect oil film between journal and bearing, but the oil carries off much of the heat generated. With forced lubrication higher bearing pressures are safe than with ordinary lubrication.

152. *Mr. Tower's experiments on collar friction.*—In a third report on experiments also carried out under the auspices of the Institution of Mechanical Engineers, results are given for a collar bearing, which are extremely interesting as showing the wide difference in friction when lubrication is less perfect than in a journal bearing. A ring of mild steel 12 ins. inside and 14 ins. outside diameter was placed between two gunmetal surfaces so as to form an annular bearing like the collar of a thrust bearing. The gunmetal surfaces rotated, while the steel ring was held fast by a lever which measured the friction. To secure lubrication, four pipes led the mineral oil to holes in its circumference which opened into grooves on the rubbing faces. The lubrication was adjusted to the minimum at which the bearing would run cool, varying from 60 to 120 drops per minute. The load on the bearing was put on by a powerful and carefully tested spring. With all except the lowest loads it was found impossible to keep the bearing cool without a little water running over it. Under these circumstances the friction was found to be almost absolutely independent of the speed, which ranged from 50 to 130 revs. per minute corresponding to rubbing velocities of 2·8 to 7·4 ft. per sec. The greatest pressure which could be carried on the bearing surface was far less than with a cylindrical journal, because a continuous oil film cannot be maintained between the surfaces. At the highest speeds the greatest pressure which could be carried was 75 lbs. per sq. in. and at the lowest speeds 90 lbs. per sq. in.

*Friction of a Collar Bearing*

| Pressure in lbs. per sq. in. of bearing surface | The coefficient of friction $\mu$ was, for speeds in revs. per min., | | | | |
|---|---|---|---|---|---|
| | 50 | 70 | 90 | 110 | 130 |
| 15 | 0·045 | ·065 | ·043 | ·054 | ·064 |
| 30 | ·037 | ·048 | ·050 | ·049 | ·048 |
| 45 | ·036 | ·040 | ·036 | ·036 | ·037 |
| 60 | ·029 | ·038 | ·036 | ·037 | ·041 |
| 67 | ·035 | ·033 | ·035 | ·036 | ·038 |
| 75 | ·035 | ·034 | ·035 | ·035 | ·036 |
| 82 | ·034 | ·032 | ·035 | — | — |
| 90 | ·031 | ·044 | — | — | — |

Here $\mu$ is practically independent both of speed and pressure, except with the smallest pressures. For all but the smallest pressures its mean value is 0·036, and for the smallest pressures about one-third greater.

It was stated in the discussion of Mr. Tower's paper that Mr. Thorneycroft limits the pressure on the collar bearings of propeller shafts to 50 lbs. per sq. in. Mr. Ford Smith also stated that the pressure which could be carried depended partly on the hardness of the surfaces. Hardened steel working on dense cast iron would carry pressure with which other bearing surfaces failed.

153. *Theory of journal proportions. Limit of bearing pressure.*—Since the friction of a journal produces heat, and the heating of the lubricant renders it less viscid and more liable to be squeezed out, it would seem that the journal proportions must in some way depend on the amount of heat produced per sq. in. of surface of journal. Further, it would seem rational at first sight to use the values of $\mu$ given by Mr. Tower's experiments to determine the amount of heat produced. No doubt those values should be used in all calculations on the friction of well-lubricated journals in normal conditions of running. But no relation can be traced between the proportions of journals adopted in practice in cases differing widely in size, intensity of pressure and speed, and the friction per sq. in. calculated by Mr. Tower's coefficients.

On the other hand there seems to be some relation, though a rough one, for given types of journal, if the friction is calculated with $\mu$ constant as it would be for very imperfect lubrication. Perhaps this might be expected, for a tendency to heat and seize must generally arise just when lubrication has become imperfect.

The work expended in friction produces a quantity of heat—

$$\text{H} = \frac{\text{T}}{\text{J}} = \mu \, \text{P} \frac{\pi \, d \, \text{N}}{12 \, \text{J}} \text{ Th.U. per min.}$$

where $\text{J} = 778$ is Joules equivalent. The rate of dissipation of heat must depend on the excess of temperature of the journal over surrounding space and on the area of surface of the journal. Let $h$ units of heat be dissipated per minute per sq. in. of the projected area of bearing $d\,l$. Then since the rate of production and dissipation of heat must be equal—

$$h \, d \, l = \frac{\mu \, \text{P} \, \pi \, d \, \text{N}}{12 \, \text{J}}$$

But $P = p\ d\ l$, and treating $h$ and $\mu$ as constants, the bearing pressure is

$$p = \frac{\beta}{N d} = \frac{\beta}{229 v} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where $\beta$ is to be regarded as a constant for a given type of bearing in given conditions of accuracy of fit, lubrication, etc., and $v$ is in ft. per sec. The length of journal for a given load is

$$l = \frac{P N}{\beta}$$

a result which implies that the length of a journal should increase as the speed is greater whatever the diameter. This is a result recently strongly contested by Dr. J. T. Nicholson. In any case these results depend on certain assumptions and must be regarded as nothing more than a means of classifying the results of experience.

*Comparison of the results of this rough theory with practical experience.*—Mr. Tower's experiments furnish some data as to the limiting value of the constant $\beta$ in the expressions just found. Taking the highest speeds and pressures at which the journal in those experiments ran without seizing when the lubricant was supplied by an oil bath, we get the following values of $\beta$. Let $p$ be the greatest intensity of pressure on the bearing surface, N the number of revolutions per minute, $d$ the diameter of the journal. Then $\beta = p \times d$.

| | Greatest pressure $p$ at which the journal ran without seizing | $\beta$ = |
|---|---|---|
| | lbs. | |
| Sperm oil . . . . . . . . | 415 | 747,000 |
| Mineral oil . . . . . . . | 625 | 875,000 |
| Rape oil . . . . . . . . | 573 | 916,000 |
| Olive and lard oil . . . . . | 520 | 936,000 |
| Mineral oil . . . . . . . | 625 | 1,125,000 |

When the lubrication was less perfect the values of $\beta$ diminished thus :—

| | Greatest pressure $p$ at which the journal ran without seizing | $\beta$ = |
|---|---|---|
| | lbs. | |
| Rape oil fed by siphon . . . . | 258 | 309,600 |
| " " with pad . . . . . | 328 | 393,000 |

These are values of $\beta$ for which the journal was just on the point of seizing. They must be divided by a 'factor of safety' in applications, to allow a margin of safety against heating. We may expect to find, therefore, that the values of $\beta$ deduced from actual working journals are lower than the values above by a varying amount depending on the varying margin of security allowed. It will be seen presently that this appears to be the case for journals loaded, like Mr. Tower's journals, by a load constant in direction and from which the heat is chiefly dissipated through one step. It appears, however, that in journals on which the direction of the load constantly changes the lubrication is more easily effected, the heat is dissipated through two steps and the tendency to heating is much less. Consequently in such cases $\beta$ has a higher value.

The following are values of $\beta$ deduced from journals in service.

| | Coefficient $\beta$ = |
|---|---|
| Outside crankpins of locomotives | 1,000,000 to 1,500,000 |
| Locomotive axle journals | 750,000 to 850,000 |
| Passenger carriage axles | 300,000 to 600,000 |
| High-speed engine crankpins | 300,000 to 400,000 |
| Marine engine crankpins | 200,000 to 300,000 |
| Stationary engine crankpins | 100,000 to 200,000 |
| Flywheel shaft bearings | 85,000 to 150,000 |
| Small engine crankpins | 60,000 |

The higher values correspond generally to cases in which the direction of the load reverses at each revolution, the heat is distributed to two brasses, and the fitting and materials used are of the highest quality. The lower values correspond to cases where the direction of the load is constant and the heat is dissipated through one brass or step. No doubt in many cases journals are of larger bearing surface than is absolutely required to obviate danger of heating, especially as no clearly rational rule of design has been available.

That the rate of conduction of heat from a bearing has a great influence on the pressure it will carry without seizing was shown by an interesting experience at Niagara. The vertical turbine shaft was carried at the turbine casing in a bronze bush made in halves (fig. 173). This bush was fitted to the cast-iron casing at three ring bearings, the intermediate part being recessed in both bush and casing, so that there were two annular air spaces round the outside of the bush. These bushes cut badly.

*At Dr. Sellers' suggestion the air spaces were filled with type-metal and the bushes lined with Babbitt metal. The bearing then ran cool without wearing. Filling the air spaces with type-metal facilitated the conduction of heat from the bearing.*

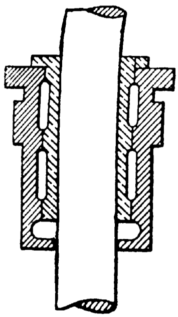


Fig. 173

154. *Test of an exceptionally large bearing.*—Very interesting experiments were made by Kingsbury for the Niagara Falls Power Company ('Trans. Am. Soc. Mech. Eng.' 27, 425). A shaft was mounted with two supporting end bearings 9 ins. diameter and 30 ins. long, and a middle loaded bearing 15 ins. diameter and 40 ins. long, and driven by an electric motor. The middle bearing was flooded with oil by a method equivalent to bath lubrication. The bearing was lined with Babbitt metal and water cooled, and the oil was cooled. To reach high speeds of 1,000 revs. per min. it was found necessary to accelerate slowly, allowing three hours for heating the outer parts. Below about 300 revs. per min. the lubrication became imperfect and the friction increased. With the precautions taken it was found possible to run the bearings at loads and speeds greatly exceeding those ordinary in practice. The following table gives a reduction of some of the results.

| | Tests with heavy machine oil | | | | Tests with paraffin oil | |
|---|---|---|---|---|---|---|
| Load on centre bearing, lbs. | 50,000 | 50,000 | 67,200 | 84,600 | 94,000 | 94,000 |
| Bearing pressure, lbs. per sq. in. | 83 | 83 | 112 | 141 | 157 | 157 |
| Shaft speed, r.p.m. | 180 | 301 | 454 | 480 | 946 | 1,243 |
| Rubbing velocity, ft. per sec. | 11·8 | 19·7 | 29·7 | 31·5 | 62·0 | 81·7 |
| Value of $p\,v$ | 970 | 1,630 | 3,330 | 4,440 | 9,734 | 12,820 |
| Friction h.p. of the three bearings | 6·4 | 10·1 | 16·0 | 17·9 | 41·9 | 47·8 |
| Calculated average coefficient of friction of bearings | ·0040 | ·0037 | ·0029 | ·0024 | ·0025 | ·0022 |
| Temperature of oil supply, F. | 49 | 53 | 58 | 62 | 71 | 69 |
| " " drip from centre bearing F. | — | 105 | 121 | 124 | 133 | 143 |
| Lbs. oil per min. to centre bearing | 3·1 | 4·0 | 3·1 | 5·8 | 10·0 | 7·6 |
| Values of $\beta$ for centre bearing | 224,180 | 373,000 | 763,000 | 1,015,000 | 2,223,000 | 2,920,000 |

The extreme result obtained was with paraffin oil, with a load of 157 lbs. per sq. in. of bearing, and a rubbing speed of 81·7 ft.

per sec., which gives $\beta = 2{,}927{,}000$. With a little greater load the bearing seized. With mineral oil the highest result was with a bearing pressure of 141 lbs. per sq. in., a rubbing speed of 31·5 ft. per sec., which gives $\beta = 1{,}015{,}000$. This last value agrees very well with Beauchamp Tower's result for mineral oil, though the size of journal and rubbing speed was so very different.

Another rather interesting special case of a bearing of much smaller proportions is that of the axle bearings of some steam cars on the Taff Vale Railway ('Proc. Inst. Mech. Eng.,' July 1908). These bearings were 6 ins. diameter and 9½ ins. long, and the bearing pressure was 466 lbs. per sq. in. At 30 miles per hour the axle made 300 revs. per min. These bearings could only be run with an undue amount of lubrication and cases of their running hot were not infrequent. Forced lubrication was substituted and the trouble ceased. Now in this case $\beta = 839{,}000$, which is not very different from the higher values at which Mr. Tower's journals ran without seizing with bath lubrication and much higher than the corresponding values for imperfect lubrication.

## PROPORTIONS OF JOURNALS

155. *Strength of end journals.*—Let P, fig. 174, be the load on a journal, assumed to be uniformly distributed; let $d$ be the diameter, and $l$ the length of the journal; $f$ the working stress suitable for the material of the journal. Then the bending moment at the fixed end of the journal is $\frac{1}{2}\,\text{P}\,l$, and equating this to the moment of resistance of a circular section, § 49, p. 67,

$$\tfrac{1}{2}\,\text{P}\,l = 0{\cdot}098\,f\,d^3$$

$$d = 1{\cdot}72\sqrt[3]{\{(\text{P}\,l)\,/f\}} \qquad . \qquad . \quad (9)$$

On many journals the direction of the load relatively to the journal reverses every revolution and the safe working stresses are those given in Table II, Case C, § 39. Thus for wrought iron $f = 5{,}000$ lbs. per sq. in.; for cast steel $f = 6{,}500$; for cast iron $f = 2{,}500$, on the average.

The bending action may be shown graphically thus: Suppose first of all that the load on a journal (fig. 174) is P acting at the centre of its length. Then the bending moment diagram is the triangle *cab*, in which $a\,b = \frac{1}{2}\,\text{P}\,l$ is the bending moment at

the root of the journal. If the load is uniformly distributed the bending moment curve is a parabola $b\ d$, which may be approximated to by drawing $b\ e$ perpendicular to $b\ c$ and describing a circular arc $d\ b$ with centre $e$. The bending moment at the root of the journal is the same as before, and on this the diameter necessary for strength depends.

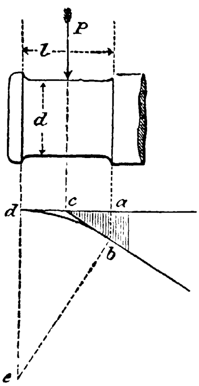


Fig. 174

*Case I. Ratio of length to diameter of journal given or assumed.*—Most commonly the length $l$ of the journal will not be given, and there are therefore two unknown quantities in Equation 9, $l$ and $d$. In a number of cases it is possible to fix beforehand the ratio $l/d$ of length to diameter of bearing. Thus in the case of journals which work intermittently, or on which the pressure only acts for part of a rotation, $l/d = 1$; for very many journals in which the speed of rotation does not exceed about 150 revolutions per minute it has been customary to make $l/d = 1·5$ if the step is of gunmetal, and $l/d = 1·75$ if the step is cast iron. For journals at greater speeds of rotation it has been usual to make the proportion of length to diameter greater.

*Rough average values of $l/d$*

| Material of journal | Revs per min. 60 | 120 | 240 | 360 |
|---|---|---|---|---|
| Wrought iron | 1·4 | 1·5 | 1·75 | 2·0 |
| Cast iron | 1·25 | 1·35 | 1·5 | — |
| Steel | 1·75 | 2·0 | 2·12 | 2·5 |

When $l/d$ is fixed Eq. 9 becomes

$$d = \sqrt{\frac{5·1}{f}}\sqrt{P}\sqrt{\frac{l}{d}} \quad . \quad . \quad (10a)$$

Where with the values above—

$$\sqrt{\frac{5·1}{f}} = 0·032 \text{ for wrought iron.}$$
$$= 0·028 \text{ for cast steel.}$$
$$= 0·045 \text{ for cast iron.}$$

*Case II. The intensity of bearing pressure fixed.*—In journals running constantly, freedom from wear and liability to heating are as important as strength, and this involves fixing beforehand a limit of bearing pressure suitable for the conditions. Values of bearing pressures in some definite cases are given below. The bearing pressure is $p = \text{P}/l\,d$, where P must be taken to be the average load when the machine is running fully loaded. Substituting in Eq. 9—

$$d = \sqrt[4]{\frac{5\cdot 1}{pf}}\sqrt{\text{P}} \quad . \quad . \quad . \quad (10b)$$

Values of $\sqrt[4]{\frac{5\cdot 1}{pf}}$.

| | For bearing pressure $p$ = | | | | | | |
|---|---|---|---|---|---|---|---|
| | 200 | 250 | 300 | 400 | 500 | 750 | 1,000 |
| Wrought iron, $f$ = 5,000 | ·0475 | ·0449 | ·0429 | ·0400 | ·0378 | ·0342 | ·0318 |
| Cast steel, $f$ = 6,500 . | ·0445 | ·0421 | ·0402 | ·0374 | ·0354 | ·0320 | ·0298 |
| Cast iron, $f$ = 2,500 . | ·0565 | ·0534 | ·0511 | ·0475 | ·0449 | ·0406 | ·0378 |

*Limits of pressure on bearings in different practical cases.*—The following values of the pressure per sq. in. of projected area of bearing may be of use, partly in showing how great the variation of bearing pressure is in different cases, and partly as a guide in selecting a suitable value when designing a bearing :

*Greatest bearing pressure on journals and slides*

| | Pressure in lbs per sq. in. of projected area of bearing |
|---|---|
| *Motion intermittent.* | |
| Bearings subject to intermittent load at slow speeds as crank pins of shearing machines . . . . . . | 3,000 |
| *Motion an oscillation, direction of load reversing.* | |
| Crosshead neck journals, slow speed . . . . . . | 1,000–1,500 |
| ,, ,, ,, quick speed . . . | 800–1,000 |
| ,, ,, ,, locomotives . . . | 1,500–2,000 |
| *Rotating journals, direction of load reversing.* | |
| Crank pins, slow engines . . . . . | 800–1,200 |
| ,, ,, medium speed engines . . . | 600–800 |
| ,, ,, high speed engines . . . . | 300–600 |
| ,, ,, large marine engines . . . . . | 400–600 |
| ,, ,, locomotives. . . . . . . | 1,200–1,800 |
| ,, ,, small land engines . . . . . | 150–250 |
| Eccentric sheaves . . . . . . . . . . . | 80–100 |

*Greatest bearing pressure on journals and slides*—continued

| | Pressure in lbs per sq. in. of projected area of bearing |
|---|---|
| *Rotating journals, direction of load constant or nearly so.* | |
| Main crankshaft bearings, slow engines | 300–450 |
| ,, ,, ,, ,, fast ,, | 200–300 |
| Flywheel shaft bearings | 150–250 |
| Railway carriage journals | 250–450 |
| Locomotive and tender axle bearings | 200–250 |
| Lying transmission shafts, gunmetal steps | 200 |
| ,, ,, ,, cast iron steps | 50 |
| Steel shaft on lignum vitæ. Water lubrication | 350 |
| *Motion a rotation, direction of load constant.* | |
| Pivots | 700 |
| Collar thrust bearings (slow) | 70–80 |
| ,, ,, ,, (fast) | 50–60 |
| *Motion a reciprocation.* | |
| Slides. Cast iron on Babbitt metal | 200–300 |
| ,, Cast iron on cast iron (slow) | 60–100 |
| ,, ,, ,, ,, (fast) | 40–60 |

*Case III. Journals for which both length and diameter are determined by rule.*—To whatever extent the rule

$$l = \frac{\beta}{d\,\mathrm{N}} \qquad . \quad . \quad . \quad . \quad (11)$$

obtained above can be trusted, there are conditions for designing a bearing rationally and determining both $d$ and $l$.

If a value of $\beta$ is selected, or better, is found by calculation from the data of a similar bearing running satisfactorily,

$$p = \mathrm{P}/d\,l = \beta/d\,\mathrm{N}$$
$$l = \mathrm{P\,N}/\beta$$

Putting this value in Eq. 10—

$$d = 1{\cdot}72\ \sqrt[3]{\frac{\mathrm{P}^2\,\mathrm{N}}{\beta f}} \qquad . \quad . \quad . \quad (12)$$

156. *Proportions of neck journals.*—The crosshead pins of engines are supported at each end and loaded uniformly. The bending moment is therefore $\frac{1}{8}$ P $l$. Hence the general equation which secures sufficient strength is

$$d = \sqrt[3]{\frac{1{\cdot}28}{f}}\ \sqrt[3]{\mathrm{P}\,l} \qquad . \quad . \quad . \quad (13)$$

As far as strength is concerned, therefore, a neck journal may be

0·63 of the diameter of an end journal for the same load and of the same length.

*Case Ia.—Neck journal, l / d given or assumed.*

$$d = \sqrt{\frac{1 \cdot 28}{f}} \sqrt{\left(\mathrm{P}\frac{l}{d}\right)} \quad . \quad . \quad (14)$$

*Case IIa.—Neck journal, bearing pressure p, given or assumed.*

$$d = \sqrt[4]{\frac{1 \cdot 28}{pf}} \sqrt{\mathrm{P}} \quad . \quad . \quad (14a)$$

$$l = \frac{\mathrm{P}}{p\,d}$$

General considerations as to the tendency to seize seem to show that a neck journal should have the same length as an end journal for the same speed and similarly efficient lubrication.

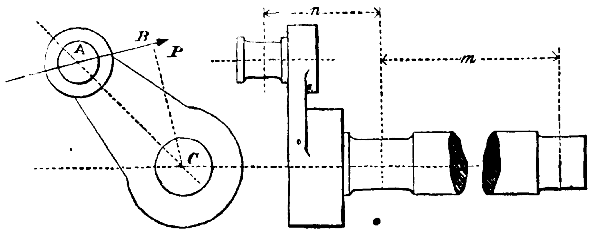



Fig. 175

157. *Proportions of journals subjected to bending and torsion.*—The proportions given for journals in the rules above provide strength to resist a load P distributed over the journal and having a resultant normal to and passing through its axis. In many cases—as, for instance, in crankshaft journals—the load is applied in a direction not passing through the journal axis, and then there is torsion as well as bending.

*Crankshaft journals.*—Let fig. 175 represent a side and end elevation of a crank and crank shaft, and let it be required to determine the dimensions of the shaft journal nearest the crank. Let $r$ be the crank radius and $n$ and $m$ the distances between the centre lines of the crank pin and two shaft journals. Let P be the thrust or pull of the connecting rod at the crank pin, and $\theta$ the angle A C B between the crank radius and the normal to P.

Imagine two opposite forces P′, P″, introduced at C equal and parallel to P. Then P and P″ form a twisting couple the moment of which is

$$T = P \times BC = Pr \cos \theta.$$

P′ is in equilibrium with parallel reactions at the shaft journals. Let R be the reaction at the nearer and Q that at the more distant journal.

$$Pn = Qm \text{ and } P + Q = R$$
$$Q = Pn/m\,;\ R = P\,(1 + n/m)$$

At the dead point when P passes through C, $r \cos \theta = 0$, and the twisting moment vanishes. If P is constant during the revolution of the crank the twisting moment is greatest when the crank and connecting rod are at right angles, and then $T = Pr$. At the same time the bending moment at the centre of the nearer journal is $M = Pn$, neglecting the small effect of the distribution of the load on the length of the journal.

By the rules generally used hitherto for combining bending and torsion, § 77, p. 126, the greatest direct stress is equal to that due to a simple bending moment—

$$M_e = \tfrac{1}{2} Pn + \tfrac{1}{2} \sqrt{\{(Pn)^2 + (Pr)^2\}}$$
$$= \frac{P}{2}\{n + \sqrt{(n^2 + r^2)}\} \quad . \quad . \quad . \quad (15)$$

Equating this to the moment of resistance of a circular section, $d$ being the diameter of the journal and $f_t$ the working stress in tension—

$$M_e = 0{\cdot}098\, d^3 f_t$$

$$d = \sqrt[3]{\frac{5{\cdot}1}{f_t}} \sqrt[3]{\{P(n + \sqrt{n^2 + r^2})\}}\,. \quad . \quad (16)$$

Let $f_t = 5{,}000$ for wrought iron, 6,000 for mild steel, 7,000 for medium steel, and 1,500 for cast iron, the stress being an alternating stress. Case C, Table II, § 39—

$$\sqrt[3]{\frac{5{\cdot}1}{f_t}} = 0{\cdot}101 \text{ for wrought iron}$$
$$0{\cdot}095 \text{ for mild steel.}$$
$$0{\cdot}090 \text{ for cast steel.}$$
$$0{\cdot}150 \text{ for cast iron.}$$

Let $n = xr$, and let the formula be in the form

$$d = K\sqrt[3]{Pr}.$$

Then for various ratios of the bending and twisting moment the values of K are as follows :—

*Values of* K

| $x =$ | Wrought iron | Mild steel | Medium steel | Cast iron |
|---|---|---|---|---|
| 0 | 0·101 | ·095 | ·090 | ·150 |
| $\frac{1}{2}$ | ·118 | ·111 | ·105 | ·176 |
| $\frac{3}{4}$ | ·127 | ·120 | ·113 | ·189 |
| 1 | ·135 | ·127 | ·121 | ·201 |
| 2 | ·164 | ·154 | ·146 | ·243 |
| 4 | ·203 | ·191 | ·181 | ·301 |
| 8 | ·254 | ·241 | ·227 | ·378 |

As explained in § 78, according to a later view the greatest shearing stress determines the safe limit of loading. In that case the maximum shearing stress is equal to that due to a simple bending or twisting moment

$$M_e = T_e = P\sqrt{(n^2 + r^2)} \quad . \quad . \quad (17)$$

If $Pn > Pr$, the diameter of the shaft is to be calculated from the bending moment with the working stress $f_t$ for tension. If $Pr > Pn$, it is calculated from the twisting moment with the working shearing stress $f_s$. Hence, either

When $Pn > Pr$, $\quad M_e = f_t \dfrac{d^3}{10{\cdot}2}$

$$d = \sqrt[3]{\frac{10{\cdot}2}{f_t}}\ \sqrt[6]{(n^2 + r^2)}\sqrt[3]{P} \quad . \quad . \quad (18)$$

or when $Pr > Pn$, $\quad T_e = f_s \dfrac{d^3}{5{\cdot}1}$

$$d = \sqrt[3]{\frac{5{\cdot}1}{f_s}}\ \sqrt[6]{(n^2 + r^2)}\sqrt[3]{P} \quad . \quad . \quad (18a)$$

The working limits of stress applicable when these formulæ are used are not well ascertained. The following values are assumed provisionally :—

| | $f_t$ | $f_s$ |
|---|---|---|
| Wrought iron . . . . | 5,000 | 3,000 |
| Mild steel . . . . | 6,000 | 4,000 |
| Medium steel . . . | 7,000 | 5,000 |

| | $\sqrt[3]{\frac{10.2}{f_t}}$ | $\sqrt[3]{\frac{5.1}{f_s}}$ |
|---|---|---|
| *Wrought iron* . . . . | 0·127 | 0·119 |
| Mild steel . . . | 0·119 | 0·108 |
| Medium steel . . . . | 0·113 | 0·101 |

Let $n = x\,r$, and let the formula be of the form

$$d = \text{K}\sqrt[3]{\text{P}\,r}$$

Then the following are values of K for different ratios of the bending and twisting moments.

| $x =$ | The value of K is | | | | | |
|---|---|---|---|---|---|---|
| | Calculated from $\text{M}_e$ | | | Calculated from $\text{T}_e$ | | |
| | Wrought iron | Mild steel | Medium steel | Wrought iron | Mild steel | Medium steel |
| 0 | — | — | — | ·119 | ·108 | ·101 |
| $\frac{1}{2}$ | — | — | — | ·123 | ·112 | ·105 |
| $\frac{3}{4}$ | — | — | — | ·128 | ·116 | ·109 |
| 1 | ·142 | ·133 | ·126 | ·133 | ·120 | ·113 |
| 2 | ·166 | ·155 | ·148 | — | — | — |
| 4 | ·203 | ·190 | ·181 | — | — | |
| 8 | ·255 | ·239 | ·227 | — | — | — |

The length of the journal is determined from the reaction R, either for an assumed bearing pressure $p$, or for an assumed value of $\beta$ (§ 153).

With the special steels now introduced which have a high elastic limit, greater working stresses may be allowed.

## Pivot Bearings

158. The limits of bearing pressure met with in practice in pivot bearings seem at first sight widely discrepant. On the pivots of the upright shafts of mills the bearing pressure sometimes reaches half a ton per sq. in. Discrepancies tend to disappear if the limit of bearing pressure is assumed to vary inversely as the velocity of rubbing.

*Proportions of pivots.*—Let P be the load in lbs. on a pivot of $d$ inches in diameter, running at N revolutions per minute. The bearing pressure is $p = \text{P}/\left(\frac{\pi}{4}d^2\right)$. The highest limit of bearing pressure usual is generally stated to be 1,500 lbs. per sq. in. for

hardened steel on gunmetal and 600 lbs. per sq. in. for unhardened steel on gunmetal. But higher bearing pressures are found in some cases. Then—

$$d > \sqrt{\frac{4}{\pi}}\sqrt{\frac{P}{p}} = 1{\cdot}13\sqrt{\frac{P}{p}} \qquad (19)$$

$$p = 1{,}500, \qquad d > 0{\cdot}029\sqrt{P}$$
$$= 600, \qquad d > 0{\cdot}041\sqrt{P}$$

If, as in the case of journals (§ 153), the bearing pressure is assumed roughly to vary inversely as the velocity of rubbing, let

$$p = \frac{\beta}{d\,N}$$

where $\beta$ is a constant to be found by experience. In some tests by Mr. Beauchamp Tower on steel pivots running on bronze and white metal, the highest pressures carried without seizing corresponded to $\beta$ = 100,000 to 169,000. But with good lubrication pivots are running with $\beta$ = 300,000. Taking this last value the following are values of the greatest permissible bearing pressure for various diameters of pivots and various speeds of rotation.

*Bearing Pressure at Different Speeds*

| Diameter of pivot in inches | Bearing pressure in lbs. per sq. in. at revs. per min. | | | | |
|---|---|---|---|---|---|
| | 50 | 100 | 150 | 200 | 250 |
| 2 | — | 1,500 | 1,000 | 750 | 600 |
| 3 | — | 1,000 | 666 | 500 | 400 |
| 4 | 1,500 | 750 | 500 | 375 | 300 |
| 5 | 1,200 | 600 | 400 | 300 | 240 |
| 6 | 1,000 | 500 | 333 | 250 | 200 |
| 8 | 750 | 375 | 250 | 187 | 150 |
| 10 | 600 | 300 | 200 | 150 | 120 |
| 12 | 500 | 250 | 166 | 125 | 100 |

159. *Pivot bearings with forced lubrication.*—In the case of steam turbines with vertical shafts, very high loads have to be carried at high speeds. In such cases oil is forced into the centre of the pivot bearing at a pressure great enough to slightly lift the shaft, and a perfect oil film is maintained between the rubbing surfaces. A circular recess is formed about half the diameter of the pivot into which the oil is forced and from which it flows

outwards. The pressure of the oil must be somewhat greater to lift the shaft, than to maintain it lifted when running. The oil pressure falls from a pressure $q$ at the central recess to zero at the outside of the pivot. The upward lifting force on the portion of the shaft over the recess is $\frac{\pi}{4}\left(\frac{d}{2}\right)^2 q = 0{\cdot}2\ d^2\ q$. The distribution of pressure between the rubbing surfaces is uncertain, but the mean pressure must approximate to $q / 2$ and the uplift on this part is then $\frac{\pi}{4}(\frac{3}{4}d^2)(\frac{1}{2}q) = 0{\cdot}3\ d^2\ q$. Hence the load supported by the pivot is approximately

$$P = 0{\cdot}5\ d^2 q \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The oil pressures used in different cases range from 250 to 800 lbs. per sq. in. The following are some examples of the footstep bearings of Curtis turbines and the calculated, bearing pressure, oil pressure, and value of $\beta = p \times d$—

| | | | |
|---|---|---|---|
| Weight of rotating part, P | 9,800 | 53,000 | 187,000 |
| Revolutions per min., N . | 1,800 | 750 | 500 |
| Diameter of pivot, $d$ . . | $9\frac{1}{4}$ | 16 | 22 |
| Pressure of oil (stated), lbs. per sq. in. . . . . . . | 180 | 420 | 650 |
| Quantity of oil, galls. per min. | 1 | $3\frac{1}{2}$ | 6 |
| Calculated oil pressure, $q$ . | 206 | 414 | 739 |
| Bearing pressure, $p$ . . . | 131 | 264 | 470 |
| Value of $\beta$ . . . . . . . | 2,300,000 | 3,170,000 | 5,300,000 |

It will be seen that higher values of $\beta$ correspond to higher velocity of flow of oil.

## Collar Bearings

160. When the load is too great to be carried by a pivot, a collar bearing is used (fig. 176). The bearing area can be increased to any extent by increasing the number of collars. Such bearings are somewhat less satisfactory than pivots partly because the friction is greater, partly because of the difficulty of securing an equal distribution of pressure amongst the collars.

### *Proportions of Collar Bearings*

Let $d_1$, $d_2$ be the outside and inside diameters of the collars.

$d_m = \frac{1}{2}(d_1 + d_2)$ the mean diameter.

$b = \frac{1}{2}(d_1 - d_2)$ radial width of collars.

$n$ = number of collars.

If P is the total load, P / $n$ is the load per collar, and

$$p = \text{P} / \left\{ \frac{\pi}{4} (d_1^2 - d_2^2)\ n \right\} = \text{P} / (\pi\ d_m\ b\ n)$$

is the bearing pressure. Usually $b = 0{\cdot}12$ to $0{\cdot}18\ d_m$. The thickness of the collars $= 0{\cdot}8\ b$ to $1{\cdot}2\ b$. The distance between collars $= b$ if the bearing is on solid gunmetal rings; $1{\cdot}5\ b$ if the bearing is on rings faced with white metal; $2{\cdot}5\ b$ if the bearing rings are hollow with water circulation.

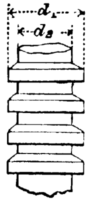


Fig. 176

Some experiments by Beauchamp Tower on collar friction are given in § 152; the mean diameter of the collars was 13½ ins. and the speeds 50 to 130 revs. per min. The greatest bearing pressure which could be carried without seizing was 90 lbs. per sq. in. at the lowest and 75 lbs. at the highest speed. If, as in other cases, it is assumed that the bearing pressure should vary inversely as the velocity of rubbing, so that

$$p = \frac{\beta}{d_m \text{N}} \quad . \quad . \quad . \quad . \quad (21)$$

then the greatest safe bearing pressures from the data just given correspond to $\beta = 60{,}000$ to $120{,}000$. If for safety $\beta$ is taken at 50,000, then the permissible bearing pressures for different diameters and speeds are given in the following table.

*Bearing Pressure on Collar Bearings*

| Mean diameter $d_m$ | Bearing pressure in lbs. per sq. in. at revs. per min. | | | | | |
|---|---|---|---|---|---|---|
| | 50 | 60 | 80 | 100 | 120 | 150 |
| 6 | 166 | 140 | 104 | 83 | 70 | 55 |
| 8 | 125 | 104 | 78 | 62 | 52 | 42 |
| 10 | 100 | 83 | 62 | 50 | 42 | 33 |
| 12 | 83 | 70 | 52 | 42 | 35 | 28 |
| 15 | 66 | 55 | 42 | 33 | 28 | 22 |
| 18 | 55 | 46 | 35 | 28 | 23 | 18 |

*Collar bearings for propeller shafts.*—It is usual to assume that two-thirds of the indicated horse-power is effective in causing propeller thrust. Let P be the propeller thrust in lbs., S the speed of the ship in knots per hour, and H the indicated horse-power. The speed of the ship in feet per sec. is 6080 S / 3600 = 1·68 S.

$$\text{P} = (217\ \text{H}) / \text{S} \quad . \quad . \quad . \quad . \quad (22)$$

The thrust bearing for propeller shafts is generally a multiple collar bearing, with horse-shoe shaped bearing rings. These are of solid gunmetal, or of cast iron or cast steel faced with white metal, and very commonly are hollow and provided with water circulation to carry off the heat. They are important and very carefully fitted bearings. In very fast warships the bearing pressure is sometimes 80 to 100 lbs. per sq. in., in ordinary warships 70 to 80 lbs. per sq. in., and in commercial vessels 40 to 70 lbs. per sq. in. But the permissible bearing pressure should decrease as the rubbing velocity is greater.

## CHAPTER VIII

### AXLES AND SHAFTS

161. The terms *axle* and *shaft* are applied rather indiscriminately to parts of machines which support rotating pieces, or which by their rotation convey and distribute motive power. They are usually cylindrical, but occasionally square or cross-shaped in section. They may be classified as follows :—

(1) Axles loaded transversely, and subjected chiefly to bending action.

(2) Transmissive shafting, subjected chiefly to torsion.

(3) Crank shafts and other shafts subjected to combined torsion and bending—

In any large factory the shafting, with its couplings, pulleys, and gearing, forms a very important, costly, and power absorbing part of the machinery. Although great attention has been paid to the economical production of power, and to its economical use in machines, the intermediate transmission is sometimes barbarous in arrangement and shows great neglect of any study of conditions of efficiency. In different machine shops it has been found that the power required to drive the shafting varies from 15 to 50 per cent. of the whole power employed.

*General equations for the strength of axles and shafts.*—If M is the bending moment, $d$ the diameter of an axle or shaft, and $f_t$ the working stress for bending—

$$M = \frac{1}{10{\cdot}2} f_t d^3 = 0{\cdot}098 f_t d^3 \quad . \quad . \quad (1).$$

$$d = 2{\cdot}169 \sqrt[3]{(M / f_t)}$$

If the shaft is subjected to a twisting moment T, and $f_s$ is the working stress in torsion—

$$T = \frac{1}{5{\cdot}1} f_s d^3 = 0{\cdot}196 f_s d^3 \quad . \quad . \quad . \quad (2)$$

$$d = 1{\cdot}722 \sqrt[3]{(T / f_s)}$$

If a shaft is subjected to bending and twisting, then an equivalent bending or twisting moment may be found by the rules for compound stress (§ 77). If the shaft is hollow or not of circular section the equations in § 57 must be used.

*Angle of twist of a shaft subjected to torsion.*—If T is the twisting moment to which a shaft is subjected, $f_1$ the stress at the surface of the shaft due to torsion, G the coefficient of rigidity of the material, $d$ the diameter of the shaft, and $\theta$ the angle of twist per unit length of shaft in circular measure, then (§ 58, p. 96)

$$\begin{aligned}\theta &= 2 f_1 / \text{G}\, d \\ &= 10{\cdot}2\, \text{T} / \text{G}\, d^4\end{aligned}$$

or if $\theta$ is expressed in degrees

$$\begin{aligned}\theta^\circ &= 114{\cdot}7 f_1 / \text{G}\, d \\ &= 578\, \text{T} / \text{G}\, d^4\end{aligned}$$

If a shaft is strained to 9,000 lbs. per sq. in. and G = 12,000,000, a shaft twists nearly $1 / (12\, d)$ of a degree per inch in length. For different shafts strained to the same working stress, the angle of twist in a given length decreases inversely as the diameter.

162. *Practical considerations.*—In applying the rules for torsional and bending strength to shafts, some practical difficulties arise. As regards bending strength the supporting forces are usually taken to act at the centres of the bearings; if the shaft is continuous over several bearings ambiguity arises, because it is hardly safe to assume the bearings to be exactly at one level and then to apply the equations for continuous beams. The error is on the safe side, if the shaft is treated as discontinuous at the bearings, at least so far as the magnitude of the greatest bending moment is concerned. The greatest bending moment on a length $l$ of shaft of diameter $d$, supported at the ends, due to its own weight only is $0{\cdot}0275\, d^2\, l^2$. As regards the twisting moment, very often only the greatest mean value in a revolution is known, and the moment varies through a more or less considerable range in each revolution. In such cases the mean value may be multiplied by a factor to allow for the variation.

It should be remembered that transmission shafting is usually turned from a bar of the nominal size of the shafting, so that the finished size of the shaft is $\frac{1}{16}$ inch less than the nominal size.

*Working stress in shafting.*—The determination of the safe working stress is a matter of some difficulty, especially because the real straining action is not usually determinable with exactness. In shafts such as those of turbines, the straining action is a twisting moment which never reverses in direction and remains generally at zero or during work near its maximum value, and there are no bending or inertia forces of much importance. In transmissive shafting the twisting moment is also constant in direction, but the bending forces are of more importance, and as the shaft revolves they produce tension and thrust alternately in every fibre of the shaft in each revolution. The variation of stress has therefore a greater range. Cases occur in which a shaft is subject to a twisting moment which reverses at times in direction and to bending, which as the shaft rotates changes its direction relatively to the shaft.

The working stress for different cases may be selected from Table II, § 39, which takes account of the range of variation of stress. The more frequently occurring cases may be distinguished as follows :—

*Case a.*—Straining action chiefly a bending moment constant in direction while the shaft revolves. Tensile stress on each fibre ranging from $+f$ to $-f$ in each revolution.

*Case b.*—Straining action chiefly a twisting moment, constant in direction in working hours. Stress varying from $f$ to 0 infrequently.

*Case c.*—Straining action chiefly a twisting moment constant in direction but varying frequently. Stress ranging from $f$ to 0 frequently.

*Case d.*—Straining chiefly a twisting moment frequently reversing in direction.

*Case e.*—Combined bending and twisting moments with large range of stress.

Axles of water wheels and some engine crankshafts correspond to Case *a*. Turbine shafts approximate to Case *b*. Shafts transmitting power and not subject to much bending action correspond to Case *b* or *c*. Shafts transmitting the power of reversing engines correspond to Case *d*.

The safe limiting working stress must be decided by examination of cases in which the conditions are similar. The following average values may be used as guides.

*Working Stress in Shafts*

| — | Case *a* $f_t$ | Case *b* $f_s$ | Case *c* $f_s$ | Case *d* $f_s$ |
|---|---|---|---|---|
| Cast iron | 2,500 | 4,000 | 2,800 | 1,400 |
| Wrought iron | 5,000 | 7,500 | 6,000 | 3,000 |
| Mild steel | 5,000 | 10,000 | 7,500 | 4,000 |
| Medium steel | 6,000 | 12,000 | 9,000 | 4,500 |
| Steel castings | 3,000 | 7,000 | 5,000 | 2,750 |

For Case *a*, the bending equation and for the other cases the twisting equation is to be used.

163. *Modern practice in the manufacture of steel shafting.*—The shafting and crank pins of large engines subjected to constantly varying torsional and bending straining actions require the utmost care in the selection of material and design. When such parts fail, generally near crank webs or flanges, they often exhibit a fracture which has begun at the surface where the stress is greatest and has gradually extended inwards. The fracture shows a fine-grained and sometimes discoloured exterior patch or ring and coarser grained centre, where final sudden fracture has taken place. That is the characteristic fracture due to '*fatigue*' or to continued repetition of a range of stress greater than the material can permanently sustain. Such fractures depend on the elastic, and not on the ultimate strength of the material, and the safe range of strength is greater for a material with high elastic strength than for a material with low elastic strength. When steel crank pins were first used very mild steel was adopted, and fractures of steel pins were as numerous as those of wrought iron. By using steel of harder quality with a yield point at 18 tons per sq. in. fractures were less frequent, the size of the pin being the same. The difficulty of forging, however, becomes greater the harder the steel, or at any rate the risk of developing in forging a coarse-grained and undesirable quality of steel, and internal fissures. The presence of defects can to some extent be ascertained by boring an axial hole, and if this is half the diameter of the shaft it reduces the strength only about 6 or 7 per cent. while it diminishes the weight by 25 per cent. The steel is improved in toughness by annealing and its elastic limit is raised by oil tempering, and this is facilitated by boring out the centre. Sometimes forging hollow on a mandril is possible in a hydraulic forging press, and this

secures high quality with the least risk of injury, because the temperature during the forging process is more uniform. Of late years much has been done to secure a steel of high elastic limit and adequate toughness by the addition of about $3\frac{1}{4}$ per cent. of nickel to the steel. This raises the elastic limit without reducing the toughness, and makes the steel more amenable to oil tempering. The following table gives the physical properties of various special qualities of forged steel.

The working stress when these special materials are employed may be greater than that for ordinary steel nearly in proportion to the increase of elastic strength.

*Properties of Steel Forgings*

| | Tenacity—tons per sq. in. | Elastic limit—tons per sq. in. | Elongation in 2 ins. per cent. |
|---|---|---|---|
| Mild steel annealed | 28 | 14 | 28 |
| Medium steel annealed | 36 | 17 | 23 |
| ,, ,, oil tempered | 40-55 | 20-35 | 23-20 |
| Medium nickel steel annealed | 40 | 26 | 25 |
| ,, ,, ,, oil tempered | 50 | 32 | 25 |
| Nickel chrome steel | 60 | 50 | 17 |

164. *Axles loaded transversely.*—In designing axles of this kind, it is convenient to determine first the dimensions of the journals. If the axle is cylindrical, its diameter at any other point can be obtained from the journal diameter, if it is remembered that the diameters at any two points should be proportional to the cube roots of the bending moments at those points. If the section is not circular, it is still convenient to design a cylindrical axle, and then to replace the cylindrical sections by equivalent sections of any other form. If the axle rotates, the cylindrical form is the only one which is of equal strength in all positions. The mode of designing axles is best explained by examples.

*Example I.*—An axle is supported on two end journals, and carries a load, P, at a point between the journals. Fig. 177 shows the axle. The load P is in equilibrium with the reactions Q, R, acting at the centres of the journals.

$$Q = P\,\frac{a}{a+b}\,; \quad R = P\,\frac{b}{a+b}$$

These are the loads for which the journals are to be calculated. From the rules in §§ 155 to 157 will be determined $d$, $d'$, $l$, $l'$, and the projections which limit the end-play. The axle diameters which it is most necessary to determine are those marked $d_1$, $d_2$

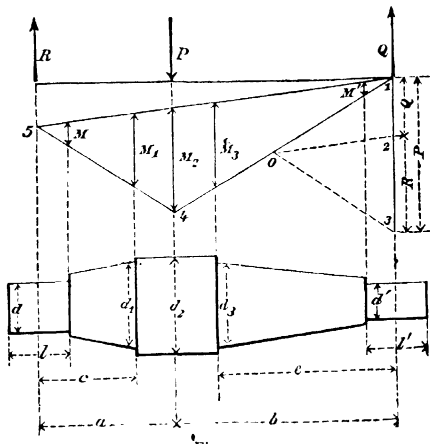



Fig 177

and $d_3$. The bending moments at those points are $M_1 = Rc$; $M_2 = Ra$; $M_3 = Qc$. The bending moments at the fixed ends of the journals are $M = R\frac{l}{2}$ and $M' = Q\frac{l'}{2}$. The journal diameters are $d = 1{\cdot}72\sqrt[3]{\{(Rl)/f_t\}}$ and $d' = 1{\cdot}72\sqrt[3]{\{(Ql')/f_t\}}$.

Since at any section the diameter must be at least equal to $\sqrt[3]{\frac{10}{f_t}}\sqrt[3]{(\text{bending moment})}$,

$$\left.\begin{aligned} \frac{d_1}{d} &= \sqrt[3]{\frac{M_1}{M}} = \sqrt[3]{\frac{2c}{l}} \\ \frac{d_2}{d} &= \sqrt[3]{\frac{M_2}{M}} = \sqrt[3]{\frac{2a}{l}} \\ \frac{d_3}{d'} &= \sqrt[3]{\frac{M_3}{M'}} = \sqrt[3]{\frac{2c}{l'}} \end{aligned}\right\}$$

The smallest values of the diameters consistent with the requirements of strength are, therefore, easily obtained from the journal diameters.

It is often convenient to measure the bending moments from the bending moment curve, which is easily drawn thus :—

Take 13 on the direction of Q produced, and = P, on any scale; choose any pole O, and draw 1 O 4, meeting the direction of P produced in 4. Join 30 and draw 45 parallel to 30, meeting the direction of R in 5. Join 51, and draw O 2 parallel to it. Then 12, 23 are the values of Q and R on the scale assumed for P. The vertical ordinates of the triangle 1 4 5 are proportional to the bending moments at the corresponding points of the axle. The values of $M$, $M_1$, $M_2$, &c., on any scale, measured on the diagram, may be used in the preceding equations, in determining the diameters of the axle.

The boss at the loaded part of the shaft is intended for keying on the wheel, or other part supported by the axle. Its projection must therefore be sufficient for cutting a keyway, § 133, even if it is then larger than is necessary for strength. If at any part the axle is not circular, it is only necessary to equate the modulus of a section of the required form, to the modulus of the circular section previously determined. Thus, if the section is to be square,

$$0.118\, s^3 = .0982\, d^3$$

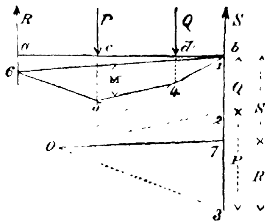


Fig. 178

will give the side of the square, the values of the moduli having been taken from Table IV, p. 70. If the axle rotates, the value of the modulus must be that which corresponds to the position in which it is weakest.

*Example II.*—The axle, fig. 178, supports two parallel loads between the journals. The bending moment curve is drawn thus: Let $a$ $b$ be the centres of the journals, $c$ $d$ the points at which the loads P Q are applied. At the points $a$ and $b$ the reactions R, S are produced by the action of P and Q. On the direction of S set off 12, 23, equal to Q and P on any scale. Choose a pole, O; join O 1, intersecting the direction of Q in 4. Join O 2 and draw 45 parallel to it intersecting the direction of P in 5. Join O 3, and draw 5 6 parallel to it, intersecting the direction of R in 6. If, now, O 7 is drawn parallel to the closing line 6 1, 3 7 and 7 1 will be equal to the reactions R and S. Also, 6 1 4 5 is the bending moment polygon, the breadth of which at any point, measured parallel to the forces, is proportional to the bending moment at the corresponding point of the axle.

Having, therefore, the bending moments, the diameters of the axle may be obtained from the journal diameters, as before.

When $P = Q$ and $a\,c = d\,b$, the case is one which occurs very commonly in practice, in which it will be found that the bending moment is uniform from $c$ to $d$. A railway carriage axle is in this position when the carriage is at rest, or moving along a straight portion of line. In passing round curves, however, it is subjected to torsion as well as bending; and in consequence of the pressure of the wheel flange against the rail, the forces are no longer parallel. Then the bending action between $c$ and $d$ is no longer uniform. It is for this reason that railway axles are tapered a little towards the centre.

If the forces acting on the axle are not parallel, resolve them into components parallel and perpendicular to the axle. The bending moment curve for the components perpendicular to the axle can be drawn in the same way.

165. *Shafts transmitting power, and subjected to torsion only.*—Rotating shafts are very extensively used, in transmitting the energy of prime-movers to the various parts of the factory or workshop in which it is applied to useful purposes. Such shafting was at one time of timber, then cast iron was adopted, and, later still, wrought iron and steel have almost entirely superseded cast iron. In transmitting power, shafts are subjected to torsion, but they are also subjected to bending action, due to their own weight, the weight of the wheels and pulleys they support, to the thrust of the gearing and the tension of the belting connected with them. This bending action is, to a great extent, indeterminate; it will, therefore, be convenient to consider, first, the torsion due to the power transmitted, and then to examine how an allowance can be made for the other straining actions.

Let H. P. be the indicated horse-power transmitted.

- N the number of revolutions of the shaft per minute.
- P the twisting force in lbs., acting on a shaft at a radius R in ins.
- $f_s$ the greatest safe stress for the material of the shaft, in lbs. per sq. in.
- $d$ the diameter of the shaft in ins.

The mean twisting moment is in statical inch lbs. (§ 57)—

$$T = P\,R = 63{,}024\,\frac{\text{H. P.}}{N} \qquad . \quad . \quad . \qquad (3)$$

The moment of resistance of a circular section with respect to torsion is $0{\cdot}196\, d^3 f_s$. Hence

$$d = \sqrt[3]{\frac{5{\cdot}1}{f_s}}\sqrt[3]{\text{P R}} \quad . \quad . \quad . \quad (3a)$$

$$= \sqrt[3]{\frac{63{,}024}{0{\cdot}196 f_s}}\sqrt[3]{\frac{\text{H. P.}}{\text{N}}} \quad . \quad . \quad (3b)$$

*Values of* $\sqrt[3]{\frac{5{\cdot}1}{f_s}}$.

| | Case *b* | Case *c* | Case *d* |
|---|---|---|---|
| Cast iron . . . . | 0·108 | 0·122 | 0·154 |
| Wrought iron . . | 0·088 | 0·095 | 0·119 |
| Mild steel . . | 0·080 | 0·088 | 0·108 |
| Medium steel . . | 0·075 | 0·083 | 0·104 |
| Steel castings . . | 0·090 | 0·101 | 0·123 |

*Values of* $\sqrt[3]{\frac{63{,}024}{\cdot 196 f_s}}$.

| | Case *b* | Case *c* | Case *d* |
|---|---|---|---|
| Cast iron . . . | 4·32 | 4·86 | 6·12 |
| Wrought iron . | 3·50 | 3·77 | 4·75 |
| Mild steel . . . | 3·18 | 3·50 | 4·32 |
| Medium steel . . . | 2·99 | 3·29 | 4·15 |
| Steel castings . . . | 3·58 | 4·01 | 4·89 |

166. *Ratio of greatest to mean twisting moment in shafts driven by steam engines.*—In the case of a steam engine the twisting moment exerted on the crank shaft varies with the variation of the steam pressure and with the variation of the leverage of the crank as it rotates. For a single engine working with little expansion $T_{max} = 1{\cdot}3\, T_{mean}$, a rule which may be used for locomotive cranks for instance. But if the engine works with much expansion the difference between the maximum and mean twisting moment is greater. Mr. Milton has found the ratio $\rho$ of the greatest twisting moment to the mean twisting moment (that given by equation 3), from actual indicator diagrams, to be as follows in certain selected actual engines :—

| | $\rho =$ | $\sqrt[3]{\rho} =$ |
|---|---|---|
| A. Single engine . . . . . . . . | 2·1 | 1·28 |
| B. Engine with equal cylinders, cranks at right angles . . . . . . . | 1·37 | 1·11 |

| | $\rho =$ | $\sqrt[3]{\rho} =$ |
|---|---|---|
| C. Compound engine cranks at right angles | 1·48 | 1·14 |
| D. Compound engine, cranks at 130° . . | 1·77 | 1·21 |
| E. Compound, cranks at 135°. Three bearings to crank shaft . . . | 1·94 | 1·25 |

The diameters of a shaft directly connected with a steam engine, such as the propeller shaft of a ship, calculated from the indicated horse-power by the rules above, must be multiplied by $\sqrt[3]{\rho}$ to allow for the difference between the mean and maximum twisting moment.

167. *Shafts subjected to torsion and bending.*—Let T be the twisting moment (calculated by equation 3), and M the bending moment, at any section of a shaft. Then, according to the view hitherto adopted, the combined straining action is equivalent to that which would be produced by a twisting moment $T_e$, given by the following equation, which is Eq. 56 *b* of § 77.

$$T_e = M + \sqrt{(M^2 + T^2)} \quad . \quad . \quad . \quad (4)$$

Let $M = k\,T$, in any given case, so that $k$ is a known fraction. Then

$$T_e = (k + \sqrt{k^2 + 1})\,T \quad . \quad . \quad . \quad (4a)$$

Then the preceding formulæ may be used in designing the shaft, if the equivalent twisting moment $T_e$ is substituted for the actual twisting moment P R in Eq. 3*a*. Hence,

$$d = \sqrt[3]{(k + \sqrt{k^2+1})}\sqrt[3]{\frac{5{\cdot}1}{f_s}}\sqrt[3]{T_e} \quad . \quad . \quad (5)$$

Or, if $d$ is the proper diameter of the shaft, calculated for the combined bending and twisting action, and $d'$ is the diameter calculated for the twisting action alone, by Eq. 3*a* or 3*b*; then

$$d = n\,d'$$

where $n$ is equal to $\sqrt[3]{(k + \sqrt{k^2+1})}$. The following table gives some values of $n$ for given values of $k$.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $k =$ | 0·25 | 0·50 | 0·75 | 1·0 | 1·25 | 1·50 | 1·75 | 2·0 | 3·0 |
| $n =$ | 1·09 | 1·17 | 1·26 | 1·34 | 1·42 | 1·49 | 1·56 | 1·62 | 1·83 |

It appears, from some calculations of Prof. Rankine, that for such cases as the propeller shafts of steam-vessels, where the straining action, additional to the torsion transmitted, is chiefly due to the weight of the shaft itself, $k = 0{\cdot}25$ to $0{\cdot}5$, and the diameter of the shaft should be 1·09 to 1·17 times the

diameter, calculated from the torsion alone. For line shafting in mills, the bending action is often much greater, and the twisting moment is not constant, but rises above the mean value, calculated from the power transmitted. Practical experience appears to show, that for ordinary light shafting, $k$ is 0·75 to 1, and the diameter of the shafting is 1·26 to 1·34 times the diameter, calculated from the mean torsion alone. For crank shafts and heavy shafting subjected to shocks, $k = 1$ to 1·5, and the diameter is 1·34 to 1·49 times that calculated from the torsion alone. Cases occur in which still greater allowance must be made.

168. *Cranked shafts of marine engines.*—In designing marine engine crank shafts allowance has to be made ($a$) for the excess of the greatest twisting moment $T_{max}$ in a revolution, calculated from the indicated horse-power, over the mean twisting moment T. Let

$$T_{max} = \rho\ T\ ;$$

($b$) for the bending action, the moment of which is $M = k\ T_{max.}$ where general average values of $k$ for different cases are given below. In calculating the diameter of the shaft for the combined action it is better to assume the newer view that the limit of safety depends on the maximum shearing stress. Then the equivalent twisting moment is

$$T_e = \sqrt{\{(\rho\ T)^2 + M^2\}} = \rho\ T\sqrt{(1+k^2)} \quad . \quad . \quad (6)$$

*Values of ρ and k for some Marine Engines*

| | Ratio of greatest to mean twisting moment $\rho$ | Ratio of bending to twisting moment $k$ | $\rho\ \sqrt{(1+k^2)}$ |
|---|---|---|---|
| Engine A . . | 2·10 | ·33 | 2·21 |
| ,, B . . | 1·37 | ·14 | 1·38 |
| ,, C . . | 1·48 | ·10 | 1·48 |
| ,, D . . | 1·77 | ·05 | 1·77 |
| ,, E . . | 1·94 | ·19 | 1·98 |

*Diameters of Shafts for given Twisting Moments*

The following tables are calculated for a working stress $f = 9{,}000$ lbs. per sq. in., which is suitable for wrought iron and in some cases for mild steel. Generally for mild steel the diameter in the table may be multiplied by 0·9; for medium steel by 0·86; for cast iron by 1·26. If the shaft is subject to frequent reversals of the larger part of the stress the diameter otherwise sufficient should be multiplied by 1·25.

The shaft diameter is found from the known mean twisting moment in the column corresponding to the assumed ratio $k$ of bending to twisting.

| Twisting moment P R in inch lbs. | Diameter for twisting moment only $d$ | Diameter for twisting and bending moment for $k$ = 0·5 | 0·75 | 1·0 | 1·5 |
|---|---|---|---|---|---|
| 125 | ·41 | ·48 | ·52 | ·55 | ·61 |
| 250 | ·52 | ·61 | ·66 | ·70 | ·73 |
| 500 | ·66 | ·77 | ·83 | ·88 | ·99 |
| 750 | ·75 | ·88 | ·94 | 1·00 | 1·12 |
| 1,000 | ·83 | ·97 | 1·05 | 1·11 | 1·24 |
| 1,500 | ·95 | 1·11 | 1·20 | 1·27 | 1·42 |
| 2,000 | 1·04 | 1·22 | 1·31 | 1·37 | 1·55 |
| 2,500 | 1·12 | 1·31 | 1·41 | 1·50 | 1·67 |
| 3,000 | 1·19 | 1·39 | 1·50 | 1·60 | 1·73 |
| 4,000 | 1·31 | 1·53 | 1·65 | 1·75 | 1·95 |
| 5,000 | 1·42 | 1·66 | 1·79 | 1·90 | 2·11 |
| 6,000 | 1·50 | 1·75 | 1·89 | 2·00 | 2·24 |
| 7,500 | 1·62 | 1·90 | 2·04 | 2·08 | 2·42 |
| 10,000 | 1·78 | 2·08 | 2·24 | 2·39 | 2·66 |
| 12,500 | 1·92 | 2·22 | 2·42 | 2·57 | 2·86 |
| 15,000 | 2·04 | 2·39 | 2·57 | 2·73 | 3·04 |
| 17,500 | 2·15 | 2·52 | 2·70 | 2·88 | 3·20 |
| 20,000 | 2·25 | 2·63 | 2·83 | 3·01 | 3·36 |
| 25,000 | 2·42 | 2·83 | 3·05 | 3·24 | 3·60 |
| 30,000 | 2·57 | 3·00 | 3·23 | 3·44 | 3·83 |
| 35,000 | 2·70 | 3·16 | 3·40 | 3·62 | 4·02 |
| 40,000 | 2·83 | 3·32 | 3·57 | 3·80 | 4·22 |
| 45,000 | 2·94 | 3·44 | 3·70 | 3·94 | 4·40 |
| 50,000 | 3·05 | 3·57 | 3·85 | 4·10 | 4·55 |
| 60,000 | 3·24 | 3·80 | 4·10 | 4·33 | 4·82 |
| 70,000 | 3·41 | 4·00 | 4·30 | 4·57 | 5·10 |
| 80,000 | 3·57 | 4·17 | 4·50 | 4·77 | 5·31 |
| 90,000 | 3·71 | 4·35 | 4·70 | 4·97 | 5·52 |
| 100,000 | 3·84 | 4·50 | 4·85 | 5·15 | 5·72 |
| 110,000 | 3·97 | 4·65 | 5·00 | 5·30 | 5·90 |
| 120,000 | 4·08 | 4·80 | 5·15 | 5·47 | 6·10 |
| 130,000 | 4·19 | 4·90 | 5·30 | 5·61 | 6·25 |
| 140,000 | 4·30 | 5·05 | 5·40 | 5·76 | 6·40 |
| 150,000 | 4·40 | 5·15 | 5·55 | 5·90 | 6·55 |
| 175,000 | 4·63 | 5·45 | 5·85 | 6·20 | 6·90 |
| 200,000 | 4·84 | 5·65 | 6·10 | 6·50 | 7·20 |
| 250,000 | 5·24 | 6·10 | 6·55 | 7·00 | 7·76 |
| 300,000 | 5·54 | 6·50 | 7·00 | 7·42 | 8·27 |
| 400,000 | 6·10 | 7·15 | 7·70 | 8·20 | 9·10 |
| 500,000 | 6·57 | 7·70 | 8·30 | 8·80 | 9·80 |
| 600,000 | 6·98 | 8·20 | 8·80 | 9·40 | 10·40 |
| 750,000 | 7·53 | 8·80 | 9·50 | 10·05 | 11·20 |
| 1,000,000 | 8·28 | 9·70 | 10·40 | 11·10 | 12·30 |
| 1,250,000 | 8·92 | 10·42 | 11·20 | 12·00 | 13·30 |
| 1,500,000 | 9·47 | 11·10 | 11·95 | 12·70 | 14·10 |
| 1,750,000 | 9·97 | 11·70 | 12·60 | 13·35 | 14·90 |
| 2,000,000 | 10·42 | 12·20 | 13·10 | 14·00 | 15·50 |
| 2,250,000 | 10·84 | 12·70 | 13·65 | 14·50 | 16·20 |
| 2,500,000 | 11·23 | 13·15 | 14·10 | 15·10 | 16·70 |
| 3,000,000 | 11·93 | 14·00 | 15·00 | 16·00 | 17·80 |
| 3,500,000 | 12·57 | 14·70 | 15·90 | 16·80 | 18·70 |
| 4,000,000 | 13·14 | 15·40 | 16·55 | 17·60 | 19·60 |
| 4,500,000 | 13·67 | 16·00 | 17·25 | 18·30 | 20·20 |
| 5,000,000 | 14·16 | 16·60 | 17·85 | 19·00 | 21·10 |

*Diameters of Shafts, when the Horse-power and Revolutions per minute are given, calculated by Eq. 3b*

Divide the H.P. by the number of revolutions per minute. The diameter will be found opposite the nearest number to the quotient in the following table.—

| Horse-power / Revolutions $\frac{H.P.}{N}$ | Diameter for twisting moment only $d$ | Diameter for combined twisting and bending moment for $k$ = 0·5 | 0·75 | 1·0 | 1·5 |
|---|---|---|---|---|---|
| ·012 | 0·753 | 0·881 | 0·948 | 1·009 | 1·121 |
| ·025 | 0·963 | 1·126 | 1·213 | 1·290 | 1·434 |
| ·050 | 1·213 | 1·428 | 1·528 | 1·625 | 1·806 |
| ·075 | 1·389 | 1·624 | 1·750 | 1·862 | 2·070 |
| ·1 | 1·529 | 1·788 | 1·926 | 2·050 | 2·277 |
| ·15 | 1·750 | 2·047 | 2·202 | 2·345 | 2·607 |
| ·2 | 1·926 | 2·255 | 2·425 | 2·58 | 2·87 |
| ·25 | 2·075 | 2·427 | 2.615 | 2·78 | 3·09 |
| ·3 | 2·205 | 2·58 | 2·78 | 2·95 | 3·28 |
| ·35 | 2·321 | 2·71 | 2·92 | 3·11 | 3·46 |
| ·4 | 2·428 | 2·84 | 3·06 | 3·25 | 3·62 |
| ·45 | 2·524 | 2·95 | 3·[illegible]8 | 3·38 | 3·76 |
| ·5 | 2·614 | 3·06 | 3·29 | 3·50 | 3·90 |
| ·6 | 2·777 | 3·25 | 3·50 | 3·73 | 4·14 |
| ·7 | 2·925 | 3·42 | 3·68 | 3·92 | 4·36 |
| ·8 | 3·06 | 3·58 | 3·85 | 4·10 | 4·56 |
| ·9 | 3·18 | 3·72 | 4·01 | 4·26 | 4·74 |
| 1·0 | 3·29 | 3·85 | 4·14 | 4·41 | 4·90 |
| 1·25 | 3·55 | 4·15 | 4·47 | 4·76 | 5·29 |
| 1·5 | 3·77 | 4·41 | 4·75 | 5·05 | 5·62 |
| 1·75 | 3·97 | 4·64 | 5·00 | 5·32 | 5·92 |
| 2·0 | 4·15 | 4·86 | 5·23 | 5·56 | 6·18 |
| 2·25 | 4·32 | 5·05 | 5·44 | 5·79 | 6·44 |
| 2·5 | 4·47 | 5·23 | 5·63 | 6·00 | 6·66 |
| 2·75 | 4·61 | 5·39 | 5·81 | 6·18 | 6·87 |
| 3·0 | 4·75 | 5·56 | 5·98 | 6·37 | 7·08 |
| 3·25 | 4·88 | 5·71 | 6·15 | 6·54 | 7·27 |
| 3·5 | 5·00 | 5·85 | 6·30 | 6·70 | 7·45 |
| 3·75 | 5·12 | 5·99 | 6·45 | 6·86 | 7.63 |
| 4·0 | 5·23 | 6·12 | 6·59 | 7·01 | 7·80 |
| 4·25 | 5·34 | 6·25 | 6·73 | 7·16 | 7·96 |
| 4·5 | 5·44 | 6·36 | 6·85 | 7·30 | 8·11 |
| 4·75 | 5·54 | 6·48 | 6·98 | 7·43 | 8·26 |
| 5·0 | 5·63 | 6·59 | 7·10 | 7·55 | 8·39 |
| 5·5 | 5·82 | 6·81 | 7·33 | 7·80 | 8·67 |
| 6·0 | 5·99 | 7·01 | 7·55 | 8·03 | 8·93 |
| 6·5 | 6·15 | 7·20 | 7·75 | 8·25 | 9·17 |
| 7·0 | 6·30 | 7·37 | 7·94 | 8·45 | 9·39 |
| 7·5 | 6·45 | 7·55 | 8·13 | 8·65 | 9·61 |
| 8·0 | 6·59 | 7·71 | 8·31 | 8·84 | 9·82 |
| 9·0 | 6·85 | 8·02 | 8·63 | 9·19 | 10·20 |
| 10 | 7·10 | 8·31 | 8·95 | 9·52 | 10·58 |
| 11 | 7·33 | 8·58 | 9·24 | 9·83 | 10·91 |
| 12 | 7·53 | 8·81 | 9·50 | 10·10 | 11·21 |
| 13 | 7·75 | 9·07 | 9·77 | 10·40 | 11·54 |

*Diameters of Shafts, when the Horse-power and Revolutions per minute are given, calculated by Eq. 3b*—continued

| Horse-power / Revolutions $\frac{\text{H.P.}}{\text{N}}$ | Diameter for twisting moment only $d$ | Diameter for combined twisting and bending moment for $k$ = 0·5 | 0·75 | 1·0 | 1·5 |
|---|---|---|---|---|---|
| 14 | 7·94 | 9·29 | 10·00 | 10·65 | 11·82 |
| 15 | 8·12 | 9·50 | 10·23 | 10·90 | 12·10 |
| 16 | 8·30 | 9·71 | 10·46 | 11·13 | 12·37 |
| 17 | 8·47 | 9·91 | 10·67 | 11·36 | 12·62 |
| 18 | 8·63 | 10·09 | 10·87 | 11·57 | 12·86 |
| 19 | 8·79 | 10·28 | 11·08 | 11·80 | 13·10 |
| 20 | 8·93 | 10·45 | 11·25 | 11·98 | 13·30 |
| 21 | 9·08 | 10·62 | 11·44 | 12·18 | 13·52 |
| 22 | 9·23 | 10·80 | 11·63 | 12·38 | 13·74 |
| 23 | 9·38 | 10·97 | 11·81 | 12·58 | 13·97 |
| 24 | 9·51 | 11·12 | 11·98 | 12·75 | 14·17 |
| 25 | 9·66 | 11·30 | 12·16 | 12·95 | 14·39 |
| 26 | 9·76 | 11·41 | 12·29 | 13·08 | 14·54 |
| 27 | 9·88 | 11·55 | 12·44 | 13·25 | 14·71 |
| 28 | 10·00 | 11·70 | 12·60 | 13·40 | 14·90 |
| 29 | 10·12 | 11·84 | 12·75 | 13·56 | 15·08 |
| 30 | 10·23 | 11·95 | 12·89 | 13·70 | 15·24 |

If the shaft is of cast iron or steel, or if bending action is to be allowed for, proceed as indicated for the preceding table.

169. *Shafting for transmitting power. Factory and mill shafting.*—Shafting in machine workshops usually runs at 120 revs. per min.; in mills for textile manufactures at 300 to 400 revs. per min.; in wood-working shops at 250 revs. per min. Such shafting when carrying pulleys is subjected to bending as well as to the torsion due to the power transmitted. It may be calculated in ordinary cases for $k = 1$; that is, the diameter is 1·34 times that which would be necessary if there were no bending. When as usual it is of wrought iron, or mild steel, we have from Eq. 3b—

$$d = 1{\cdot}34 \sqrt[3]{\frac{63{,}024}{{\cdot}196 f_s}} \sqrt[3]{\frac{\text{H.P.}}{\text{N}}} \quad . \quad . \quad (7)$$

and for $f_s = 9{,}000$

$$d = 4{\cdot}41 \sqrt[3]{(\text{H.P.} / \text{N})} \quad . \quad . \quad (7a)$$

From this the following table has been computed, giving the values of H.P./N for the most ordinary sizes of shaft.

*Horse-power transmitted by Mill Shafting*

Multiply the tabular number in the column H.P./N by the number N of revolutions per minute, the result is the horse-power which the shaft will transmit. $f = 9{,}000$; $k = 1$.

| Diameter of shaft in ins. | $\frac{\text{H.P.}}{\text{N}}$ | Diameter of shaft in ins. | $\frac{\text{H.P.}}{\text{N}}$ |
|---|---|---|---|
| 1¾ | 0·0623 | 5 | 1·4536 |
| 2 | 0·0930 | 5½ | 1·9344 |
| 2¼ | 0·1325 | 6 | 2·5112 |
| 2½ | 0·1817 | 6½ | 3·1944 |
| 2¾ | 0·2418 | 7 | 3·9888 |
| 3 | 0·3139 | 7½ | 4·9056 |
| 3¼ | 0·3993 | 8 | 5·9536 |
| 3½ | 0·4986 | 8½ | 7·1440 |
| 3¾ | 0·6132 | 9 | 8·4800 |
| 4 | 0·7442 | 10 | 11·6288 |
| 4¼ | 0·8930 | 11 | 15·4752 |
| 4½ | 1·0600 | 12 | 20·0896 |
| 4¾ | 1·2470 | | |

Shafts in mills which are merely transmitting shafts and do not carry pulleys may be of rather less diameter. In that case

$$d = 3{\cdot}3 \text{ to } 3{\cdot}6 \sqrt[3]{\frac{\text{H.P.}}{\text{N}}}.$$

All transmission shafts must have an arrangement for preventing end motion. Generally two collars are placed at a bearing in the head length which receives the power.

Fig. 179 shows the ordinary arrangement of shafting. $l$ is the distance between bearings which should be equally spaced if possible. Collars to prevent end motion are on the first shaft. If the second shaft is reduced in diameter, the end of the first is turned down to the size of the second to receive the coupling. The first shaft should have two bearings, and the coupling should be near a bearing.

| Diameter of first shaft | Length $a$ | Diameter of second shaft | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 2 | 2¼ | 2½ | 2¾ | 3 | 3½ | 4 | 4½ | 5 |
| | | Distance $c$ | | | | | | | | |
| 2 | 4½ | 11 | — | — | — | — | — | — | — | — |
| 2½ | 5⅝ | 12 | 12¼ | 12¾ | — | — | — | — | — | — |
| 3 | 6¼ | 13 | 13¼ | 13¾ | 14⅛ | 14⅝ | — | — | — | — |
| 3½ | 7⅞ | 14 | 14¼ | 14¾ | 15⅛ | 15⅝ | 16½ | — | — | — |
| 4 | 9 | 15 | 15¼ | 15¾ | 16⅛ | 16⅝ | 17½ | 18¼ | — | — |
| 5 | 11⅛ | — | — | 17¾ | 18⅛ | 18⅝ | 19½ | 20¼ | 21¼ | 22⅛ |
| 6 | 13¼ | — | — | — | 20⅛ | 20⅝ | 21½ | 22¼ | 23¼ | 24⅛ |

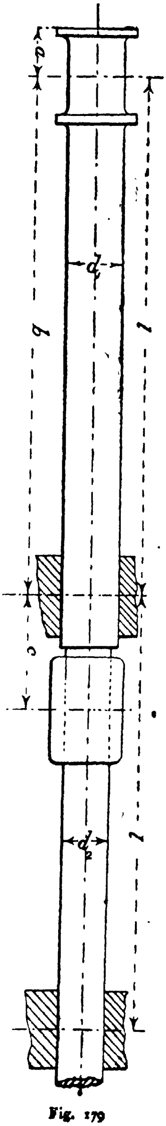


Fig. 179

170. *Flexible shafts.*—A steel wire rope will transmit a twisting moment, if the tension at 45° with the axis is roughly along the twisted wires and the compression which is normal to the tension is roughly at right angles to them. Wire ropes can therefore be used to transmit a twisting moment in one direction and they are extremely useful from their flexibility in driving portable tools. Let $d$ be the diameter of the rope, H.P. the horse-power transmitted, at N revolutions per min. Then,

$$d = 8 \text{ to } 11 \sqrt[3]{\frac{\text{H.P.}}{\text{N}}}$$

where the smaller coefficient is for ropes 0·5 inch in diameter and the larger for ropes 3 inches in diameter.

171. *Loose collars.*—To prevent endways movement of shafts collars at a bearing are used. These, when possible, are forged on the shaft. In other cases loose collars are used, fig. 180, which may be undivided or divided for facility of fixing. Hardened set screws with sunk heads fix the collars to the shaft. Let the internal diameter $= d$. Then,

$$\text{External diameter} = 1{\cdot}33\, d + \tfrac{3}{4}$$
$$\text{Thickness} = 0{\cdot}15\, d + 1\tfrac{1}{4}$$

172. *Hollow shafts.*—At the present time the use of compressed steel permits shafts to be made hollow; in this way, since the least effective portion of the section of the shaft is removed, the weight is diminished in a much greater ratio than the strength. Let $d$ be the diameter of a solid shaft, and $d_1$, $d_2$ the external and internal diameters of a hollow shaft of the same material. Then the shafts will be of equal strength when the moduli of the two sections with respect to torsion are equal, that is, when (§ 57)—

$$d^3 = \frac{d_1{}^4 - d_2{}^4}{d_1}$$

Let $d_2 = x\, d_1$

$$d_1 = d\sqrt[3]{\left\{\frac{1}{1 - x^4}\right\}} \quad . \quad . \quad . \quad (8)$$

which gives the external diameter of a hollow shaft in terms of the diameter of a solid shaft of the same strength, when the fraction of the diameter removed at the centre is fixed. Suppose a 10-inch shaft has a hollow 4 ins. diameter. Its weight will be 16 per cent. less than that of a solid 10-inch shaft, but its strength is only 2·56 per cent. less. A hollow shaft with a hole $\frac{4}{10}$ths of

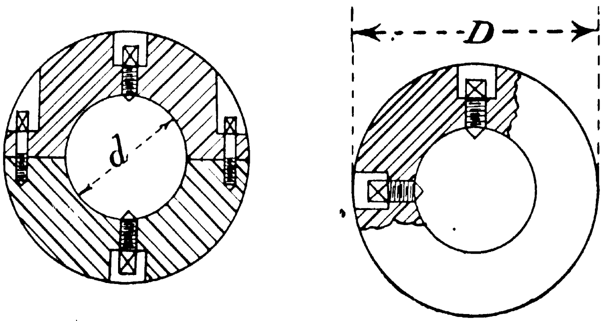



Fig 180

the diameter, and equal in strength to a 10-inch solid shaft, would have a diameter of

$$d_1 = 10\sqrt[3]{\frac{1}{1 - 0{\cdot}4^4}} = 10{\cdot}09 \text{ inches.}$$

173. *Shafts of uniform and of varying diameter. Stiffness of shafts.*—Most line shafts in factories drive machines at various points distributed along their length. Hence the amount of power transmitted across any section varies from zero at the free end to a maximum at the driving end. Considering strength alone, a line of shafting should diminish in diameter from the driving towards the free end, or, if each length is of uniform diameter, the successive lengths should be of decreasing diameter. On the other hand, there are great practical conveniences in making the diameter uniform. The hangers, couplings, &c., are

then of uniform pattern and interchangeable, and can be shifted as occasion arises. The additional cost of the uniform shaft is in many cases not great and its erection is easier. Neither the principle of varying the diameter nor that of keeping the diameter uniform can be pushed to an extreme. The extent to which the diameter should be varied is a matter of judgment, and experience shows that within limits it is desirable to maintain uniformity.

Small shafts often give trouble from insufficient stiffness, although they have ample strength. For such shafts, $\frac{3}{8}$ in. to $\frac{5}{8}$ in. may be added to the diameter, which is sufficient for strength, in order to secure stiffness and freedom from vibration.

The permissible twist is taken in some cases at 0·006° per inch length or a twist of 1° in 14 feet. If this value is substituted for $\theta$ in the equation above (§ 161), then for stiffness

$$d \overset{=}{>} 0{\cdot}3 \sqrt[4]{\mathrm{T}} \overset{=}{>} 4{\cdot}7 \sqrt[4]{(\mathrm{H.P.}/\mathrm{N})} \quad . \quad . \quad (9)$$

The span between the bearings of shafting should be so arranged as to limit the deflection of the shaft to a fixed proportion of its length. Let L be the span between the bearings, in inches. Then, if the deflection is to be a fixed fraction of the span,

$$\mathrm{L} = \gamma \sqrt{d} \quad . \quad . \quad . \quad . \quad (10)$$

where $\gamma = 60$ to 75 at most for shafting supporting its own weight only, and $\gamma = 50$ to 55 for shafting carrying the ordinary proportion of pulleys or gearing. It is desirable in mill and factory shafting, if possible, to take $\gamma$ not more than 40 to 45 to reduce the deflection due to bending actions. Very commonly the bearings of mill shafting are 8 to 10 feet apart.

*Planished wrought-iron shafting.*—The round bars which come from the rolling-mill are rough and slightly crooked. Shafting made from such bars must be turned from end to end in the lathe, to obtain uniformity of diameter and smoothness of surface. A process has, however, been introduced which supersedes turning in many cases. By passing the bar while still hot between rapidly revolving bevelled rollers, the scale is cleaned off and the bar rendered so straight and regular that it may be used for shafting, after having been merely polished with a file and emery stick, either in the lathe or in place.

174. *Expansion of shafts.*—If the shaft is of great length. especially if it carries bevel wheels, at distances of more than 40 feet from the journal with collars which prevent end motion, its alteration of length from changes of temperature becomes troublesome. The adjustment in gear of the bevel wheels is altered by the longitudinal movement. In such cases, other collar bearings are provided near the wheels, and an expansion coupling is introduced at some intermediate point. Ordinary claw couplings are sometimes used as expansion couplings, or a box coupling is used. One shaft end is fixed in the coupling box by a taper key. The other is secured from rotating relatively to the box by two parallel keys on opposite sides fitted accurately but easily, and allowing free expansion.

The expansion of iron is about 0·0012 of its length for a rise of temperature of 180° Fahr. Hence if $t_1$, $t_2$, are the highest and lowest temperatures to which a shaft is exposed, a point $l$ inches from the bearing with collars will move a distance

$$0{\cdot}0012 \frac{t_1 - t_2}{180} l \text{ inches},$$

in consequence of variation of temperature.

175. *Centrifugal whirling of shafts.*—Prof. Rankine showed that the centrifugal force of a slightly bent shaft and the elastic stress tending to straighten it become equal for a certain speed and for a given length between bearings. That speed may be termed the *critical* speed. If a shaft is from any cause out of balance or bent to a very small extent, then if the critical speed is exceeded the shaft revolves in a bent form and the bending is liable to increase to a dangerous extent. Prof. S. Dunkerley experimentally investigated the conditions in which centrifugal whirling is liable to occur in unloaded and loaded shafts.[1] The theory is far too difficult to be given here, but the following results are taken from Prof. Dunkerley's paper. For an unloaded shaft the relation of length between bearings in feet, diameter $d$ in inches, and revolutions N per minute at which centrifugal whirling occurs is given by the relation

$$\text{N} = a\, d \,/\, l^2$$

where $a$ is a constant depending on the mode of support of the shaft. For the case of an unloaded shaft supported at the

[1] *Trans. Royal Soc.* vol. 185A, pp. 279–360. Also a paper on 'The Whirling and Vibration of Shafts,' Liverpool Engineering Soc. 1894.

ends, or an unloaded shaft on three bearings in equal spans, the length at which danger of whirling arises is about

$$l = 180 \sqrt{(d / \text{N})} \text{ feet.}$$

If the shaft is loaded with pulleys or wheels the critical speed is lowered. Consider first the critical speed for a pulley of weight W lbs., at a distance $c$ from the nearest support, on a shaft supported on equidistant bearings $l$ feet apart. Let the shaft be so light compared with the pulley that its influence on the critical speed is insignificant. Then the speed at which whirling occurs is

$$\text{N} = b\, d^2 / \sqrt{\text{W}\, c^3}$$

where $b$ is a coefficient depending not only on the manner in which the shaft is supported, but also on the dimensions of the pulley. For any particular mode of support of the shaft $b$ is a function of $c / l$ and also of $c / k$, where for cases usual in practice $k = \sqrt{\{\text{A}\, g / 2\, \text{W}\}}$. A is the mass moment of inertia of the pulley about the axis of the shaft in gravitation units.

For ordinary cases of shafts not necessarily light and carrying pulleys, the critical speed may be found approximately by calculating the critical speed of N the shaft, taking account of the mode in which it is supported, and the critical speed $\text{N}_1$ for one pulley given by the second equation above. Then the critical speed of shaft and pulley is

$$\text{N}' = \left(\text{N}_s \text{N}_1\right) / \sqrt{\left(\text{N}_s^2 + \text{N}_1^2\right)}$$

If now the action of a second pulley is to be taken into account, let $\text{N}_2$ be its critical speed by the equation above. Then the critical speed of the shaft and two pulleys is

$$\text{N}'' = \left(\text{N}' \text{N}_2\right) / \sqrt{\left(\text{N}'^2 + \text{N}_2^2\right)}$$

In this way any number of disturbing masses may be taken into account. The values of the constant $a$ for unloaded shafts are: (1) Overhanging shaft fixed in direction at one end, $a = 11645$. (2) Shaft supported in bearings at each end, $a = 32864$. (3) Shaft supported at one end and fixed in direction at the other, $a = 51340$. (4) Shaft supported on three bearings, the spans

being $l_1$, $l_2$, the latter being the larger, and $r = l_1/l_2$. Then for the larger span,

| | | | | | | |
|---|---|---|---|---|---|---|
| $r =$ | Very small | 0·1 | 0·17 | 0·25 | $\frac{1}{2}$ to $\frac{3}{4}$ | $\frac{3}{4}$ to 1 |
| $a =$ | 50654 | 47125 | 44312 | 43289 | 36884 | 32864 |

(5) Shaft fixed in direction at each end, as when there are two bearings near together at each end, $a = 74971$. (6) Continuous shaft with bearings $l$ feet apart, $a = 32864$. A selection of values of $b$ is given in the following table for the case of a pulley on an intermediate span of a shaft carried by three or more bearings.

| Values of $c/k$ | Values of $c/l$ | | | |
|---|---|---|---|---|
| | Very small | $\frac{1}{10}$ | $\frac{1}{4}$ | $\frac{1}{2}$ |
| | | | Values of $b$ | |
| ·25 | 9037 | 10035 | 11481 | 17800 |
| ·50 | 8102 | 9318 | 11071 | 17800 |
| 1·00 | — | 7028 | 10051 | 17800 |
| 1·50 | — | 5669 | 9380 | 17800 |
| 2·00 | — | 5191 | 9052 | 17800 |

176. *Fencing of Shafting.*—To protect workmen from accident, shafting and gearing not out of reach should be fenced by covers of tin plate. Shafting between detached buildings should be at a considerable height above the ground, or in a covered trench below it.

### CRANK SHAFTS

177. *Forged cranked shafts.*—Forged cranked shafts are very extensively used, for inside cylinder locomotives and for marine engines of all kinds. Fig 181 shows the ordinary form of such a cranked shaft. In marine-engine practice the diameter may be obtained thus:—

H.P. = indicated horse-power.
N = number of revolutions per minute.
$d$ = diameter of shaft.

$$d = 4{\cdot}55 \sqrt[3]{\left(\frac{\text{H. P.}}{\text{N}}\right)} \qquad . \quad . \quad . \quad (11)$$

which allows for bending as well as twisting actions. The arms of the crank may be so proportioned that

$$b h^2 = c d^3$$

where $$c = 0{\cdot}9 \text{ to } 1{\cdot}0.$$

The objection to the use of cranked shafts is their liability to fracture. On this point Mr. Milton makes the following remark :[1] ' Most of the flaws for which cranked shafts are condemned occur at the angle between the web of the crank and the journal or crank-pin, at the place where the forging is most likely to be defective, and they are evidently produced more by the bending than by the twisting strains. At these places the change of form of the shaft throws great local stresses on the material and there can be no doubt that, if these parts are made with a large radius, the strength of the shaft is materially increased. Among the causes which tend to throw great strains on the shafting may

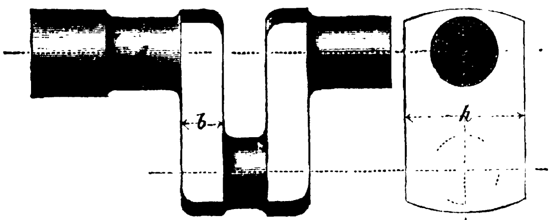


Fig. 181

be mentioned, the presence of water in the cylinders, slackness of the brasses, and the bearings being out of line.'

178. *Graphic method of determining the straining actions in crank shafts.*—The crank arm and crank shaft are subjected to bending and twisting actions which are very conveniently dealt with graphically. Suppose the crank and shaft provisionally designed so as to fix the positions of the bearings. The moment equivalent to the bending and twisting moments at each section can then be determined and the necessary dimensions calculated.

*Case I.*—Taking a simple overhung crank, shown by its centre lines in fig. 182. Let A be the crank-pin centre and B and C the centres of the bearings. In general the total action will be greatest when the thrust or pull, P, of the connecting rod is perpendicular to the plane of the figure—that is, when the

[1] 'Strains on Crank Shafts' (*Proc. Inst. of Naval Architects*, 1880).

connecting rod is at right angles to the crank arm. P will be supposed to be acting so, although drawn in the plane of the figure.

Suppose P given, then the reactions Q and R at B and C can be determined by taking moments, or more simply by drawing the diagram of forces. Take 1 2 on P's direction equal to P, and a polar distance 2 *o* at right angles of any convenient number of inches. Join 1 *o* cutting Q's direction in *a*. Join *a b* and draw *o* 3 parallel to it. Then 2 3 = R and 1 3 = Q. Also 1 *a b* is the diagram of bending moments on the crank shaft. So

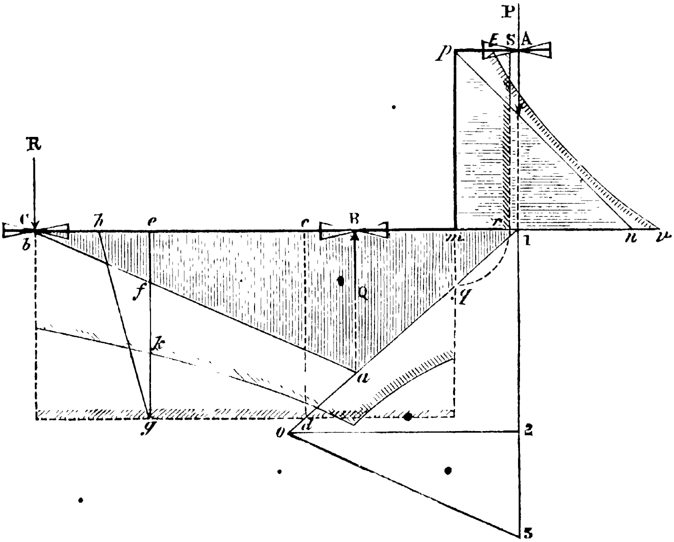



Fig. [illegible]

that the ordinate *e f* on the scale of loads multiplied by *o* 2 on the scale of lengths is the bending moment at *e*, or *e f* is proportional to the bending moment.

Take 1 *c* = 1 A. Draw *c d* at right angles to the shaft, cutting 1 *o* or 1 *o* produced in *d*. Then since *c d* : *c* 1 :: 1 2 : *o* 2, ∴ *c d* × *o* 2 = *c* 1 × 1 2 = P × 1 A. But P × 1 A is the twisting moment on the shaft. Consequently *c d* represents the twisting moment on the same scale as the diagram of bending moments. Drawing a line through *d* parallel to the shaft we get the diagram of twisting moments.

To combine the bending and twisting moments at any point into an equivalent bending or twisting moment, let $e f = M$ be the bending and $e g = T$ the twisting moment at $e$. Take $e h = e f$ and join $h g$; then $h g = \sqrt{(e h^2 + e g^2)} = \sqrt{(M^2 + T^2)}$. Consequently $e f + h g$ is $M + \sqrt{(M^2 + T^2)}$, the equivalent twisting moment, and half this is the equivalent bending moment. The shaded curve is the curve of equivalent bending moments obtained by repeating this process at several points, and drawing a curve through the points so found. From the ordinates of this curve the dimensions of the shaft at any point necessary for strength can be calculated.

When P acts perpendicular to the plane of the figure, $P \times I A$ is the bending moment at the point $m$ of the crank arm. But this has already been found to be represented by $c d$. Take $m n = c d$ and join $n p$. Then $p n m$ is the diagram of bending moments on the crank arm. The twisting moment on the crank arm is $P \times A p$ or equal to the bending moment on the crank shaft at $m$. Take $m r = m q$ and complete the rectangle $s m$. This is the diagram of twisting moments on the crank arm. Combining the bending and twisting moments by the same process as before we get the curve of equivalent bending moments on the crank arm $t v$.

*Case II.*—An engine crank shaft, fig. 183, is subjected at the crank pin to a pressure P of 3 tons, and carries a spur flywheel at Q weighing 5 tons. Hence only the part of the shaft between P and Q is subject to torsion. The forces are supposed normal to the plane of the drawing, fig. 183, although shown for convenience in the plane of the drawing. The comparatively small pressure at the spur-wheel teeth is neglected. This case is conveniently treated partly by calculation and partly graphically.

The moment of P about $R_2$ is $-213$; that of Q is $-135$ inch tons. Hence the moment of $R_1$ is $213 + 135 = 348$ inch tons. Hence $R_1 = 348/55 = 6{\cdot}327$ tons and $R_2 = 8 - 6{\cdot}327 = 1{\cdot}673$ tons. Now take any line $a b$, $a$ being on P's direction and $b$ being on $R_2$'s direction. Set off $b c$ equal to the moment of P and join $a c$; then $a b c$ is P's moment area, so that any vertical intercept between $a b$ and $a c$ is the moment of P at that point. Let $d$ be on $R_1$'s direction, set off $c e = R_1$'s moment and join $d e$, then $d e c$ is $R_1$'s moment area. Let $f$ be on Q's direction, set off $e b =$ Q's moment and join $f b$, then $f e b$ is Q's moment area.

The negative moments are measured downwards and the positive moments upwards. It will now be seen that, in the areas *f e b* and *d g b c*, a positive and negative moment balance. There remains the shaded area *a d g f b*, which is the bending moment diagram for the shaft.

The twisting moment of P is $3 \times 15 = 45$ inch tons. Set this off downwards and draw *h k* parallel to *a b*. This is the torsion moment area. Combining the torsion moment with the

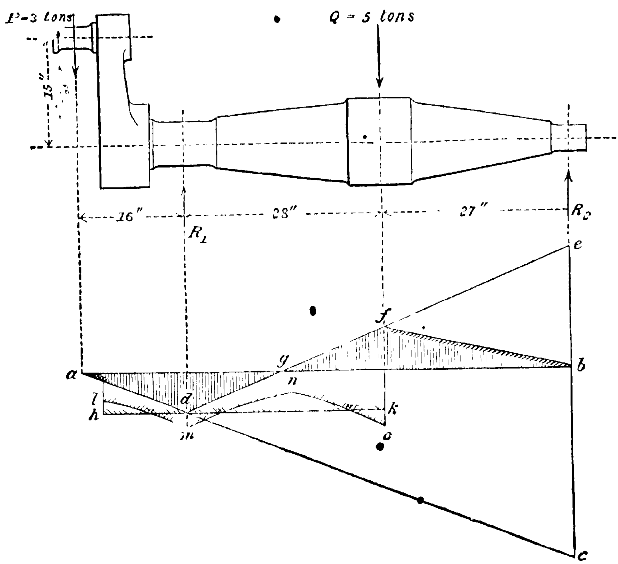



Fig. 183

bending moment at each point, as in the previous case, we get the curve *l m n o* as the curve of equivalent bending moments. The bending moment curve previously drawn gives the bending moments for the remainder of the shaft.

179. *Flywheel shaft of compound engine.*—Fig. 184 shows the bending moment and twisting moment curves for the flywheel shaft of an engine of about 1,000 H.P. The crank radii are taken at $3\frac{1}{2}$ feet, and the piston loads at 30 tons when either engine is at the dead centre and 23 tons at half stroke. The fly-

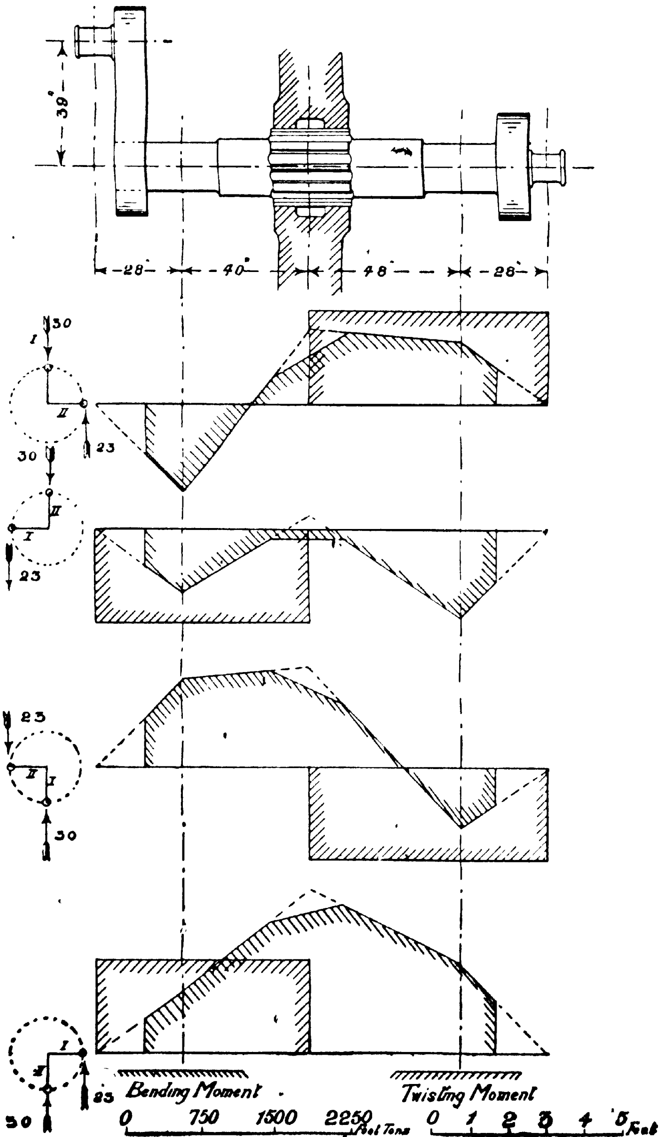

39"
28"
40"
48"
28"
30
23
Bending Moment
Twisting Moment
0 750 1500 2250 Foot Tons
0 1 2 3 4 5 Feet

Fig. 164

wheel weight is taken at 40 tons. The reactions are approximately as follows, in tons.

| Position | Left crank | Journal centre | Flywheel centre | Journal centre | Right crank |
|---|---|---|---|---|---|
| I. | 30 | -69 | 40 | 22 | -23 |
| II. | 23 | -43 | 40 | -50 | 30 |
| III. | -30 | 25 | 40 | -58 | 23 |
| IV. | -23 | -1 | 40 | 14 | -30 |

The straining action varies greatly as the shaft revolves. Hence the bending and twisting moments have been examined for four positions of the cranks shown in the diagrams on the left.

# CHAPTER IX

## SHAFT COUPLINGS

LINES of shafting are made of lengths of 12 feet and upwards coupled together. The couplings should be placed near bearings and on the side farthest from the driving-point. Then if a length of shaft is disconnected the running part is supported.

We may distinguish (*a*) couplings for shafts having a common axis of rotation. (*b*) Couplings for parallel shafts. (*c*) Couplings for shafts the axes of which intersect. Next as to the description of coupling we may distinguish: (1) fixed or permanent couplings, which can only be disconnected by unscrewing bolts or slacking keys. (2) Loose couplings or clutches, provided with arrangements for throwing part of the shafting out of gear as often as necessary. (3) Friction couplings, which are loose couplings having the special peculiarity that they put the driven shaft into gear gradually and slip if the resistance becomes excessive.

### FAST OR PERMANENT COUPLINGS

180. *Box couplings.*—There are three forms of couplings, known as 'box' or 'muff' couplings. In figs. 185 and 187 the coupling is termed a butt coupling, and relative movement of the shafts is prevented by a wrought-iron key. Fig. 186 is a half-lap coupling, the shaft ends overlapping, so as to prevent relative motion independently of the key, whose chief function is to fix the box rigidly in place. In fig. 186 the key is a saddle key. The other dimensions may be obtained from the proportional numbers. The half-lap coupling is an excellent coupling for shafts not exceeding 5 ins. in diameter, but is now very rarely used. The butt coupling is cheaper, but less secure. Both forms are free from projections likely to catch the clothes of a workman.

181. *Flange coupling.*—Fig. 188 shows a flange or face-plate coupling used chiefly for shafts over four inches in diameter. It consists of two parts of cast iron, which should be bored out 0·004 inch per inch diameter of shaft less than the shaft diameter, then heated and shrunk on or forced on the shaft ends by hydraulic pressure, and turned true in place. Keys are added. If a flange coupling is fitted so as to slide off the shaft ends

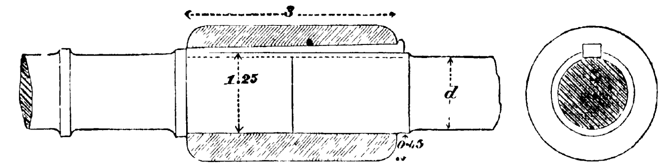



Fig. 185

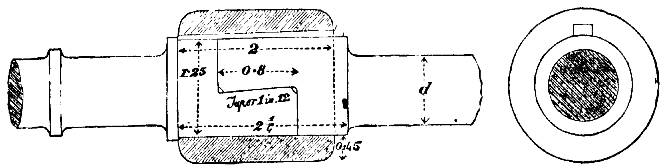
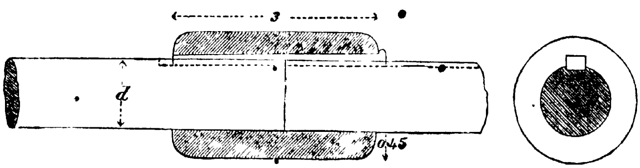



Fig. 186




Unit = $d + \frac{1}{2}$

Fig. 187

easily, and fixed by a taper key, it is very difficult to make it run true. As the pressure due to the key is on opposite sides of the shaft only the coupling will eventually work loose. The flanges are connected by turned bolts in reamed holes. The bolts transmit the torsion by their resistance to shearing. As a precaution to keep the shafts in line, one shaft may enter the coupling on the other $\frac{3}{8}$ inch, or a projection on one coupling may fit a recess

on the other, but this makes dismounting more difficult. The number of bolts may be as follows :—

| | | | |
|---|---|---|---|
| Shaft diameter = | up to 4 | 4 to 8 | 8 to 12 inches |
| No. of bolts = $n$ = | 4 | 6 | 8 |

Let R be the radius of the bolt circle, $d$ the shaft diameter, $\delta$ the bolt diameter, and $n$ the number of bolts. Then, when the shaft is strained to a stress $f$, the shearing force on the bolts is—

$$P = \frac{f d^3}{5 \cdot 1 \, R}$$

where $f$ is the working stress allowed in the shaft. The resistance of the bolts to shearing is $\pi n \, \delta^2 f_s / 4$, where $f_s$ is the mean shearing stress on the bolt sections. Hence, if the bolts and shaft are of the same material, and $f_s$ is taken equal to $0 \cdot 8 f$, then

$$\delta = 0 \cdot 56 \sqrt{\frac{d^3}{n R}}$$

Usual proportions for the bolt diameter are—

$$\delta = 0 \cdot 5 \sqrt{\frac{d^3}{n R}} + 0 \cdot 4$$

This allows for the liability to overstraining of small bolts. Length of coupling parallel to shaft, $2\tfrac{1}{4}d + 3$. Diameter of coupling, $2\tfrac{1}{2}d + 6$.

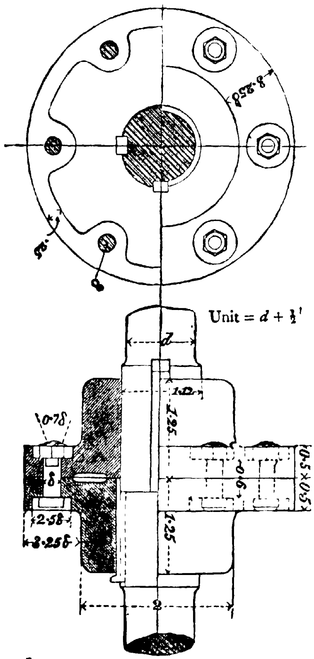



Fig. 188

When shafting is not closely calculated for strength, the flange coupling may be fixed as shown in fig. 189, the shaft diameter being reduced ⅛ to ¼ inch where the coupling is seated. Where a change of diameter occurs it is usual to reduce the end of the larger shaft to the size of the smaller, so that a coupling suitable for the smaller shaft can be used.

182. *Propeller shaft coupling.*—Flange couplings are used both for hollow and solid propeller shafts, the flanges being

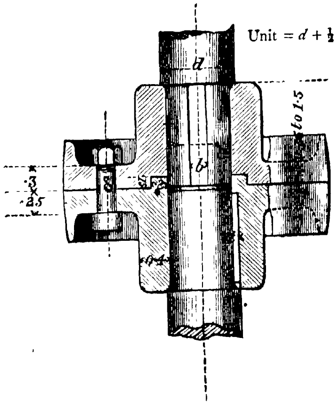



Fig. 189

forged on the shaft ends (fig. 190). For a solid shaft, let $d$ be the diameter of shaft, $\delta$ the diameter and $n$ the number of bolts,

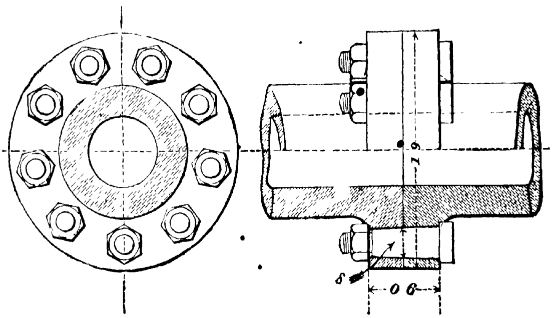



Fig. 190

and R the radius of bolt circle. Then, $n = d/3 + 2$, the nearest even number being taken.

$$\delta = 0{\cdot}56\sqrt{\frac{d^3}{n\,\mathrm{R}}} = 0{\cdot}56\,\frac{d}{\sqrt{n}}\text{ nearly.}$$

Diameter of flange, $d + 3\delta + 1\frac{1}{2}$.
Thickness of flange, $0{\cdot}3d + \frac{1}{8}$.

For a hollow shaft let $d_2$ be the external and $d_1$ the internal diameter. Then, the unit for the proportions in fig. 190 is

$$d = \sqrt[3]{\left(\frac{d_2^4 - d_1^4}{d_2}\right)}$$

For $n$ bolts, the bolt diameter is $\delta = \sqrt{\{d^3/2nR\}}$.

183. *Sellers' double cone vice coupling.*—With box couplings it is generally necessary to forge bosses on the shaft ends, to receive the couplings. This prevents pulleys and wheels being put on the shafts from the ends. Mr. Coleman Sellers introduced a coupling which obviates these difficulties, and which does not require such perfect fitting. Fig. 191 shows this coupling in longitudinal section, end elevation and cross section. It consists of an outer cylindrical muff, or barrel, enclosing the ends of the shafts. The inside of this is turned to a double conical form. Between the barrel and the shaft are two sleeves, the outsides of which are conical, and fit the box, and the insides are cylindrical, and fit the shaft. These sleeves are pressed together by three screw-bolts, parallel to the shaft. The bolts are square or round in section, and rest in slots cut into the sleeves and the barrel. To give elasticity to the sleeves, they are completely cut through on one side, at the bottom of one of the bolt slots. Each sleeve is drawn inwards with equal force

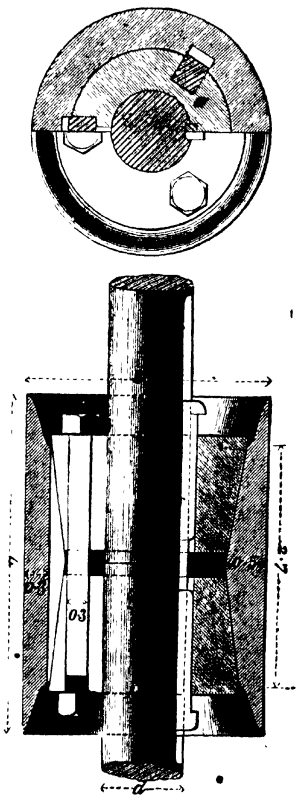


Fig. 191

and grasps the shaft with equal tightness, and the sleeves do this perfectly, even when the shaft ends are only approximately

of the same size. A key is driven into each shaft end, as an additional precaution, but these keys should fit sideways only, and not at top and bottom. They do not then exercise any bursting force on the coupling. Absolute equality of size of the shafts is unnecessary. When two shafts of unequal size are connected, the larger is turned down, at the end, to the size of the smaller. If the parts are well oiled before they are put together, there is no great difficulty in disconnecting. The bolts are taken out, and the coupling struck with a wooden mallet, or a wedge is driven into the split through the sleeves. If there is any difficulty from rusting, a bolt with a hooked end can be inserted through one of the bolt holes and screwed up against a bar placed across the end of the coupling.

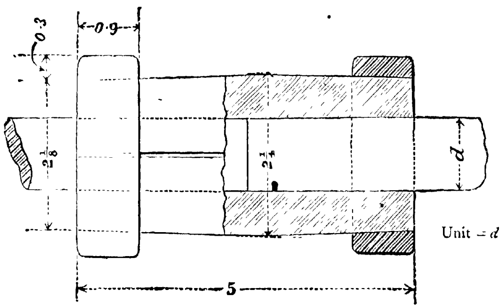



Fig. 192

The proportional figures give roughly the usual proportions of the coupling, the unit being the shaft diameter. But preferably the length of coupling may be $3d + 2\frac{1}{2}$ and the diameter $2\frac{1}{4}d + 1\frac{1}{8}$.

184. A simple and effective friction coupling, suitable for small shafts, is shown in fig. 192. The coupling consists of a pair of semicircular cast-iron clips. These are placed together with a thin plate or sheet of paper interposed, and bored out to the diameter of the shaft. When placed on the shaft the clips are held by two wrought-iron rings shrunk on. The ends of the shafts are turned, but not polished. The usual proportions are given on the figure, the outside of the clips being turned slightly taper. Sometimes bolts are used to connect the two clips instead of rings, and then the clips have a somewhat different form This coupling is chiefly used for small shafts.

Supposing the shaft transmitting a twisting moment only,

$$d = a\sqrt[3]{\text{P R}};$$

or for wrought-iron shafts, P estimated at the circumference of the shaft is

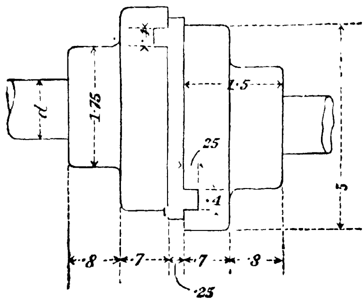
$$\text{P} = 3530\,d^2.$$

Let $p$ be the pressure of the clip on each shaft end per unit of circumference ; $\mu =$ the coefficient of friction. Then

$$\mu\,\pi\,d\,p = \text{or} > 3530\,d^2.$$

Taking $\mu = 0{\cdot}2$,

$$p = \text{or} > 5616\,d.$$




Unit = $d + \frac{1}{2}$

Fig. 193

Such a bursting pressure produces in the ring a tension of $\frac{1}{2}\,p\,d$ lb. Taking the safe stress in the wrought-iron rings at 9,000 lbs. per sq. inch, each ring must have a section

$$\frac{p\,d}{2 \times 9000} = \frac{5616\,d^2}{18000} = 0{\cdot}312\,d^2\,;$$

or if the width of the ring is $0{\cdot}9\,d$, its thickness must be at least $0{\cdot}35\,d$. The thickness given above is a little less than this ; but, on account of allowance for bending, the torsion transmitted is rarely as great as that assumed above. With a stress of 9,000 lbs. the ring will extend

$$\frac{l}{\text{L}} = \frac{f}{\text{E}} = \frac{1}{3200}$$

of its circumferential length. Consequently its internal diamcter

before shrinking on must be $\frac{1}{3200}$th less than the external diameter of the coupling.

185. *Oldham coupling.*—For two shafts which are parallel the coupling shown in fig. 193 may be used. A disc is keyed on each shaft end, and between these lies a third disc, which has a diametral feather on each side fitting in a slot in the corresponding shaft disc. The two feathers are at right angles. The middle disc revolves round an axis parallel to the shafts and midway between them. The shafts and the middle disc have all equal velocities at every period of the rotation.

186. *Coupling permitting longitudinal expansion of shafts.*—Fig. 194 shows a coupling made by the Berlin-Anhaltische Maschinenbau Gesellschaft in Dessau. It forms an expansion joint in long lengths of shafting, transmitting the torsion but allowing the shaft on one side to expand without displacing the other. It is formed like a clutch, but the two parts of the clutch

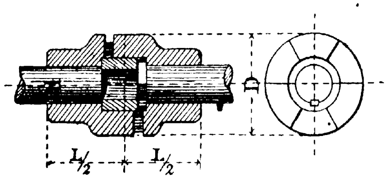



Fig. 194

are fixed on their respective shaft ends. That the shafts may have a common axis, the turned ring shown in section is provided. Empirical dimensions are as follows:

$$L = 4\,d + 0{\cdot}8$$
$$D = 2{\cdot}5\,d$$

*Raffard coupling.*—A flexible coupling, or one which will connect two shafts when not quite exactly in line, is useful in some cases. The Raffard coupling, which has been a good deal used on the Continent for connecting motors and dynamos, is of this kind, and it has the further advantage in certain cases of electrically insulating the dynamo from the motor. The coupling consists of two discs connected by indiarubber bands, which have a small initial tension when the shafts are at rest. Fig. 195 shows the bands in the resting and driving positions.

The coefficient of elasticity of the rubber may be taken at $E = 119$ lbs. per sq. in. The limiting stress when driving should

not exceed 50 lbs.per sq. in. The coupling has been used for transmitting large amounts of power between shafts running at 250 or more revs. per min. The indiarubber is liable to decay from oxidation, and leather bands are sometimes used instead.

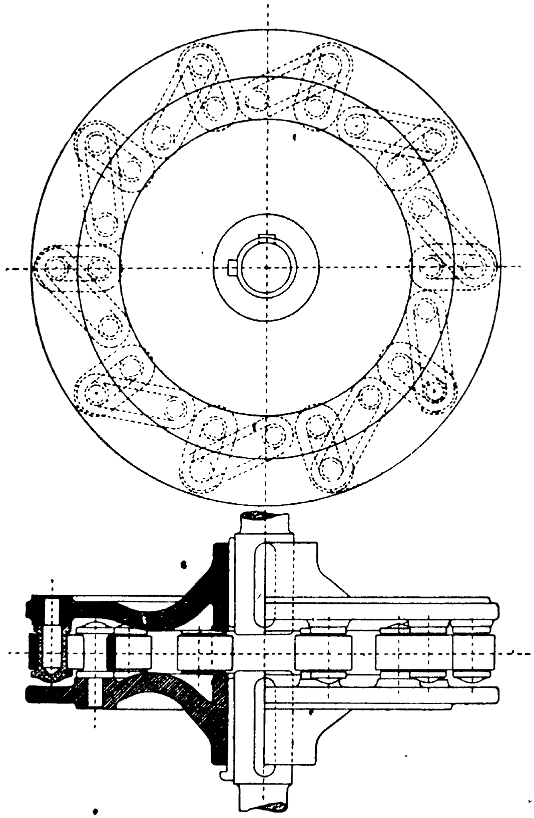


Fig. 195

Fig. 196 shows a flexible coupling introduced by Mr. Brotherhood. Two parts of the coupling are connected by a hard, thin steel plate, which bends slightly if the two lengths of shaft are not exactly in line.

*Ramsbottom's friction-coupling.*—This is a coupling used for rolling mills, which acts as a fast coupling ordinarily, but slips

if the resistance becomes excessive, and saves the machinery from fracture. Discs are keyed on each shaft, and these are bolted together with a force which creates sufficient resistance

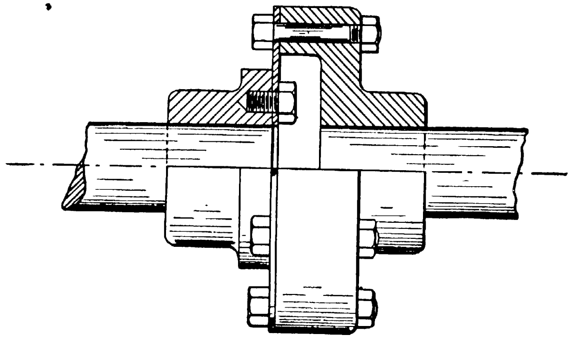


Fig. 196

to slipping with the normal load. Wood faces are interposed between the rubbing surfaces. The adjustment of the bolts is

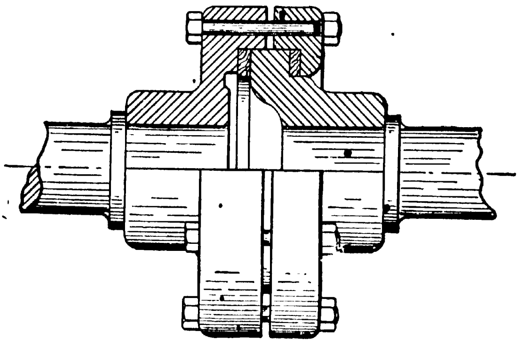


Fig. 197

such that there is no slipping when the effort is normal, but the coupling slips if, in consequence of a check in rolling, the effort becomes abnormal.

### Loose or Disengaging Couplings

187. *Claw coupling or positive clutch.*—For very large slowly rotating shafts it is desirable that the coupling should have a slight amount of play, so that, when the shafts are a little

out of line, the coupling accommodates itself to the obliquity without straining the shafts. The claw coupling may then be used either as a fast coupling or so arranged that one half can be slid back and the shafts thrown out of gear. Fig. 198 shows this coupling arranged for disengaging. It consists of two parts, like the face-plate coupling; but each part has projections, which fit in recesses in the opposite coupling. In the coupling shown, the left-hand part is prolonged, and has a groove cut round it. In this fit the jaws of a lever, for sliding it back. The right-hand part is firmly keyed on its shaft. The left-hand part slides on one or two fixed feathers, which are not tapered.

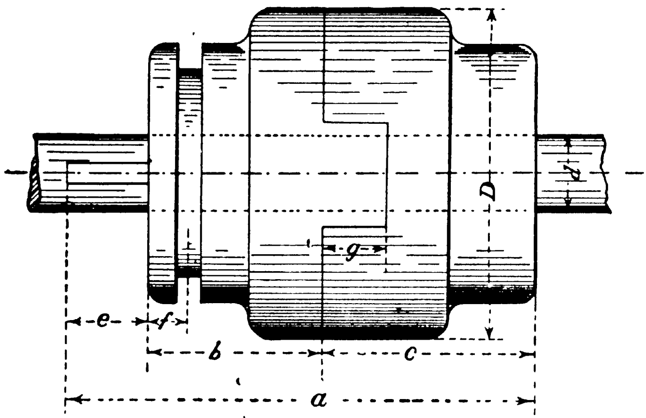


Fig. 198

The claws of one coupling fit a little loosely in the recesses of the other, so as to permit a small amount of play. This clutch requires to be held up to its work as it has a tendency to jar out of gear. Sometimes the planes of contact are inclined so much that the clutch has a strong tendency to slide out of gear. It is then held in gear by force, and the pressure can be so arranged that it automatically disengages if the shaft is overloaded. Ordinary proportions are as follows:—

$$D = 2{\cdot}6\,d + 0{\cdot}5$$
$$b = 1{\cdot}2\,d + 1{\cdot}45$$
$$c = 1{\cdot}4\,d + 1{\cdot}6$$
$$e = 0{\cdot}4\,d + 0{\cdot}85$$
$$f = 0{\cdot}16\,d + 0{\cdot}375$$
$$g = 0{\cdot}4\,d + 0{\cdot}5$$

*Hildebrand's coupling.*—In the ordinary clutch the whole straining action transmitted is resisted by a feather, on which one part of the clutch must slide somewhat loosely. This is not a very strong or satisfactory arrangement. In the coupling shown in fig. 199 two parts, A and B, are firmly keyed on their respective shafts. A third piece C, sliding on B, carries segments Z, which engage in recesses E in both A and B when the clutch is driving. The straining action therefore comes on fixed keys in

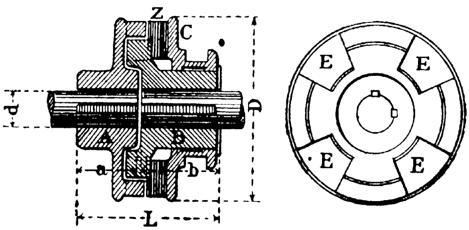



Fig. 199

both shafts, instead of a key on one shaft and a feather on the other. Empirical proportions are as follows:

$$\begin{aligned}\text{Length } \mathrm{L} &= 2{\cdot}4\,d + 4\tfrac{1}{2} \\ \text{Diameter } \mathrm{D} &= 3{\cdot}8\,d + 4 \\ a &= 1{\cdot}1\,d + 1{\cdot}8 \\ b &= 1{\cdot}26\,d + 2{\cdot}7\end{aligned}$$

188. *Friction couplings.*—The simplest form of friction coupling consists of a cone, keyed rigidly on one shaft, against which a movable cone sliding on a feather on the other shaft can be pressed (fig. 200). If the axial pressure is sufficient, the friction on the surface of the cones is greater than the resistance of the driven shaft. The movable part should be on the driven shaft so as to be at rest when the coupling is out of gear. The mean cone radius $r$ may be two or three times the shaft diameter, or more if the resistance of the driven shaft is considerable. The cones are sometimes faced with leather.

If M is the moment of the effort required to drive the shaft and 2 N is the total normal pressure between the cones of mean radius $r$, the coefficient of friction being $\mu$, then (fig. 201),

$$2\,\mathrm{N}\,\mu r = \text{or} > \mathrm{M}$$

$$2\,\mathrm{N} = \text{or} > \frac{\mathrm{M}}{r\,\mu}$$

But if P is the axial force pressing the cones together and $a$ the angle of the cones,

$$P = 2N\left(\sin\frac{a}{2} + \mu\cos\frac{a}{2}\right)$$

Consequently

$$P = \text{or} > \frac{M}{r\mu}\left(\sin\frac{a}{2} + \mu\cos\frac{a}{2}\right).$$

When once the coupling is in gear it will retain its position with a less axial pressure. Apart from vibration, which pro-

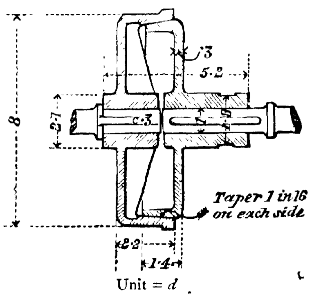



Fig. 200

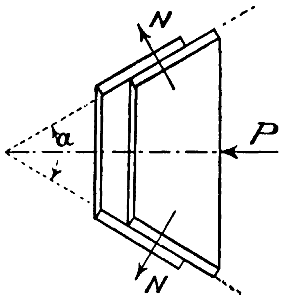



Fig. 201

bably has the same effect as diminishing $\mu$, the coupling would continue to drive until the axial pressure fell to

$$P' = \frac{M}{r\mu}\left(\sin\frac{a}{2} - \mu\cos\frac{a}{2}\right).$$

And if the angle of the cone is small enough it may require a reversed axial force to disengage it. For iron on iron $\mu$ may be taken 0·15, and for iron on leather 0·25. The angle at the vertex of the conical surface is usually $7\frac{1}{2}°$ to $12\frac{1}{2}°$.

It is of very great importance if friction cones are used on fast running shafts, that they should be able to adjust themselves so as to seat exactly in contact. Hence a Hooke's joint is introduced on one of the shafts, or some form of flexible coupling.

*Weston friction coupling.*—A simple form of friction coupling is shown in fig. 202, which can be used either as a shaft coupling or, as here shown, to couple at will a spur wheel to a shaft. The wheel $a$ has a long boss with two feathers on which are strung wrought-iron rings. Between these are wood rings, $b$,

carried by six feathers inside the coupling-box, D, which slides on a feather on the shaft, *c*. If the coupling box is pressed to the left there is friction at each face between the wood rings.

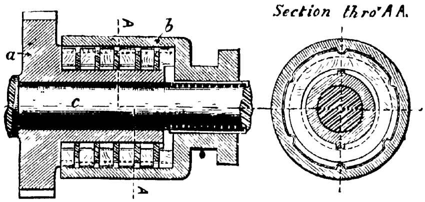
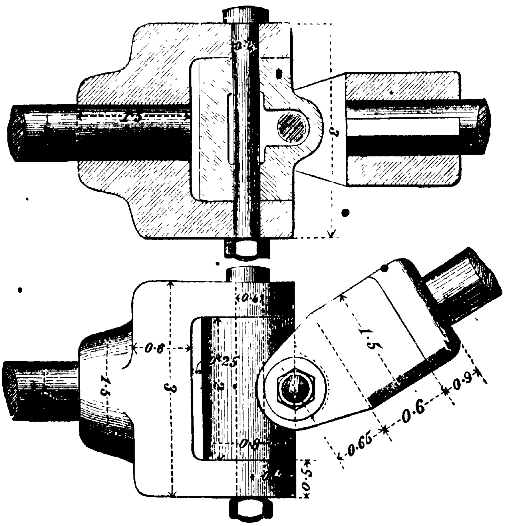



Fig. 202

and the iron rings. With $n$ iron rings there is friction at $2n + 1$ faces. There is therefore $2n+1$ times the friction there would be on a single face. The figure is taken from Towne on Cranes.




Unit $= d+1$

Fig. 203

He states that clutches of this kind have been used to transmit the power of a 1,000 H.P. steam engine to a train of rolls.

For most purposes both sets of rings may be of iron or steel.

If M is the moment transmitted, $r$ the mean radius of the

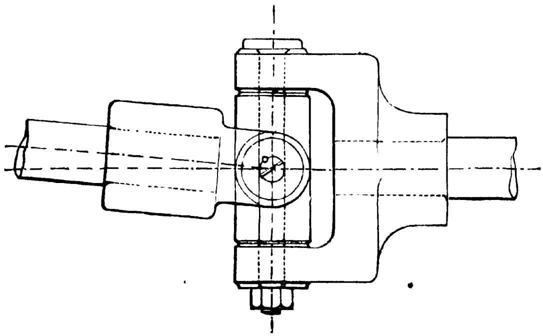


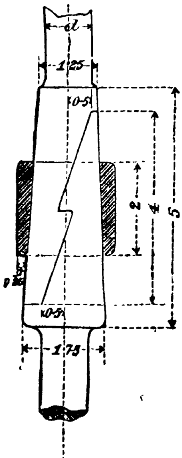
Fig. 204

rubbing surfaces, P the axial force pressing the rings together and $n$ the number of rings,

$$(2\,n + 1)\,\mu\,P\,r = \text{or} > M$$

$\mu$ has the values given above.

189. *Universal coupling.*—When the axes of two shafts which are not in line intersect, they may be connected by a Hooke's joint, or universal coupling, shown in fig. 203. The velocity ratio of the shafts is then variable, but if their directions make a small angle, the variation is not great, and is generally unimportant. In any case the angle between the shafts should not exceed 30°. The proportional unit for the dimensions is $d + \frac{1}{2}$ or $d + 1$.




Unit = $d$

Fig. 205

The form in fig. 203 is often used in rough machinery where uniformity of motion is not of great importance. The irregularity of motion is less with the form of Hooke's joint shown in fig. 204, in which the two axes of the cross are in one plane.

Let P R be the twisting moment transmitted by the shaft, and let R be measured to the middle of one of the

journals in the forks. Then the maximum pressure on the journal is

$$\tfrac{1}{2}\,(\mathrm{P} / \cos i)$$

where $i$ is the angle between the axes of the shafts. For this pressure its diameter is calculated.

If the driving shaft has a constant angular velocity $w_1$, the velocity of the driven shaft varies from $w_1 / \cos i$ to $w_1 \cos i$.

*Disengaging coupling for transmitting an axial thrust or pull.*—Fig. 205 shows a form of coupling often used where two links have to be connected so as to be easily disengaged. The ring shown in section holds the parts together when in gear and is kept in place by gravity.

# CHAPTER X

## BEARINGS FOR ROTATING PIECES

### PEDESTALS

190. The simplest form of journal bearing is a cylindrical hole in the frame supporting the rotating piece. Such a hole wears oval in the course of time, and does not admit of re-adjustment. The hole may be lined with a gunmetal sleeve or bush, or with soft metal; it can then be restored to its original condition by a new brass bush, or a new lining of soft metal, or by forced lubrication the wear can be very greatly reduced. What is in ordinary cases a better though still a very simple form of bearing is shown in fig. 206. In this the bearing is still in part formed in the frame of the machine, but it is in two parts, which are so arranged that the upper part can be tightened down on the journal by bolts, as it wears. The projections on the cap prevent any horizontal movement. The wear, being most often due to the weight of the pieces supported, takes place vertically. When this is not the case, the division of the bearing should be at right angles to the direction of the resultant pressure on the journal.

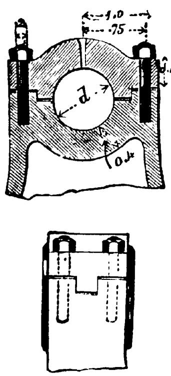
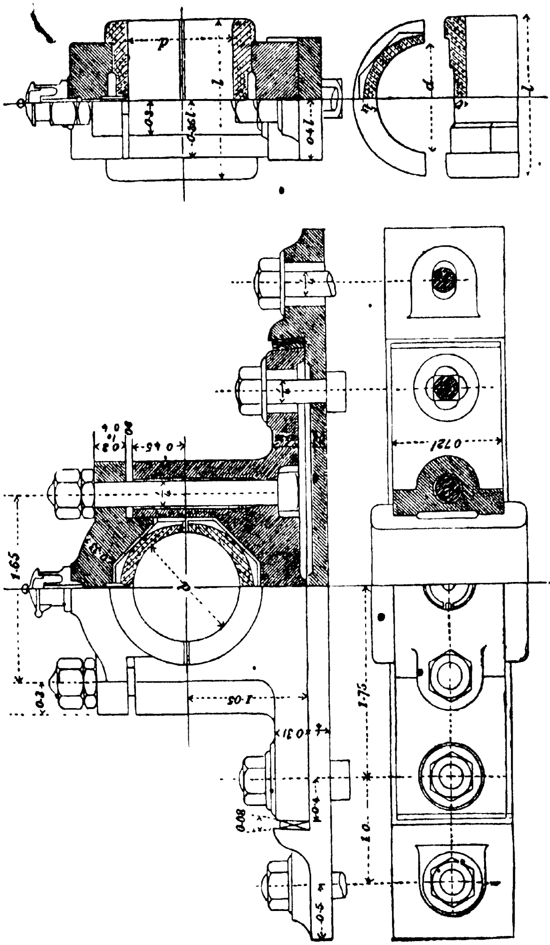


Fig. 206

By making the bearing separate from the framing of the machine, a means of initially adjusting its position is secured. Further, by lining the bearing with brasses or steps it is made practically independent of wear. The steps are divided so as to permit adjustment from time to time, and they can be removed and replaced by new ones when so much worn that their adjustment is no longer sufficient to keep the journal steady. When such a bearing instead of being fixed on the framing of a machine has

to be fixed on masonry or brickwork, a wall plate or foundation plate is commonly used.

Fig. 207 shows a typical pedestal, with the foundation, or




Unit $d+\frac{1}{4}$

Fig. 207

wall plate, on which it is fixed. This wall plate spreads the pressure of the pedestal over a larger area, and affords a levelled surface, on which the pedestal can be adjusted with less trouble

than on the rough masonry of a wall. The gunmetal steps are shown externally of octagonal form, the shape most convenient for hand fitting. They are often cylindrical, and are then turned in the lathe, and the pedestal is bored out to receive them. The steps have flanges, to prevent lateral movement. The under surface of the pedestal, and the upper surface of the wall plate have narrow chipping strips, to facilitate the adjustment of level. The bolt holes in the wall plate and pedestal base are oblong, so that the pedestal can be shifted laterally in either direction. When adjusted to its true position, it is fixed by hard wood or iron wedges, driven between the ends of the pedestal and jaws cast on the wall plate.

The diameter and length of the steps are the same as those of the journal. The other dimensions may be obtained from the proportional figures, the unit for which is $d + \frac{1}{2}$.

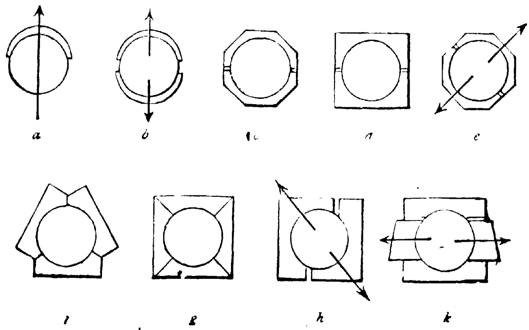


Fig. 208

191. *Brasses or steps.*—Fig. 208 shows some ordinary forms of steps for journal bearings. In many cases the pressure of a journal on its support acts always in one sense. In such cases a single step is sufficient, as at *a*, which represents the arrangement adopted for railway axles. The part of the journal not in contact with the step is then protected from dust by a shell cap. More commonly the pressure of a journal acts alternately in opposite senses, and then two steps are required as shown at *b*, *c*, *d*, *e*. The dividing plane of the steps is placed in the direction in which the wear is least, or at right angles to the direction of the resultant pressures. When the direction of the pressure varies, more complex adjustment is necessary, and three or

four separately adjustable steps may be used, as shown at *f* and *g*; *h* is an arrangement adopted where the pressure of the journal is inclined, but where the adjustment can be most conveniently made horizontally; *k* is an arrangement permitting horizontal adjustment. Sometimes brasses are fitted 'loose,' or with a gap permitting them to be adjusted by merely tightening the cap bolts. It is better to fit them 'brass and brass,' to prevent the possibility of screwing them so tight that they heat. They should be bored out slightly larger than the journal in order that there may be room for lubricant and to permit expansion due to change of temperature. Without such a small difference of dimension the journal would seize and continuous relative motion would be impossible. The Standards Committee have published a Table of such differences of dimension for running fits for three classes of workmanship.[1] They divide the whole difference into a necessary part termed the *allowance*, which must exist when the journal is largest and the bearing smallest; and into additional parts termed *tolerances* providing for inaccuracies of workmanship. The following short table gives a sample of these differences.

RUNNING FITS—INCHES

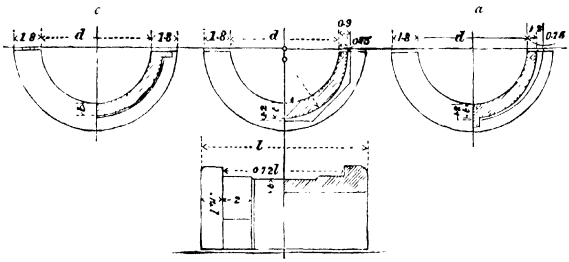
| Grade of workmanship | Diameter of journal | Journal tolerance | Allowance | Bearing tolerance |
|---|---|---|---|---|
| I. | 3 | ·0018 | ·0018 | ·0017 |
| ,, | 9 | ·0030 | ·0030 | ·0030 |
| II. | 3 | ·0035 | ·0035 | ·0035 |
| ,, | 9 | ·0060 | ·0060 | ·0060 |
| III. | 3 | ·0053 | ·0070 | ·0070 |
| ,, | 9 | ·0090 | ·0120 | ·0120 |

Fig. 209 shows sections of three ordinary forms of brass or gunmetal steps, and a half-plan, half-longitudinal, section of a step. The composition of the gunmetal, white brass and phosphor bronze, used for steps, is given in §§ 15, 16. When anti-friction metal is used, it is usually applied as a lining to a gunmetal step, and is cast in shallow recesses formed to

[1] Report on Standard Systems for Limit Gauges (Running Fits). No. 27.

receive it. The unit for the proportional figures is $t = 0{\cdot}08\,d + 0{\cdot}125$.

*Bearing metal* should usually be softer than the shaft, so that the wear occurs on it rather than on the shaft. It is easier to replace the bearing than the shaft. A soft bearing metal of 85 per cent. lead and 15 per cent. antimony may be



Unit $t = {\cdot}08\,d + \frac{1}{8}$

FIG. 209

used for low pressures and moderate speeds. Babbitt metal 85 to 89 per cent. tin, 2 to 5 per cent. copper, and 7 to 10 per cent. antimony is widely used. Some lead is often added, but lead is a bad conductor of heat, which is unfavourable. Prof. R. C. Carpenter found an alloy of 50 per cent. aluminium, 25 per cent. zinc, and 25 per cent. tin to have good properties as a bearing metal.

Fig. 210 shows two good methods of applying antifriction metal or white brass in pedestal steps. The shallow recesses in

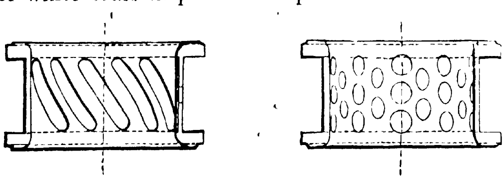


FIG. 210

which the white metal is placed are either helical shallow slots or circular shallow depressions. It appears that from some difference of expansion of the white metal and brass, channels for distributing the lubricant are formed when the white metal is applied in this way, and the bearing wears better than when a complete white metal lining is applied.

The thickness of the steps at the bottom, where the wear is greatest, may be

$$t = 0{\cdot}07\,d + \tfrac{1}{8} \text{ to } 0{\cdot}1\,d + \tfrac{1}{8}.$$

At the sides the thickness may be $\frac{3}{4}\,t$. The proportional unit for the dimensions of the steps is $t$.

*Table of Pedestal Proportions*

| Diameter of journal, in ins | Length of bearing, in ins. | Height to centre | Diameter of bolts | Size of bolt holes | Length of base | Centres of cap bolts | Centres of base bolts | Thickness of step at bottom |
|---|---|---|---|---|---|---|---|---|
| $1\frac{1}{2}$ | $2\frac{1}{2}$ | $2\frac{1}{8}$ | $\frac{1}{2}$ | $\frac{5}{8} \times 1$ | $8\frac{7}{8}$ | $3\frac{1}{2}$ | $7\frac{1}{4}$ | $\frac{1}{4}$ to $\frac{5}{16}$ |
| 2 | 3 | $2\frac{3}{4}$ | $\frac{5}{8}$ | $\frac{3}{4} \times 1\frac{1}{4}$ | 11 | $4\frac{3}{8}$ | 9 | $\frac{5}{16}$ to $\frac{3}{8}$ |
| $2\frac{1}{2}$ | $3\frac{1}{2}$ | $3\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{7}{8} \times 1\frac{1}{2}$ | $13\frac{1}{4}$ | $5\frac{1}{4}$ | $10\frac{7}{8}$ | $\frac{5}{16}$ to $\frac{7}{16}$ |
| 3 | 4 | $3\frac{3}{4}$ | $\frac{7}{8}$ | $1 \times 1\frac{5}{8}$ | $15\frac{1}{2}$ | $6\frac{1}{8}$ | $12\frac{5}{8}$ | $\frac{3}{8}$ to $\frac{1}{2}$ |
| $3\frac{1}{2}$ | $4\frac{1}{2}$ | $4\frac{5}{16}$ | 1 | $1\frac{1}{8} \times 1\frac{3}{4}$ | $17\frac{1}{2}$ | 7 | $14\frac{3}{8}$ | $\frac{3}{8}$ to $\frac{1}{2}$ |
| 4 | 5 | $4\frac{7}{8}$ | $1\frac{1}{8}$ | $1\frac{1}{4} \times 2$ | 20 | $7\frac{7}{8}$ | $16\frac{1}{4}$ | $\frac{7}{16}$ to $\frac{9}{16}$ |
| 5 | 6 | 6 | $1\frac{3}{8}$ | $1\frac{3}{8} \times 2\frac{1}{4}$ | 24 | $9\frac{5}{8}$ | $19\frac{7}{8}$ | $\frac{1}{2}$ to $\frac{3}{4}$ |
| 6 | 7 | 7 | $1\frac{5}{8}$ | $1\frac{7}{8} \times 2\frac{1}{2}$ | $28\frac{1}{2}$ | $11\frac{3}{8}$ | $23\frac{3}{8}$ | $\frac{9}{16}$ to $\frac{13}{16}$ |
| 7 | 8 | $8\frac{1}{8}$ | Two $1\frac{1}{4}$ | $1\frac{3}{8} \times 2\frac{1}{4}$ | — | $12\frac{1}{4}$ | — | $\frac{5}{8}$ to $\frac{7}{8}$ |
| 8 | 9 | $9\frac{1}{8}$ | ,, $1\frac{1}{2}$ | $1\frac{3}{4} \times 2\frac{1}{2}$ | — | 14 | — | $\frac{11}{16}$ to 1 |
| 9 | 10 | $10\frac{1}{4}$ | ,, $1\frac{5}{8}$ | $1\frac{7}{8} \times 2\frac{1}{2}$ | — | $15\frac{3}{4}$ | — | $\frac{3}{4}$ to 1 |
| 10 | 1 | $11\frac{1}{2}$ | ,, $1\frac{3}{4}$ | $2 \times 2\frac{3}{4}$ | — | $17\frac{1}{2}$ | — | $\frac{7}{8}$ to $1\frac{1}{8}$ |
| 12 | 13 | $13\frac{1}{2}$ | ,, $2\frac{1}{8}$ | $2\frac{3}{8} \times 3\frac{1}{8}$ | — | 21 | — | 1 to $1\frac{1}{8}$ |

From 7 ins. upwards, the pedestals have two bolts on each side, both in cap and base plate.

For a fairly strong pedestal with two bolts on each side of the journal the following proportions are suitable: Length of journal $2d$; height of centre above base, $1\frac{5}{8}\,d + \frac{3}{8}$; length of base plate $= 4{\cdot}4\,d + 0{\cdot}4$; breadth of base plate $= 1{\cdot}6\,d$; distance between bolts in base plate $3{\cdot}6\,d$; distance between bolts in base plate and cap, parallel to journal, $0{\cdot}8\,d + 0{\cdot}4$; diameter of bolts $0{\cdot}25\,d$.

Large steps when heated by the friction of the journal tend to grip the journal at the sides. They should therefore be eased so as to fit the journal a little loosely at the sides. At one or more bearings of a shaft, collars are used to prevent longitudinal motion, and the steps at these bearings sometimes give trouble from heating. Generally a very small amount of clearance between the collars and steps is sufficient. But in shafts driven by belting, where a small amount of longitudinal motion is unobjectionable, the clearance may be one-tenth of the length of

the journal. The longitudinal play tends to make the journal wear uniformly.

At times a pedestal requires to be contracted in dimensions. Fig. 211 shows a very neat and compact pedestal, designed by Mr. Arthur Rigg, C.E. The cap fits in a cylindrical recess in the body, which can be turned out in the lathe. The bolt-holes are bored out, and the recess for the steps also. The pedestal may be still further contracted, by making the cap bolts double-

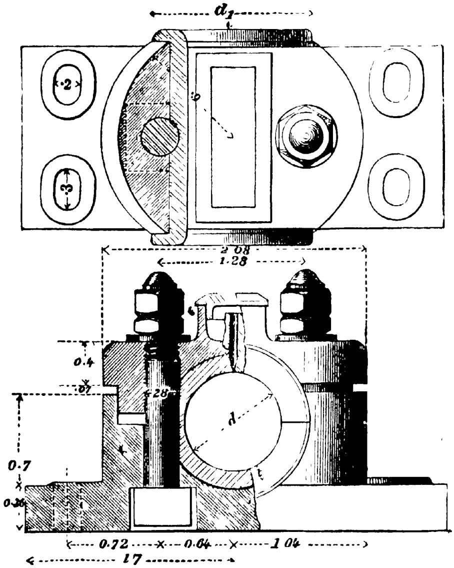



Fig. 211

ended, and using them both for attaching the cap to the body and the body to its support. The base is then absent.

192. *Long bearings for high-speed shafts.*—When a shaft runs at a high speed, the bearings must be long, to secure durability. The steps are then often of cast iron, which answers well as a support for wrought iron or steel if sufficient bearing surface is given. But the longer the bearings are the more important it becomes that they should be exactly concentric and in line. For long shafts it is then desirable to give the steps a spherical seat, so that they may, to some extent, adjust themselves to

the position of the shaft. In America, fast-running shafts, supported on cast-iron bearings four diameters long, have been

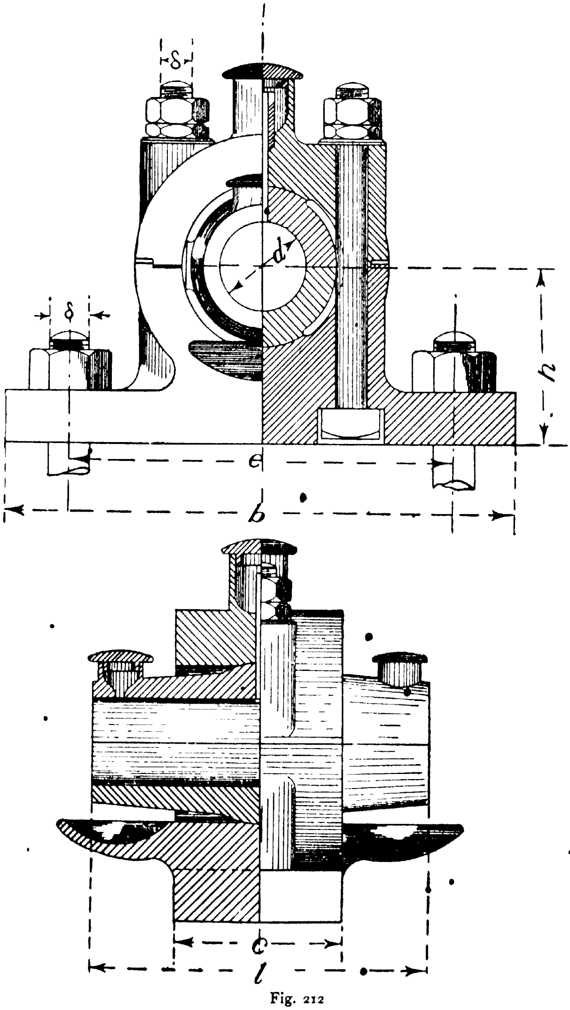



Fig. 212

extensively used; and for carrying these shafts Mr. Sellers has introduced the pedestal shown in fig. 212. The steps are

supported on the spherical parts, and can rotate slightly, either horizontally or vertically, adjusting themselves to the shaft. The following are suitable proportions :

$$\begin{aligned} l &= 4d \\ h &= 1\tfrac{1}{8}d + 1 \\ b &= 3{\cdot}6d + 3\tfrac{1}{4} \\ c &= 1{\cdot}6d + 1 \\ e &= 2{\cdot}9d + 2\tfrac{1}{4} \\ \delta &= \tfrac{1}{4}d + \tfrac{1}{4} \end{aligned}$$

For line shafts the pressure on bearing surface is limited to 50 lbs. or at most 100 lbs. per square inch when the steps are of cast iron. The ordinary lubrication is at the centre of the pedestal; in addition to this, two cup-shaped hollows are formed near the ends of the top step. These are filled with a mixture of tallow and oil, which is solid at ordinary temperatures, and melts at about 100° F. If the step heats from failure of the ordinary lubrication the tallow melts, and prevents injury to the shaft. A drip cup is provided under each end of the pedestal.

Fig. 213 shows a similar pedestal in which the height of the axis is adjustable by screws, the ends of which form the spherical surfaces on which the steps have a limited motion in all directions The proportions may be as follows :

$$\begin{aligned} l &= 4d \\ h &= 2d + 1\tfrac{3}{4} \\ a &= 4\tfrac{1}{4}d + 4 \\ b &= 1\tfrac{3}{8}d + 1\tfrac{3}{8} \\ c &= d + \tfrac{7}{8} \\ e &= 2\tfrac{3}{4}d + 1\tfrac{1}{2} \\ \delta &= \tfrac{1}{4}d + \tfrac{1}{4} \end{aligned}$$

193. *Self-lubricating pedestals.*—Many pedestals have been designed with oil reservoirs, which enable the pedestal to run six months without additional lubrication. Fig. 214 shows Möhler's pedestal. This has a lower brass only divided into two portions by a collar on the shaft. The lower part of the pedestal is hollow, and forms a reservoir, into which the collar dips. As the shaft revolves, the collar lifts the oil and distributes it to the shaft on either side. The surplus oil flows back into the reservoir. An objection to these pedestals is that they require a large supply of oil at first, which is liable to absorb oxygen

and to become viscid. However, in various forms the use of self-lubricating bearings of this type has extended. A fixed collar on the shaft prevents end motion, and it is replaced by

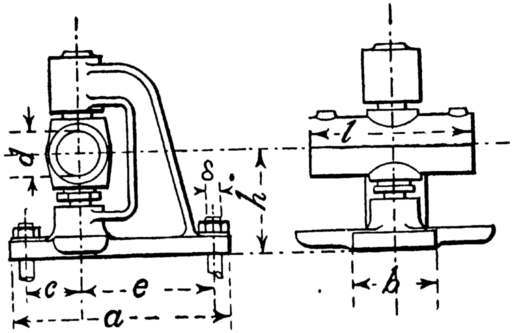



Fig. 213

a loose ring or chain, dipping into the oil in the reservoir, and which is carried round by the shaft as it rotates. At high speeds transfer of the oil to the shaft is hindered by the centrifugal

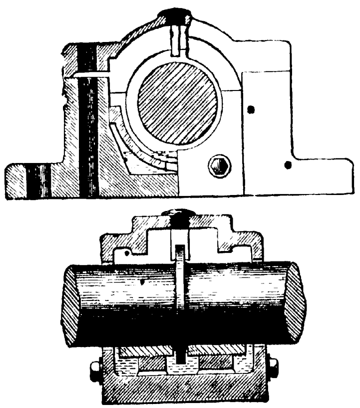


Fig. 214

force, and scrapers are added which divert the oil to the shaft. Felt rings are sometimes used at the ends of the bearing to prevent escape of oil from the reservoir.

194. *Lignum vitæ bearings.*—In a few cases where ordinary oil lubrication is difficult, lignum vitæ bearings are used. Thus the stern tube bearing of propeller shafts consists of strips of

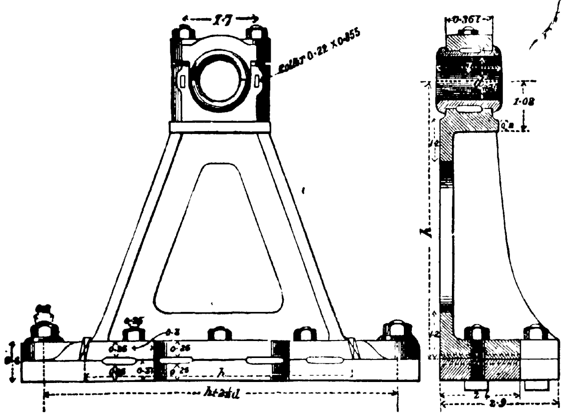



Fig. 215

lignum vitæ, 1 to 4 inches wide, placed parallel to the shaft. The strips are fixed in a cast-iron tube, into which they are dovetailed. They project $\frac{1}{4}$ to $\frac{3}{8}$ inch, and have spaces of about $\frac{1}{2}$ to 1 inch wide between them. Through these spaces the sea

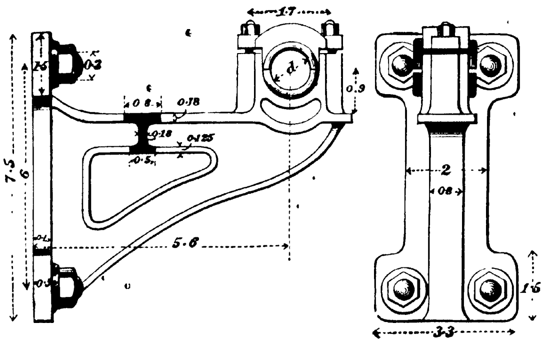



Fig. 216

water finds free access to the rubbing surfaces, acting as a lubricant and keeping the surfaces cool. The shaft is cased with gunmetal in order that there may be no corrosion of the rubbing

surface. The bearing varies in length from 2 feet for engines of 100 I.H.P. to 6 feet for engines of 1,000 I.H.P.

195. *Brackets and Hangers.*—When a pedestal requires to be elevated above its support, the form shown in fig. 215 is used. The proportions of the steps, cap and cap bolts, are the same as for an ordinary pedestal. The other dimensions are given on the figure, the proportional unit being, as before, $d + \frac{1}{2}$.

Sometimes a pedestal has to be fixed to a wall. Then the bracket pedestal shown in fig. 216 is used. The unit for the proportions is $d + \frac{1}{2}$. An ordinary pedestal may be used, fixed on a bracket such as that shown in fig. 217. The recess under

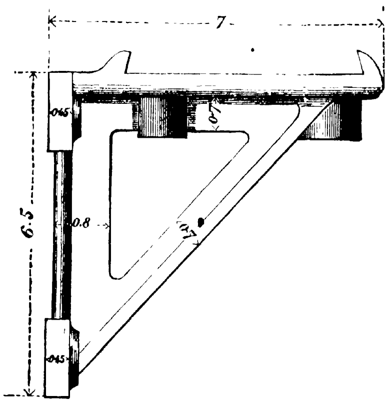



Fig. 217

the bearing in fig. 216 serves to receive a tin dish, which catches the oil drippings.

*Hangers.*—When a shaft is supported from the ceiling girders, the pedestal is modified in form, and is termed a hanger. Two forms are used : in fig. 218, the pedestal base is bent round and upwards, and attached to the ceiling on both sides ; in the other, the pedestal is supported on one side only. The latter arrangement facilitates the dismounting of the shaft, but requires more metal in the hanger.

Fig. 219 shows a hanger for two shafts, whose directions intersect. This happens when one shaft drives another by bevil gearing. The cap of the upper pedestal is kept in place by keys. The steps, bolts and caps of this hanger may be

designed as for ordinary pedestals. The proportional unit for the remainder of the pedestal is

$$d_2 + 0{\cdot}4\, d_1 + \tfrac{1}{2},$$

where $d_2$ is the diameter of the greater, and $d_1$ that of the smaller of the two shafts.

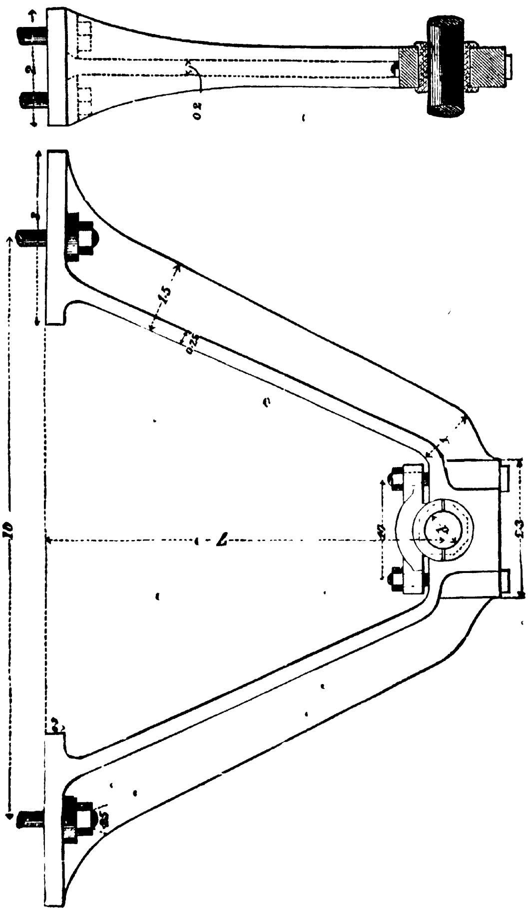


Fig 218

*Sellers' ball and socket hanger.*—Fig. 220 shows a form of hanger now largely used. The steps are of cast iron, with spherical

seatings as described in § 192. The steps rest in the plungers *d* and *e*, which are screwed into the frame of the hanger. These plungers permit the vertical adjustment of the axis of the bearing, and greatly facilitate the erection of a series of hangers in line. *a a* are boxes for solid grease, which melts if the oil

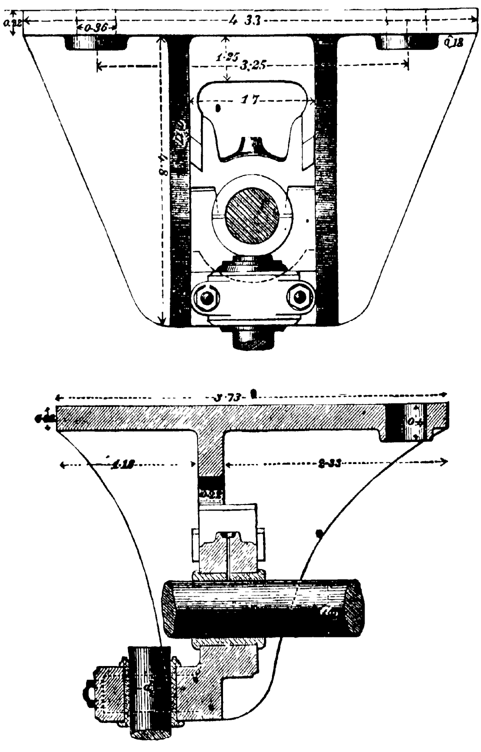


Fig. 219

supply fails. A pressure of 50 lbs. per sq. in. on the projected area of bearing is recommended as the limit for line shafting, the steps being of cast iron four diameters long. With the swivelling hanger longer steps can be used than are suitable with a rigid hanger, and the lubricant is less likely to be squeezed out.

196. *Weight of pedestals.*—The approximate weight of the cast iron in pedestals is given approximately by the following equation :—

$$w = 1 \cdot 1\, d^3 + 18 \text{ lbs.}$$

and the weight of a pair of steps is

$$w = 0 \cdot 23\, d^3 + 6 \text{ lbs.}$$

Fig. 221 shows an ordinary wall plate to carry a pedestal. The proportions may be as follows, the diameter of the shaft being $d$ :—

$$a = 5\,d + 8$$
$$b = 1 \cdot 5\,d + 1 \cdot 5 \text{ to } 2\,d + 2$$
$$g = 0 \cdot 35\,d + 0 \cdot 5$$
$$f = \tfrac{3}{8}\,d + \tfrac{3}{4}$$
$$e = 4\tfrac{1}{2}\,d + 5$$
$$\delta = 0 \cdot 2\,d + \tfrac{3}{8}$$

197. *Wall fixings.*—When a pedestal is fixed in a wall, a wall box is used. These wall boxes (fig. 222) not only give a firm and level support to the pedestal, but they carry the wall over the opening, and give a regular form to the opening.

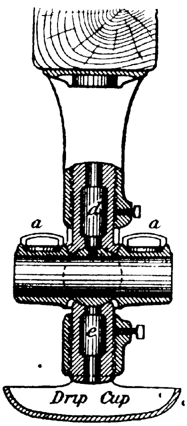


Fig. 220

When wall boxes are provided and built into the wall during its erection, they may have broad outside flanges on each face of the wall to prevent movement. They are also often conveniently built of four separate plates, instead of being in a single casting (fig. 223). All wall boxes giving a clear opening through a wall should be provided with internal flanges, to which wrought-iron plates can be bolted to form a fireproof barrier.

Figs. 223, 224, taken from Mr. Sutcliffe's paper,[1] are examples of a modern type of wall fixings, for carrying shafts of considerable size in well-arranged factories. In fig. 224 a small wall box is shown, which forms a support for a bracket fixing, on which a pedestal is to be placed. The wall box is built into

[1] Sutcliffe, *Proc. Inst. C.E.* vol. lviii.

the wall between two ashlar stones, which carry the pressure of the ironwork better than the brickwork of the wall. The face of the iron wall box is planed to receive the base plate of the bracket, and is provided with two vertical grooves which receive the T-shaped heads of the bolts which fix the bracket in place. Thus a very accurate vertical adjustment of the bracket can

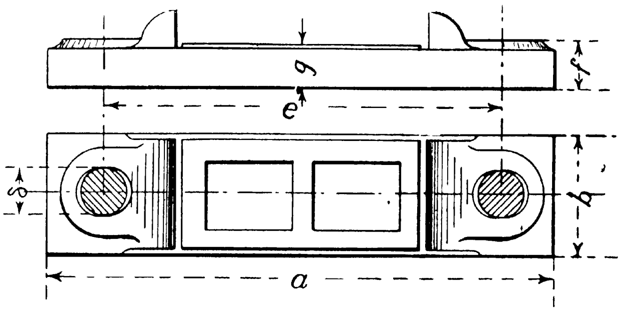



Fig 221

be effected. When this is done, the bracket is additionally secured by wedges or floats driven between the ends of the bracket and snugs on the wall box. The horizontal adjustment of the pedestal can be similarly effected, and thus the adjustment of the position of the shaft is completely provided for.

In most mills the engines drive a horizontal shaft, which gives motion to a vertical shaft passing up through all the floors of the mill, and in turn driving the shafting on each floor.

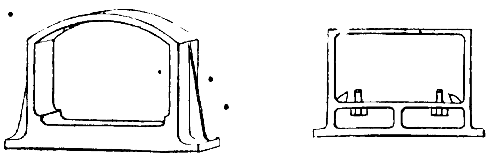


Fig. 222

The vertical shaft, therefore, carries the whole power of the engines, and any disarrangement of its action affects the whole factory. Great care, therefore, is taken in the fixing of this shaft. The shaft is now often placed in a separate walled tower with stages or floors at each pair of wheels. The footstep is carried on a massive plate, which rests on a masonry and brick pier

carried up from a solid foundation. Openings to the tower are closed by iron doors to prevent the communication of fire from floor to floor. Fig. 223 shows the arrangement of the fixing at the foot of such an upright shaft on a scale of $\frac{1}{8}$th of an inch to the foot. The drawing shows the horizontal shaft, with its

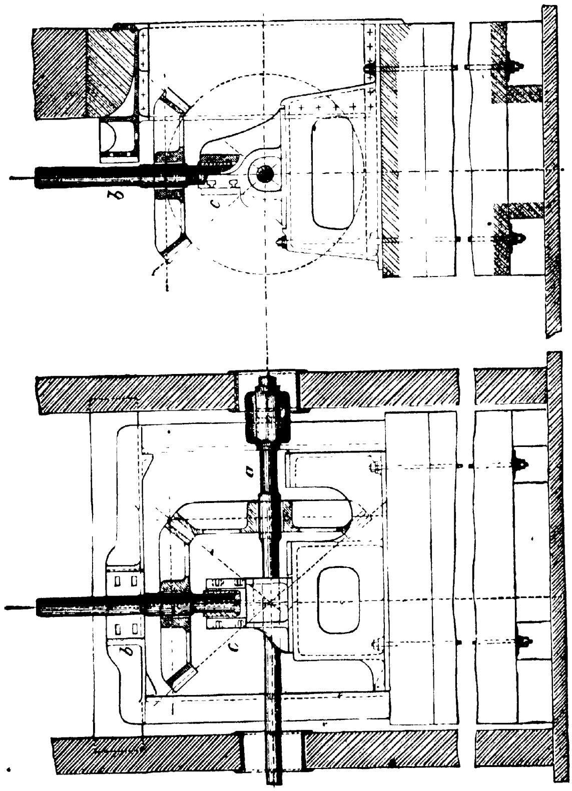


Fig. 223

coupling, and the vertical shaft with its pivot. At *a* is the position of the pedestal for the horizontal shaft; at *b*, that of the pedestal for the upright shaft. The bevil wheels work in the aperture formed by the wall box. The casting on which the footstep is fixed is partly built into the wall, partly fixed by long foundation bolts. The front plate of the footstep *c* is fixed

by T-headed bolts. When this is removed the footstep can be taken out.

## Axle Boxes

198. Axle-boxes are peculiarly formed journal-bearings, by which the weight of locomotive engines and railway carriages is transmitted to the axles. The axle-boxes of carriages are of

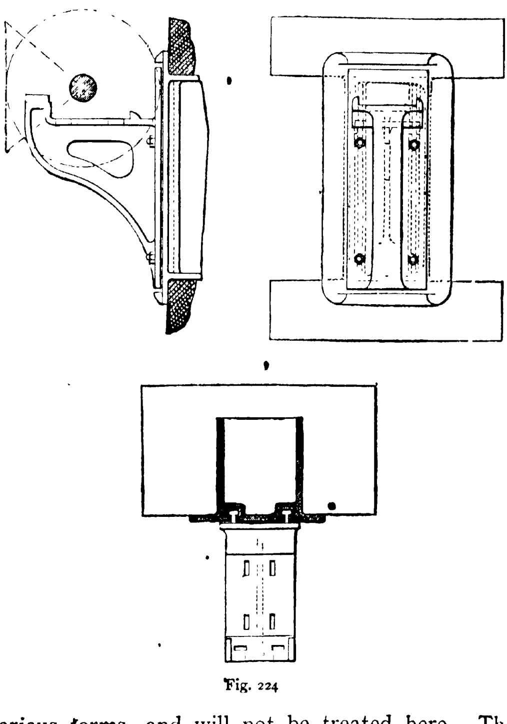


Fig. 224

very various forms, and will not be treated here. The axle-boxes of locomotive engines are more simple, and may be briefly considered, as illustrations of modified pedestals. In a good axle-box the lubrication should be constant, and not wasteful; the journal should be protected from grit; and should fit easily in the step, with a moderate amount of end play. Axle-boxes (figs. 225, 226) consist of an outer casing, a step of gunmetal or other alloy, and a hollow shell, closing the under side of the

box, and receiving the surplus oil. The outer casing is accurately faced on both sides, to fit the space prepared for it in the horn plates, between which it slides vertically, and it is provided with flanges, which permit a small amount of lateral play. The upper part is formed into an oil box, from which copper tubes conduct the oil to the journal It has also usually a socket, to receive the end of the spindle, through which the weight of the engine is transmitted to the box. This rod bears on a plate spring at its upper end. External axle-boxes are closed in front,

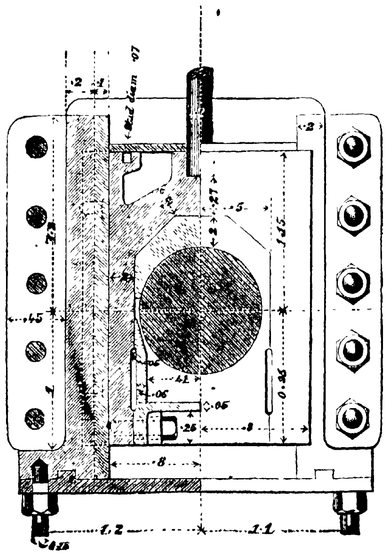


Fig 225.

and being of rather complex form, are of cast iron. Axle-boxes for internal framings are simpler, and are of forged wrought iron, or of cast steel. In the older axle-boxes, wedges were provided on each side, to compensate for wear, but these are now generally omitted. The step is like that of an ordinary pedestal, and is sometimes lined with soft metal.

Let $d$ be the diameter, $l$ the length of a journal. Then the product $d\ l$ is called the bearing surface of the journal. Let P be the load on the journal, then $P \div d\ l$ is the intensity of the pressure on the bearing surface.

The following table gives the dimensions of some actual axle journals, the ordinary speed and the pressure allowed per unit of bearing surface.

| Material of axle | | Size of journal | Revolutions per minute | Load per unit of bearing surface in lbs. |
|---|---|---|---|---|
| | | ins. ins. | | |
| Steel | Passenger carriage . | 6 × 3 | 240 to 360 | 340 |
| ,, | Goods waggon . . | 6 × $3\frac{1}{2}$ | 180 | 380 |
| ,, | Tender axle . . . | 7 × $3\frac{1}{2}$ | 240 to 360 | 330 |
| ,, | Locomotive axle . | 6 × $4\frac{1}{2}$ | 240 | 210 |
| Wrought iron | Goods engine axle . | 7 × $5\frac{1}{2}$ | 180 | 250 |
| ,, | Goods waggon . . | 6 × $3\frac{1}{2}$ | 180 | 380 |

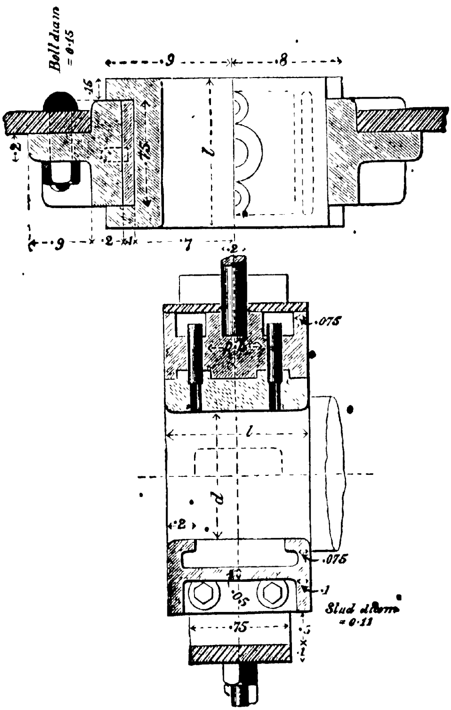



Fig. 226

The axle-box shown in figs. 225, 226, is a trailing axle-box of cast iron. Let D be the diameter of the cylinder. The bearing

surface on each side may be $0{\cdot}4\ \text{D}^2$. The thickness of the step is $\frac{d}{5}$ to $\frac{d}{6}$, where $d$ is the journal diameter. Lengthways, the step may be $\frac{1}{16}$th inch shorter than the journal, to allow a little end play. The unit for the proportional figures is $d + \frac{1}{2}$.

## Footstep Bearings

199. When a shaft is vertical, its lower end rests in a kind of pedestal, termed a footstep. The ordinary arrangement is shown in fig. 227. The end of the shaft is steeled or has a steel end welded to it. Lateral motion is prevented by a brass bush, fitting in a cast-iron fixing, and end movement, by a tough bronze or steel, slightly cup-shaped, disc, on which the shaft pivot revolves. The thickness of the brass may be $t = 0{\cdot}07\,d + \frac{1}{8}$. The unit for the other dimensions is $d + \frac{1}{2}$.

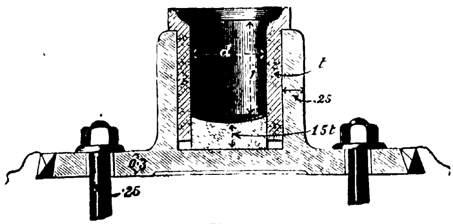



Fig. 227

In the case of important upright shafts of factories, the weight of the shaft is so great that the brass disc is liable to split. To prevent this, it may be hooped with a wrought-iron ring shrunk on. Fig. 223 shows a pivot footstep in position, and has already been described.

When exact vertical and lateral adjustment of the footstep is necessary, the arrangement in fig. 228 is adopted. The lateral adjustment is effected by four set screws, and the vertical adjustment by a single set screw. The horizontal screws are tapped into the casting, and fixed by lock nuts. The vertical screw has two nuts. Unit $d + \frac{1}{2}$.

When a footstep works under water, there is difficulty in insuring proper lubrication of the pivot. Fig. 229 shows a turbine pivot, enclosed in an oil casing, through which a flow of oil can be insured. Two small copper pipes are connected with

the casing, and these are conducted to points above the water level. The shaft passes into the casing through a stuffing box. The end of the shaft is provided with a steel disc, working on a similar disc fitted in the casing.

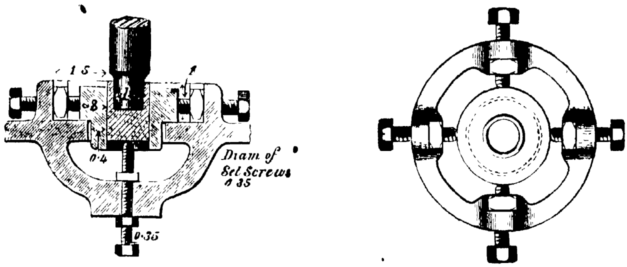



Fig. 228

Another arrangement, which is very effective, is to dispense with the ordinary metal pivot, and replace it with a pivot of lignum vitæ. For metal on wood, water is an excellent lubricant, and such bearings work with very little wear under great pressures. The pivot, fig. 230, is inverted, so that grit is less likely

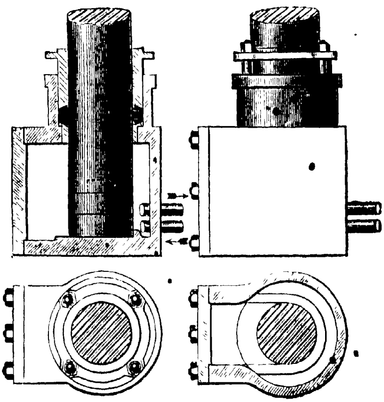


Fig 229

to enter. The pivot is adjusted vertically by a set screw. The end of the shaft is enlarged, bored out, and fitted with a brass step. A groove is cut round the pivot, which, being always filled with water, insures proper lubrication.

200. *Pivot bearing for suspended shaft.*—In the construction of turbines which have a vertical shaft, the difficulty of lubricating the pivot under water has led most Continental constructors to adopt the arrangement shown in fig. 231. A is a fixed wrought-iron pillar carrying at its top the footstep C, which is in any position above the water in which the turbine works convenient for access. D is the wrought-iron shaft carrying the power of the turbine which rests on the pivot, and which is coupled to a hollow cast-iron shaft B B surrounding the fixed pillar A, and carrying at its lower end the turbine wheel. The lower hollow shaft is, therefore, suspended on the pivot. The diameter of the upper wrought-iron shaft $d$ is calculated by the rules for wrought-iron shafting. The pillar has a diameter $d_3$ which is often the same as $d$; but it must be considered if this diameter is sufficient to carry the weight of the turbine and gearing when the pillar is treated as a long column virtually rounded at the ends (Case II, Table VIII, § 59). The inside diameter, $d_1$, of the hollow shaft must be sufficient to allow room for the brass sleeve which steadies the top of the pillar A. Usually $d_1 = 1·35\ d_3$. Lastly, $d_2$ is calculated so that the section of the hollow shaft is equivalent in torsional resistance to the solid shaft of diameter $d$. If the two shafts were of the same material we should have

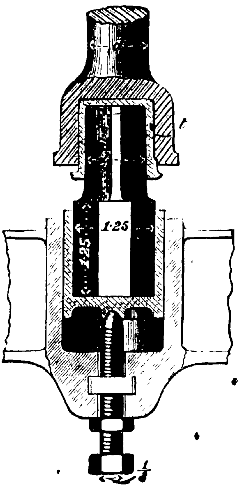


Fig. 230

$$\frac{d_2^4 - d_1^4}{d_2} = d^3.$$

Taking the shearing resistance of wrought iron to be three times as great as that of cast iron, we must have

$$3\,\frac{d_2^4 - d_1^4}{d_2} = d^3.$$

Generally $d_1$ is known. Then

$$d_2^4 - \tfrac{1}{3}\delta_2 d^3 - d_1^4 = 0,$$

an equation difficult to solve directly. Let $\delta_2$ be any approximation to the value of $d_2$. Then

$$\delta_2 - \frac{\delta_2^4 - \frac{1}{3}\delta_2 d^3 - d_1^4}{4\delta_2^3 - \frac{1}{3}d^3}$$

is a nearer approximation. By repeating the calculation with the new value a still nearer approximation is obtained.

Fig. 232 shows a suspension bearing used by the late Mr. Emile Geyelin in the United States for some very large vertical turbine shafts. The turbine shaft is fixed in the upper part A, and rests on the lower frame. This is dish-shaped to contain oil. The fixed bearing surfaces are glass discs B B cemented into recesses in the frame and ground to a true plane surface.

*Suspension bearing with forced lubrication.*—In some of the later turbines of large size, erected on the Continent, suspension bearings with forced lubrication have been used. Fig. 233 shows a bearing of this kind designed by Messrs. Ganz & Co., of Buda-Pesth. The weight of the shaft and turbine is transmitted through two nuts $a$ $a_1$ to the bell-shaped forging $b$, which turns with the shaft. This rests on a fixed gunmetal face $c$, supplied with oil under pressure by the oil inlet and distributing passages shown. The framing below the gunmetal step has passages for cooling water to be used in emergencies. The oil

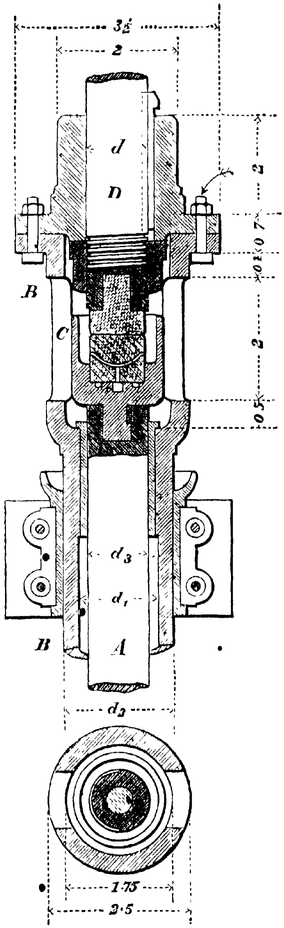


Fig 231

passing down the shaft or thrown off from the footstep bearing

finds its way to a collector *d*, from which it drains to a filter. Thence it is pumped back through the bearing. In the case of the Assling Sava turbines, where the total load on the suspension footstep is 17¼ tons, the pressure on the footstep is 230 lbs. per sq. in.; the shaft runs at 140 revolutions per minute, and the mean velocity of rubbing is 8·8 ft. per sec. (See 'Engineering,' lii. p. 307.)

*Pivots with loose discs.*—Sometimes loose washers or discs are used to distribute the pressure and wear, and if the lubrication of any one surface becomes temporarily imperfect the rubbing occurs only at the other surfaces. Fig. 234 shows an arrangement with loose washers. Here provision is made to

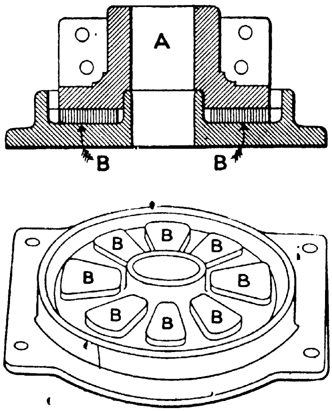



Fig. 232

introduce oil at the centre of the discs, whence it distributes itself over the rubbing surfaces by centrifugal action.

201. *Collar bearings.*—An axial thrust tending to displace a shaft longitudinally is resisted by some form of collar bearing as described in § 152. Fig. 235 shows an interesting collar bearing on the vertical shafts of the Niagara turbines, the bearing in this case being a suspension bearing, carrying part of the weight of the turbines and long vertical shaft. As originally designed, the gunmetal lines supporting the collars were four-parted. It proved, however, to be necessary to line them with Babbitt metal to prevent cutting. The figure shows a modification of the earlier form, designed by Dr. Coleman Sellers. The shaft is

held in alignment by the cylindrical surfaces at the bottom of the collars, not by the outside of the collars. The bearing consists of gunmetal blocks faced with segments lined with white metal, so made that when worn they can be detached, relined with white metal, and dressed to template in a lathe.

*Carboid bearings.*—Attempts have been made to produce a bearing which will run without lubrication for use in conditions where lubrication is difficult or likely to be neglected. A material termed fibre graphite has been used in the United States, consisting of plumbago and wood fibre pressed in moulds and saturated with drying oil. Mr. Killingworth Hedges has used a mixture of carbon and steatite. Experiments on a bearing of this kind with pressures of 15 to 170 lbs. per sq. in. of bearing and speeds up to 500 revolutions per minute showed that the coefficient of friction was very constant, but increasing with temperature and diminishing with increase of speed to a certain extent. The bearing suffered no injury even when running very hot, and never seized. The coefficient of friction is, however, high.

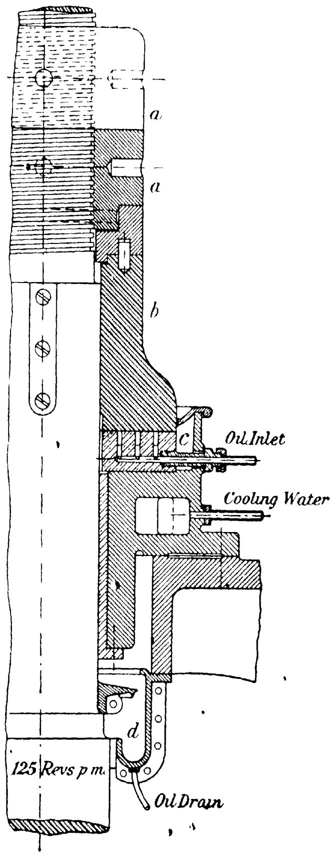



Fig. 233

## Roller and Ball Bearings

202. *Roller Bearings.*—With a perfectly formed journal and step, and perfect lubrication, a fluid layer of oil is formed between the journal and step, and the friction is reduced to the viscous resistance of the oil and is very small. It is not always possible to secure such favourable conditions. If the fluid layer is more or less discontinuous, there is sliding friction of the shaft on the step, and the work wasted and wear are much larger than they would be in the best conditions. Hence attempts have been made to substitute rolling for sliding friction by

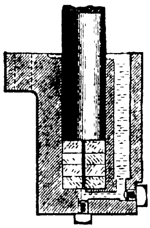


Fig. 234

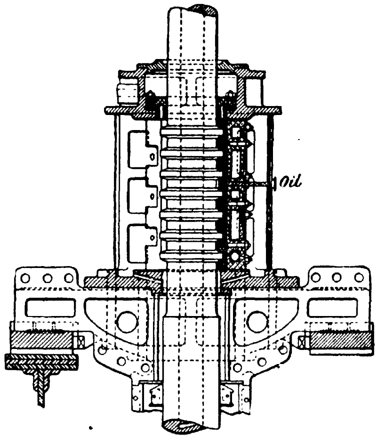



Fig. 235

introducing solid rollers between the journal and step. The earlier bearings of this kind were unsuccessful, probably because either the rollers or the seatings were too soft, and were excessively deformed by the pressure. Of late bearings of this type have been reintroduced and used with success, when heavy pressures have to be supported at slow speeds.

In the simplest type of roller bearing, free rollers are placed between the shaft S and bearing B (fig. 236). Then, although there is rolling contact with S and B, there is some sliding friction between the rollers themselves. Further, as the rollers cannot exactly fill the annulus between S and B, there is risk that the rollers may be not strictly parallel to the axis of the shaft, and

there is then a grinding action. Next, rollers with the ends turned down to form small journals carried in a pair of rings forming a cage have been used (fig. 237). This has not proved of much advantage. Lastly, rollers carried in a solid cast gunmetal cage (Mossberg system) (fig. 238) have been used. There is then, of course, some sliding friction between rollers and cage, but it does not seem to amount to much. Roller bearings of this kind have been used with advantage for rolling mills and to a limited extent for carriage axles. It does not appear that the frictional resistance in running is much less than that of

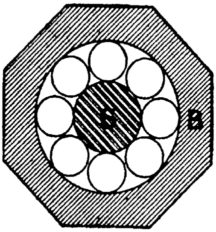



Fig. 236

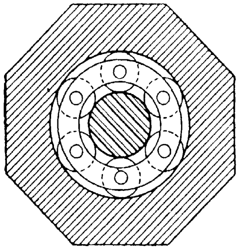


Fig. 237

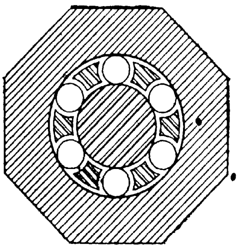


Fig. 238

good journals and steps, but the resistance when starting is much less. A very curious form of roller bearing is that known as the Meneely bearing (fig. 239). In this there are three sets of hollow rollers, the outer rollers having their centres half-way between the centres of the inner rollers. Segmental spaces are thus formed through which small solid rollers are introduced, which maintain the spacing of the rollers, and roll inside the larger rollers. Messrs. Kynoch have used hollow rollers ingeniously made by rolling up a steel plate so that there is a

spiral division along the roller (fig. 240). Such rollers have a certain amount of elasticity which helps to spread the pressure over the length of the roller in spite of small inaccuracies of form of rollers or seating.

Rollers should be of hardened steel on hardened seatings in cases where the speed of the shaft is not very slow. The modern roller bearing can be run at high speeds, and requires but little lubrication, but in this case ball bearings are more suitable.

In Mossberg bearings the length of the journal is usually one and a half times the diameter, and the bearing pressure $p = \text{P}/d\,l$ is 600 to 1,000 lbs. per sq. in. Hence,

$$d = \text{or} > 0{\cdot}033 \text{ to } 0{\cdot}026 \sqrt{\text{P}}$$

The diameter $\delta$ of the rollers is about

$$\delta = 0{\cdot}057\, d + 0{\cdot}1$$

and the number of rollers in a bearing is about $0{\cdot}8\, d + 18$. The rollers are of hardened spring steel; the journal of steel of medium spring temper; the box of high-carbon steel tempered as hard as possible. The cage is of gunmetal. If the shaft is of soft steel a hardened sleeve is fitted over it.

In the case of rollers for turntables of bridges, &c., at slow speeds of rotation, let $l$ be the length and $\delta$ the mean diameter of the rollers (usually conical). Then the safe load for one roller is

$$\text{P} = c\, l\, \delta.$$

$c = 350$ for hard cast-iron on hard cast-iron
850 for hard steel on steel or steel castings.

203. *Roller thrust bearing.*—The arrangement of a roller thrust bearing with conical rollers is shown in fig. 241. The vertices of the conical surfaces must be at the intersection of the axes of rotation A A and R R. The angle of the cones should be small to reduce the tendency to slide radially. From some data for Mossberg thrust bearings with hardened steel rollers on hardened steel races, it appears that the bearing pressure should decrease as the speed of rotation increases. Let $l$ be the length, $\delta$ the mean diameter, and $z$ the number of rollers. Then

$l \delta z$ is the bearing surface, and the greatest safe load on the bearing, at N revs. per min., is

$$P = 140{,}000 \frac{l \delta z}{N} \text{ lbs.}$$

204. *Ball Bearings.*—Ball bearings (fig. 242) are now used in many cases with great advantage, even for large loads and

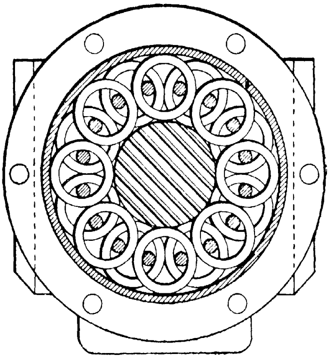


Fig. 239

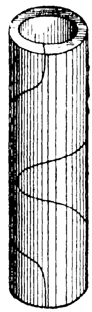


Fig. 240

high speeds. In the best of these the balls have two points of contact with the races, and the surface of the races is plane or concave. Professor Stribeck has shown that when the races

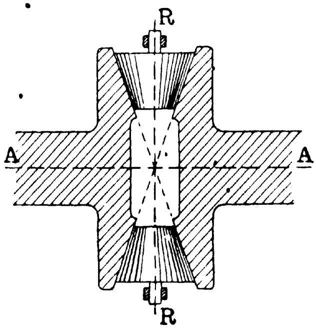



Fig. 241

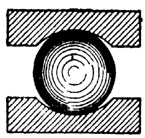


Fig. 242

are curved to a radius equal to two-thirds of the ball diameter the balls carry safely a greater load and there is less friction in running. The balls and races are of hardened steel, very great

uniformity in the size of the balls is essential, an accuracy to 1/10,000th inch is usual, and the balls and races should be polished and free from scratches. A groove is in some cases cut in one or both the races for introducing the balls. These bearings must be lubricated.

*Crushing and working load.*—Let $\delta$ be the diameter of a ball. Then when placed between two other balls the crushing load is 40,000 $\delta^2$ to 80,000 $\delta^2$, and when crushed between planes is greater. The working pressure must be much less, or continual deformation rapidly destroys the ball and races. From experiments by Stribeck and Bach the working load for each ball of diameter $\delta$ is $P = c\,\delta^2$.

| | $c =$ |
|---|---|
| Hard cast-iron balls on plane surfaces . . . | 36 |
| Hard steel balls on plane, conical, or cylindrical surfaces :— | |
| (*a*) intermittent running . . . . . . | 1400 |
| (*b*) continuous running . . . . . . | 700 to 1000 |
| Hard steel balls on races concave to radius $\frac{2}{3}\delta$ running continuously . . . . . . . | 1400 to 2000 |

For a ring of balls the pressure is unequally distributed to different balls. Let Q be the total load and $n$ the number of balls. Then, according to Stribeck, the greatest load on any one ball is $P = 5\,Q/n$. Hence the total safe load the bearing will carry is

$$Q = \frac{n\,c\,\delta^2}{5} \text{ lbs.}$$

There are usually 10 to 20 balls in a bearing and the equation is true within these limits.

205. *Friction of ball bearings.*—The friction of a ball bearing reckoned at the shaft surface is $\mu\,Q$. The coefficient $\mu$ for loads ranging from the greatest to half the permissible load is 0·0013 to 0·0017, and is constant over a wide range of speeds. For smaller loads it is somewhat greater.

*Example of a ball bearing for radial pressure.*—Fig. 243 shows an ordinary ball bearing for radial pressure. The inner race is a force fit on the shaft and is firmly gripped to the shaft by a nut. The outer race is free to slide in the casing or if fixed for the bearing at one end of the shaft should be free to slide at the other. In that case a small end-thrust will be resisted by the fixed bearing.

206. *Ball bearings with three or four points of contact.*—Various esigns of such bearings have been used. In most there is not

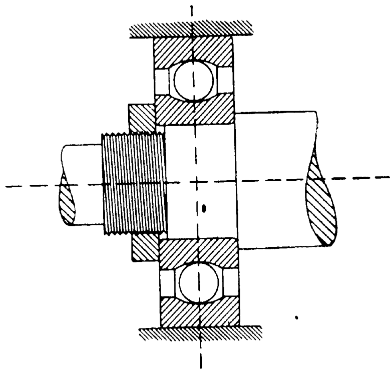


Fig. 243

pure rolling contact. The condition which should be fulfilled s that the lines joining the contact points on each race should

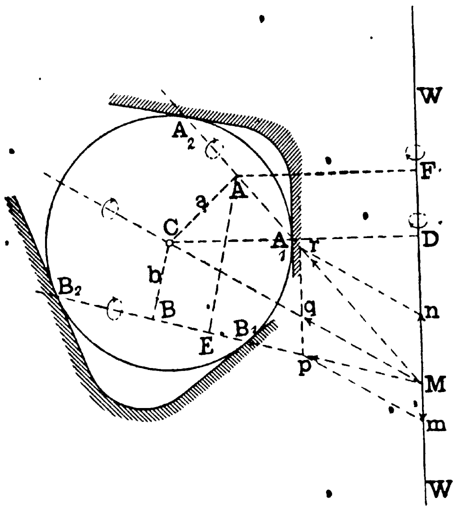



Fig. 244

meet in the same point M on the axis of rotation W W of the rotating race or be parallel to it.

Let fig. 244 represent a ball touching the two races at $A_1$, $A_2$, $B_1$, $B_2$, W W being the axis round which the centre of the ball C and the upper race revolve. Then the ball is rotating

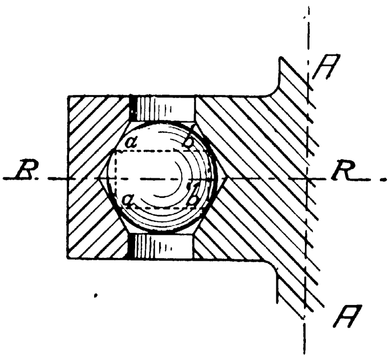



Fig. 245

relatively to the lower race about an instantaneous axis $B_1$ $B_2$ and relatively to the upper race about an instantaneous axis $A_1$ $A_2$. Draw C A, C B perpendicular to $A_1$ $A_2$, $B_1$ $B_2$; C D, A F perpendicular to W W, and A E perpendicular to $B_1$ $B_2$. That the

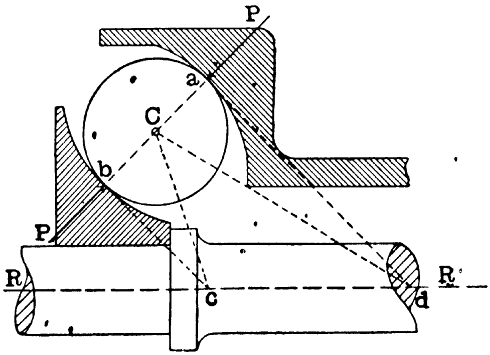



Fig. 246

motion may be as nearly as possible a rolling motion, $A_1$ $A_2$ and $B_1$ $B_2$ must meet in the same point, M on W W.

Let $\omega$ be the angular velocity of the upper race relatively to the lower race round W W; $\omega_0$ the angular velocity of the

ball centre C round W W; $\omega_a$, $\omega_b$, the angular velocities of the ball round the axes $A_1 A_2$, $B_1 B_2$ relatively to the races. Let C A $= a$; C B $= b$; A E $= e$; A F $=$ R; C D $= R_o$. The distance of the ball centre from $B_1 B_2$ is $b$ and of $B_1 B_2$ from A is $e$. The velocity of A is R$\omega$ and that of C, $R_o\ \omega_o$. Then

$$\omega_b = R\ \omega/e$$
$$= R_o\ \omega_o/b$$
$$\omega_o = \frac{R}{R_o}\frac{b}{e}\omega$$

If the upper race is considered at rest the ball centre moves relatively to it with the angular velocity $\omega_o - \omega$

$$\omega_a = -\ \{R_o(\omega - \omega_o)\}/a$$
$$= -\ \frac{R_o}{a}\left(1 - \frac{R}{R_o}\frac{b}{e}\right)\omega$$

Take M $m = \omega_o$; M $n = \omega - \omega_o$; and complete the parallelogram $m\ p\ r\ n$. M $p = \omega_b$; M $r = \omega_a$; M $q = \omega_o$.

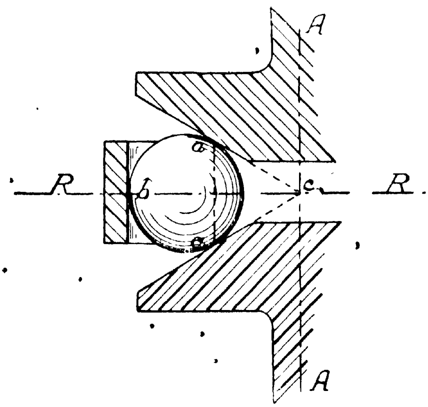



Fig. 246A

Fig. 245 shows a ball bearing with four-point contact for radial pressure. The lines $a\,a$, $b\,b$, joining the contact points are parallel to the axes of rotation A A.

207. *Ball bearing to carry a radial and end thrust.*—Ball bearings have been constructed as in fig. 246, where the lower race is fixed on the shaft and the upper rotates round the shaft

axis. As the pressure and reaction P, P, which are oblique to the axis R R, must be equal and opposite, the contact points *a b* must be at the ends of a diameter of the ball. Draw *a d b c* perpendicular to *a b*. Now a small surface of contact at *a* may be regarded as part of a cone having a vertex at *d* and axis *d* C and a similar contact surface at *b* as part of a cone having a vertex at *c* and an axis C *c*. For rolling of the ball at *a* its axis of rotation must be C *d* and for rolling at *b* it must be C *c*. As the motion of the ball cannot be a simple rotation about two different axes, there cannot be simple rolling at *a* and *b*. The

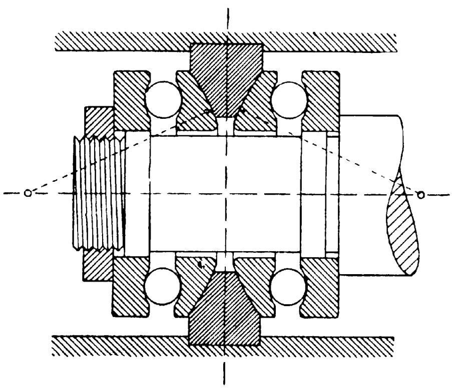


Fig 247

motion of the ball must be rolling at *a* or *b* or both, and a spin round *a b*. Hence there is prejudicial grinding at the points of contact.

208. *Ball thrust bearing.*—Fig. 246 A shows the simplest form of ball thrust bearing. The tangents to the ball at the contact points *a a* meet the axis of rotation of the shaft at *c*. A loose ring prevents the radial displacement of the balls.

Fig. 247 shows a ball bearing for thrust in either direction with two rings of balls. The seatings are concave. It is desirable that the inner seatings should have spherical contact surfaces with the fixed thrust ring so that they can adjust themselves to any slight defect of parallelism of the outer seatings.

## CHAPTER XI

### ROLLING CONTACT

### Friction and Wedge Gearing

Gearing is a general term for the means of transmitting motion, but it is especially employed to denote the wheels by which motion is transmitted from one shaft to another. The wheels employed for transmitting motion are almost always toothed wheels, but it is convenient to study first the action of toothless rollers, because each kind of toothed wheel is equivalent cinematically to a toothless roller.

In this and the following chapter, the notation and units employed will be as follows :—

$r$ = radius, $d$ = diameter of wheel in inches.
N = number of rotations per minute.
$p$ = pitch in inches. $h$ = height of tooth in inches.
$b$ = width of face of wheel in inches.
$t$ = thickness of tooth in inches.
T = number of teeth.
P = pressure of one wheel on another, measured in direction of motion or along tangent to pitch line, in lbs.
$n$P = load on one tooth in lbs.
$f$ = safe stress in lbs. per sq. in.
H = horse-power transmitted.
V = velocity of pitch line in ft. per sec.

209. *Rolling and sliding contact.*—Let two pieces rotating about fixed axes A, B, be in contact at P (fig. 248). Then, if the upper piece rotates counter clockwise, it will drive the other. Let T T be the common tangent and N N the common normal at P. Join A P, B P. The point P of the upper piece is moving in the direction P $a$ at right angles to A P, and the point P of the lower piece in the direction P $b$ at right angles to B P. Let P $a$ =

$v_1$, P $b = v_2$ be the velocities at the moment at the point of contact. If $\omega_1$, $\omega_2$ are the angular velocities of A and B, and $r_1 =$ A P, $r_2 =$ B P, then $v_1 = \omega_1 r_1$ and $v_2 = \omega_2 r_2$. In order that contact may continue, the normal components $b c$, $a d$ of $v_1$ and $v_2$ must be equal, otherwise one piece would be leaving or penetrating the other. In general the tangential components P $c$, P $d$ need not be equal, and if not $c d$ is the velocity of sliding at P.

Drop perpendiculars A $e$, B $f$ on the common normal N N. Then the triangles A $e$ P, $a d$ P are similar, and also the triangles

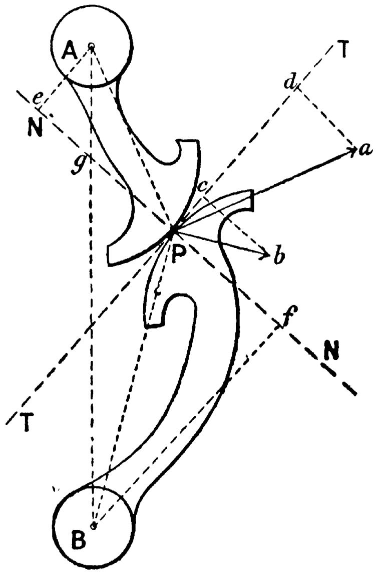



Fig. 248

B P $f$ and P $b c$. Then $\omega_1 = v_1/r_1 =$ P $a$/P A $= a d$/A $e$ ; $\omega_2 = v_2/r_2$ $=$ P $b$/P B $= b c$/B $f$. Since $a d = c b$,

$$\frac{\omega_1}{\omega_2} = \frac{\mathrm{B}f}{\mathrm{A}e} = \frac{\mathrm{B}g}{\mathrm{A}g}$$

That is, the angular velocities of two rotating pieces, one of which drives the other by contact, are inversely as the segments into which the common normal at the point of contact divides the line of centres. Let T P $a = \alpha$, T P $b = \beta$. Then $v_1 \sin \alpha = v_2$

sin $\beta$. The tangential components of the velocities are $v_1 \cos \alpha$, and $v_2 \cos \beta$, and the velocity of sliding is $v_1 \cos \alpha - v_2 \cos \beta$. As the pieces move, the point $g$ will trace out two curves, one on a plane rotating with A, the other on a plane rotating with B. These curves are termed pitch lines, and their point of contact at any moment on the line of centres is termed the pitch point. If the velocity ratio $\omega_1/\omega_2$ is constant, $g$ must be a fixed point on the line of centres and the pitch lines are circles with centres at A and B. This is the case with ordinary gearing. If the velocity ratio varies elliptic and spiral pitch lines are possible.

For rolling contact without sliding the tangential components $v_1 \cos \alpha$ and $v_2 \cos \beta$ must be equal, as well as $v_1 \sin \alpha$ and $v_2 \sin \beta$.

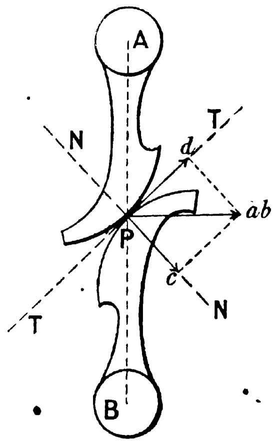


Fig. 249

Hence, $v_1 = v_2$, and they must have the same direction. This can only happen when, as shown in fig. 249, the point of contact is on the line of centres. Then if the surfaces are of suitable form they roll in contact.

Neglecting friction the pressure between the pieces must be along the normal N N. For positive driving the pressure of the driver A must act in the direction P $f$, fig. 248, on the side of the tangent T T on which the driven piece is. This involves that during contact the radius A P of the driver at the point of contact is increasing. In the case of rolling contact with constant velocity ratio, A P (fig. 249) is constant. Then driving is only possible if forces are introduced creating a frictional resistance to slipping. Hence altogether these cases arise:

(*a*) Rolling contact, $\omega_1/\omega_2$ constant, pitch lines circles. Friction gear.

(*b*) Sliding contact, $\omega_1/\omega_2$ constant, pitch lines circles Ordinary toothed gear.

(*c*) Sliding contact, $\omega_1/\omega_2$ variable. Gear with non-circular pitch lines.

## Communication of Motion by Rolling Contact
## Constant Velocity Ratio

210. *Parallel shafts.*—Let two accurately turned cylindrical rollers (fig. 250) be keyed on the shafts, of such a size that they

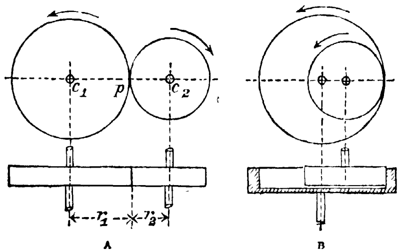



Fig 250

are in contact. Then, if one shaft revolves the other must revolve also, unless the resistance to motion, reckoned at the point of contact, is greater than the frictional resistance to slipping. The contact may be external contact, A, or internal

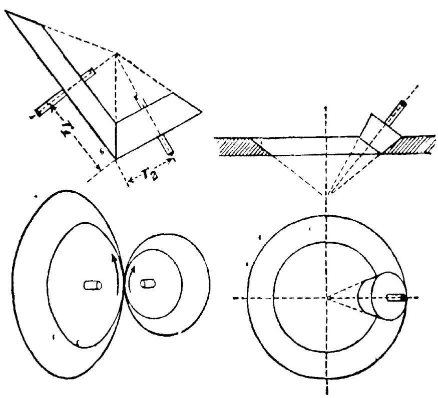


Fig. 251

contact, B. External contact is more common in ordinary cases. Let $N_1$, $N_2$, be the speed of the shafts in revs. per min., $r_1$, $r_2$, the radii, $d_1$, $d_2$, the diameters of the rollers. The velocities at the

point of contact are $2\pi r_1 N_1$ and $2\pi r_2 N_2$. Since these are equal, if there is no slipping,

$$\frac{N_1}{N_2} = \frac{r_2}{r_1} = \frac{d_2}{d_1} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the velocity ratio is constant. Toothed wheels corresponding to rollers of this kind are called spur wheels. The surfaces of the rollers are termed *pitch surfaces*. Planes normal to the shafts intersect these surfaces in circles termed *pitch lines*. The point $p$ is the *pitch point*.

*Shafts the directions of which intersect.*—If two conical rollers, having vertices at the intersection, are placed on the shafts

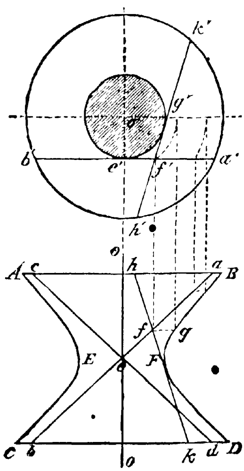



Fig. 251

(fig. 251), one shaft will drive the other with constant velocity ratio, as in the last case, the contact being external or internal. Toothed wheels corresponding to these rollers are termed bevil wheels. In practice the shafts are in most cases at right angles. The radius or diameter of bevil wheels is conventionally measured at the larger ends of the cones. Internal bevil wheels are hardly ever used on account of practical difficulties in constructing them.

211. *Shafts the direction of which are not parallel and do not intersect.*—Let $o\ o$ (fig. 252) be a vertical axis. Let $a\ b$ (in plan

$a'b'$) be a line inclined to the axis and not in the same plane. If $a\ b$ revolves round the axis $o\ o$, it will describe the hyperboloid of revolution A B C D, the edges of which in elevation are hyperbolas A E C and B F D having vertices at E and F. $c\ d$ is a projection in elevation of a second line, also represented in plan by $a'b'$, and equally inclined to the axis which by rotation would describe the same hyperboloid.

Any point such as $f'$ describes a circle $f'g'$ as $a'b'$ rotates round $o$. Project $f'$ to $f$, draw $f\,g$ horizontal, and project $g'$ to $g$. Then $g$ is a point on the outline of the solid. If $o'e'$ is the perpendicular from $o'$ on $a'b'$ it is the shortest distance between the axis and generator and is the radius of a circle called the gorge circle, and corresponds to the section of the solid through the vertices of the hyperbolas at E F. Also $a\ b$ and $c\ d$ are asymptotes to the hyperbolas A E C and B F D.

If through $f'$ a tangent $h'k'$ is drawn to the gorge circle, this is the plan of a generator passing through $f'$, which lies wholly in the surface of the hyperboloid. Its projection in elevation is $h\ k$.

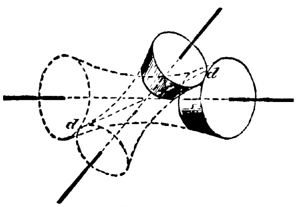


Fig. 253

Two hyperboloids (fig. 253) can be constructed by the rotation successively of the same line round two axes which are not parallel and do not intersect. These two hyperboloids touch always along the straight line which is the common position of the generator in its two successive rotations. A frustum of one of these hyperboloids will communicate motion to a frustum of the other, but not by pure rolling. There is always sliding in the direction of the common generator $d\ d$. Toothed wheels corresponding to these rollers are called skew bevil wheels.

Let figure 254 represent projections of two hyperboloids, in contact at the line $a\ b$. Let the upper figure be the projection on a plane parallel to the axes and the line of contact; the lower figure on a plane normal to $o\ o$, the left-hand figure on a plane normal to $p\ p$. Let $o_1\ p_1$ and $o_2\ p_2$ be the projections of the common normal to the surfaces at $b$. Then in the lower figure $o_1$ is the projection of the axis $o\ o$ and $p_1\ p_1$ the projection of the axis $p\ p$. In the left-hand figure $p_2$ is the projection of the axis $p\ p$ and $o_2\ o_2$ the projection of the axis $o\ o$. At the point

of contact of the surfaces at $a$, $c\,d$ perpendicular to $a\,b$ is the projection of the common normal. Projecting $d$ on the axis $p\,p$ to $d_1$, and $c$ on the axis $o\,o$ to $c_2$, then $d_1\,o_1$ and $c_2\,p_2$ will be projections of the normal $c\,d$. Projecting the point of contact $a$ to $o_1\,p_1$ and $c_2\,p_2$, $a_1$ and $a_2$ will be projections of $a$ and will be on the projections of the circles of rotation passing through $a$. Also $a_1\,b_1$ perpendicular to $o_1\,p_1$ and $a_2\,b_2$ perpendicular to $o_2\,p_2$ are projections of $a\,b$ and mark off the radii $o_1\,b_1$ and $p_2\,b_2$ of the gorge circles.

Let $b$ (fig. 255) be the point of contact at the gorge circles, then the point $b$ of the upper hyperboloid is moving along $b\,v$

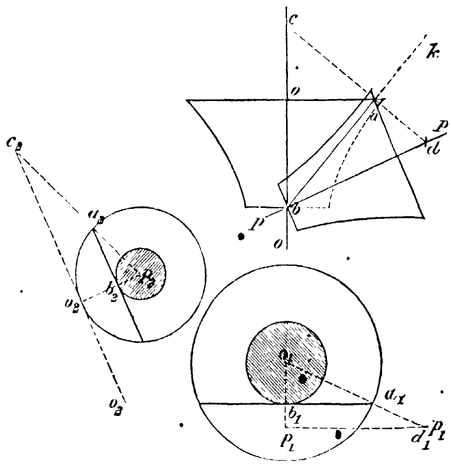


Fig. 254

perpendicular to $b\,p$ and the point $b$ of the under hyperboloid is moving along $b\,w$ perpendicular to $b\,o$. Let $b\,v$ and $b\,w$ be the velocities. Then the angular velocity of the upper hyperboloid is $\omega_1 = b\,v/b\,e$ and that of the lower hyperboloid is $\omega_2 = b\,w/b\,f$.

$$\frac{\omega_1}{\omega_2} = \frac{b\,v}{b\,w} \cdot \frac{b\,f}{b\,e}$$

If these hyperboloids have teeth in contact along $a\,b$, $b\,v$ and $b\,w$ must have a common component $b\,x$ at right angles to $a\,b$, the tangential velocities parallel to $a\,b$ being $v\,x$, $w\,x$. But $b\,v\,w$ is similar to $b\,d\,c$. Hence $b\,v/b\,w = b\,d/b\,c$. Comparing

the previous figure with this, $b f / b e = o_1, b_1 / b_1 p_1 = o_1 a_1 / a_1 d_1 = c a / a d$. Substituting these values,

$$\frac{\omega_1}{\omega_2} = \frac{b d}{b c} \cdot \frac{c a}{a d} = \frac{b d}{a d} \cdot \frac{c a}{b c} = \frac{b a}{a p} \cdot \frac{o a}{a b} = \frac{o a}{a p}$$

consequently the velocity ratio of skew bevil wheels is inversely proportional to the projections of their radii at the point of contact on a plane parallel to both axes.

Suppose the directions of two axes given, their perpendicular distance apart, and their required velocity ratio. Take

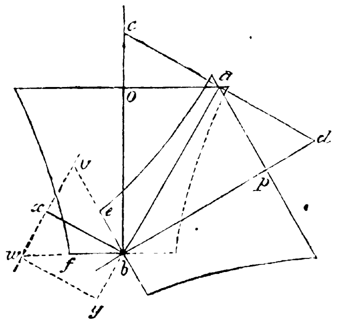



Fig. 255

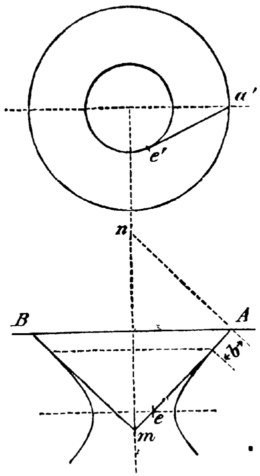



Fig. 256

any two distances in that ratio, and setting them off from the axes draw lines parallel to the axes intersecting in $k$, fig. 254. A line from $b$ through $k$ is then the line of contact, and the construction of the hyperboloids by the methods given above is easy.

When hyperboloidal or skew bevil wheels have a comparatively narrow width of face, $b$, fig. 256, a conical surface, tangent to the hyperboloidal surface, may be substituted as a pitch surface for the curved surface. Draw $a' e'$ tangent to the gorge circle. Then $a' e'$ is a plan of one position of the generator whose elevation is found by projecting $e'$ to $e$ and joining A $e$. Produce A $e$ to $m$. Then $m$ is the apex of a conical surface B $m$ A tangent

to the hyperboloidal surface in the circle A B. If A $n$ is drawn perpendicular to A $m$, $n$ is the apex of the normal cone or cone which cuts the surface B $m$ A normally in the circle A B.

### Velocity Ratio Variable

212. Non-circular rollers may be used to transmit motion with a variable velocity ratio.

*Lobed wheels.*—By taking alternately reversed logarithmic spirals, lobed wheels are constructed capable of continuous rotation and having a series of maximum and minimum velocity ratios, fig. 257.

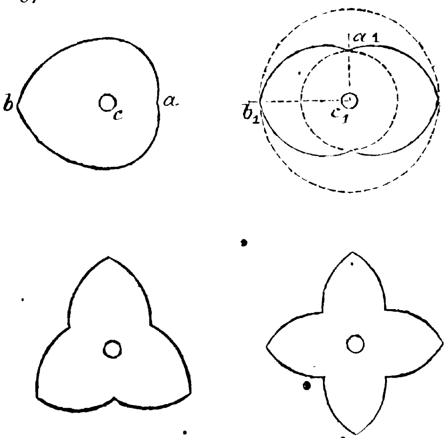



Fig. 257

Equal unilobes or equal multilobes will work together. But also a unilobe will work with a bilobe or trilobe.

213. *Elliptical wheels.*—Let two equal and similarly placed ellipses (fig. 258) be centred about two foci $c_1$ $c_2$, $d_1$ $d_2$ being the other foci, and the point of contact $p$. Let T T′ be the common tangent at $p$. Then T T′ makes equal angles with lines drawn from $p$ to the foci. Hence $d_1\, p$ T $= d_2\, p$ T′ and $c_1\, p$ T′ $= c_2\, p$ T, and $c_1\, p$ and $p\, c_2$, and also $d_1\, p$ and $p\, d_2$, must be in the same straight line. Also $d_1\, p + p\, c_1 = c_2\, p + p\, d_2$; as the ellipses are equal, $= c_1\, p + p\, c_2 = p\, d_1 + p\, d_2$. Hence in rolling the distance $c_1\, c_2$ remains constant, and the distance $d_1\, d_2$ between the free foci remains constant also. Also $c_1\, p = p\, d_2$, and the arcs

measured from the ends of the major axis, $e\,p, f\,p$, which roll in contact, are equal also.

If we put $c = c_1\,c_2$ and $a$ and $b$ for the major and minor semi-axes of the ellipse, then the radii from the centres of rotation to the point of contact vary from

$$r_{max} = a + \frac{c}{2} \text{ to } r_{min} = a - \frac{c}{2}.$$

Hence the velocity ratio varies from

$$\left(a + \frac{c}{2}\right) \Big/ \left(a - \frac{c}{2}\right) \text{ to, } \left(a - \frac{c}{2}\right) \Big/ \left(a + \frac{c}{2}\right).$$

The ratio of the greatest and least velocity ratios is

$$\rho = \left(\frac{2\,a + c}{2\,a - c}\right)^2$$

$$c = 2\sqrt{(a^2 - b^2)}$$

Consequently

$$\frac{b}{a} = \frac{2\sqrt[4]{\rho}}{1 + \sqrt{\rho}}.$$

An equation which gives the ratio of the semi-major and minor axes for a given range of velocity ratio.

For a fuller account of wheels giving a varying velocity ratio, the reader should consult Maccord, 'Kinematics' (New York, 1883).

## Friction Gearing

214. If two wheels of any of the forms described are pressed together by a force acting normally to the surfaces at the line of contact, there is a frictional resistance to the slipping of one wheel on the other.

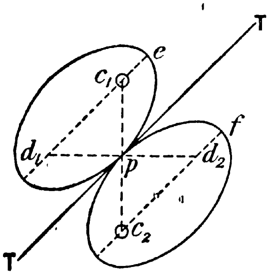


Fig. 258

Hence, if one wheel is rotated, the other will rotate also, provided the resistance to motion, measured at the pitch surfaces, is less than the frictional resistance to slipping. With toothless rollers it is difficult, however, to transmit much force without causing a slipping of the rollers, and in consequence noise and wear.

**Fig.** 259 shows simple friction gearing of this kind with circular pitch lines. The wheels at A are for parallel shafts, those at B for shafts at right angles. In both these cases, if the arrangements are perfect there is simple rolling contact of the wheels. Such wheels may be used (*a*) when the power to be transmitted is not very great; (*b*) when the speed is so high that toothed wheels would be noisy; (*c*) when the shafts require to be frequently put into or out of gear. At C, fig. 259, is shown another form of friction gearing, which has been used with a different object. Here the wheels take the form of discs, and the shafts are at right angles. By moving the smaller wheel towards or away from the axis of the larger wheel the velocity ratio is varied, and that while the gear is in motion and without any abrupt change. As, however, the smaller wheel must have

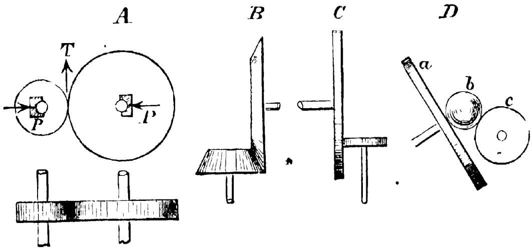



Fig. 259

a sensible thickness, its edge is in contact with parts of the larger disc having different velocities. Consequently there must be sliding and wear of the surfaces in contact. To avoid this, the arrangement shown at D has been invented by Prof. James Thomson. *a* is a disc, *b* a heavy metal sphere, and *c* a cylinder. The disc rotates the sphere by friction at the point of contact, and this in turn communicates motion to the cylinder. The contact is simple rolling contact. By moving the sphere across the face of the disc, the velocity ratio is altered. Except at very low velocities the inertia of the ball gives rise to trouble.

Let P be the pressure acting between two friction wheels normally to the surfaces at the line of contact; $\mu$ the co-efficient of friction; T the tangential resistance to motion of the driven wheel, at the pitch surface. Then

$$\mu P = \text{or} > T \text{ or } P = \text{or} > \frac{T}{\mu}$$

Let V be the velocity of the pitch surfaces in feet per second, H the horse-power transmitted. Then

$$T = \frac{550\ H}{V} \text{ and } P = \text{or} > \frac{550\ H}{\mu\ V}.$$

This gives the magnitude of the external force which must be applied to prevent slipping. The ordinary values of $\mu$ for dry surfaces are

| | |
|---|---|
| For metal on metal . . . . | 0·15 to 0·20 |
| ,, wood on metal . . . . | 0·25 to 0·30 |
| ,, millboard on metal . . . | 0·20. |

In the case of bevil wheels, B, fig. 259, the pressures acting on each wheel may be resolved into a force Q parallel and a

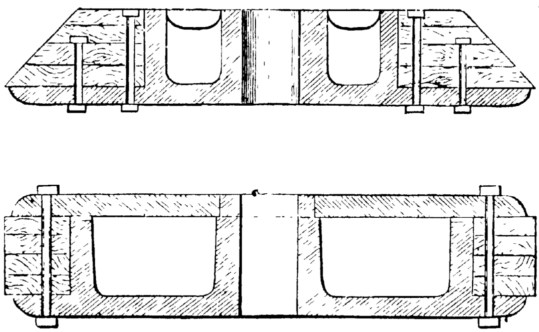


Fig. 260

force N normal to the shaft. Putting $N_1$ $Q_1$ for the forces acting on one wheel, and $N_2$ $Q_2$ for those acting on the other; P for the normal pressure at the pitch surfaces of the wheels; $r_1$, $r_2$, for the radii of the wheels; and $\delta$ for the angle the tangent of which is $r_1/r_2$; we get

$$Q_1 = P \sin \delta = N_2$$
$$Q_2 = P \cos \delta = N_1$$

in which equations, P has the value given above, and the friction of the supports of the shafts is neglected. $Q_1$, $Q_2$, are the external forces which must be applied along the shafts to prevent the slipping of the wheels.

Friction wheels may be of metal, but generally one of the pair has a surface of wood, of millboard, or of leather, to secure greater resistance to slipping.

Fig. 260 shows the construction of bevil and spur wheels with wood faces; and millboard may be also used in the same way. It is best to make the wheel with wood or millboard face the driver, as it is then less liable to wear irregularly if slipping occurs. The tangential force which can conveniently be transmitted is about 30 lbs. per inch width of face for maple wood and about 15 to 20 lbs. for pine wood.

Since a very slight movement puts friction wheels out of gear, they are convenient when rapid disconnecting is necessary.

215. *Experiments on the power transmitted by friction gear.*— Prof. W. F. M. Goss has made some experiments on the power transmitted by friction gear. In a first memoir (Trans. Am. Soc. Mech. Eng., xviii. 102) on millboard driving and cast iron driven wheels similar to A, fig. 259, he found that the coefficient of friction depended directly on the amount of relative slip of the two wheels. With 1 per cent. slip the coefficient was 0·14; with $1\frac{1}{2}$ per cent., 0·18; with 2 per cent., 0·20, and it did not much increase above this value. With more than 3 per cent. the action of the wheels became uncertain. The coefficient of friction appeared to be independent of the pressure between the surfaces in contact, up to pressures of 150 or 200 lbs. per inch width of face. Variations of peripheral speed between 400 and 2,800 ft. per min. did not affect the value of the coefficient of friction. Further experiments described in a later memoir (Proc. Am. Soc. Mech. Eng., xxix. 35) generally confirm these results. The driving wheels were of millboard, leather fibre, tarred fibre, leather and sulphite fibre. The driven wheels were of cast iron and other metals. Taking the results with 2 per cent. slip as representing normal conditions the following were the coefficients of friction on cast iron. Also the limit of pressure between the surfaces carried without failing for 15,000 revolutions:

| Driving wheel | Coefficient of friction | Pressure lbs. per inch width of face |
|---|---|---|
| Millboard | 0·41 | 750 |
| Leather fibre | 0·51 | 1,200 |
| Tarred fibre | 0·24 | 1,200 |
| Leather | 0·22 | 750 |
| Sulphite fibre | 0·55 | 700 |

From these results Prof. Goss selects the following for the

working values of the coefficient of friction, and safe working pressure P, per inch width, between the faces.

| Driving wheel | $\mu$ | P |
|---|---|---|
| Millboard | ·255 | 150 |
| Leather fibre | ·309 | 240 |
| Tarred fibre | ·150 | 240 |
| Leather | ·135 | 150 |
| Sulphite fibre | ·330 | 140 |

The energy transmitted at a peripheral velocity $v$ in feet per second is $\mu$ P $v$ foot pounds per second or $(\mu \text{ P } v)/550$ horse-power, per inch width of face of wheel.

216. *Wedge gearing* (fig. 261) is a modification of friction gearing intended to secure a given resistance to slipping with less pressure at the pitch surfaces, and therefore with less wear of the supports of the wheels than ordinary friction gear. The circumferences of the wheels are cut into wedge-shaped projections by circumferential grooves. Let N be the normal pressure on all the wedge surfaces in contact, P the force pressing the wheels together, T the tangential force transmitted—

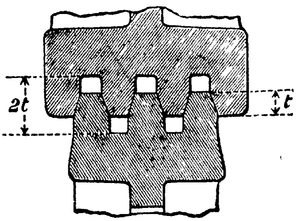



Fig. 261

$$\text{P} = \text{N} (\sin \alpha + \mu \cos \alpha)$$

where $2\,\alpha$ is the inclination of the sides of the wedges

$$\mu \text{ N} = \text{or} > \text{T}$$

$$\text{P} = \text{or} > \frac{\text{T}}{\mu} (\sin \alpha + \mu \cos \alpha)$$

The objection to these wheels is that the contact is sliding contact, and therefore the wheels grind, and in some cases are noisy. To diminish this evil, the depth of the surface in contact should be made small. The inclination of the sides of the wedge projections is usually 30° to 40°. The number of projections on each wheel is usually 1 to 6, but sometimes a greater number are used. The number of projections has no influence on the power transmitted, but only on the durability. The depth $t$ of the acting surface may be taken about

$$t = 0{\cdot}025 \sqrt{\text{T}}.$$

The rim and arms may be of the same strength as those of a spur wheel transmitting the same power.

An interesting record of experience with large wedge friction wheels transmitting a circumferential effort of nearly 4,000 lbs. will be found in 'Proc. Inst. Mechanical Engineers,' 1888, p. 363. The most serious evil was found to be a tendency to wear at the soft spots, leading to slipping.

Some extremely ingenious arrangements of friction gear were invented by the late Prof. Fleeming Jenkin and termed *nest gearing*. (See 'Report of the British Association,' 1883, p. 387.) Practical experience, however, seems to have shown that in all these forms of gear, where motion is communicated by frictional contact at a line or point of contact, slipping occurs

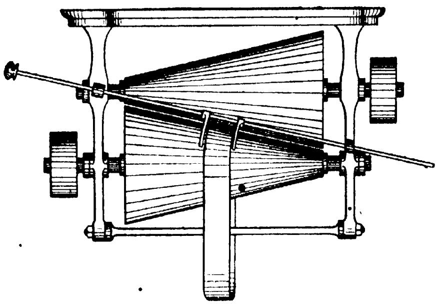


Fig. 262

and the wear and disintegration of the surfaces prove to be a very serious evil.

217. *Variable speed friction gear.*—Fig. 262 shows an arrangement for transmitting power with a variable velocity ratio. Two cones with pulleys for receiving and transmitting the power have between them a leather band, which can be shifted along by a fork and belt shifting arrangement. The cones are pressed together with force enough to produce the necessary resistance to slipping. The action is the same as that of a pair of friction wheels, but the leather band supplies a replaceable friction surface and at the same time gives the means of varying the speed.

The arrangement has been used to transmit not inconsiderable amounts of power.

# CHAPTER XII

## SLIDING CONTACT. TOOTHED GEARING

218. A pair of well-formed toothed wheels have a relative motion identical with that of a pair of friction wheels rolling in contact. The surfaces of the friction wheels correspond to surfaces in the toothed wheels, which are termed *pitch surfaces.* Hence, if $r_1\, r_2$ are the radii, $d_1\, d_2$ the diameters measured to the pitch surfaces, $N_1\, N_2$ the rotations per minute, $\omega_1$, $\omega_2$, the angular velocities,

$$\frac{N_1}{N_2} = \frac{r_2}{r_1} = \frac{d_2}{d_1} = \frac{\omega_1}{\omega_2} \quad . \quad . \quad . \quad (1)$$

In designing the wheels the pitch surfaces are first drawn and the teeth are constructed with reference to the pitch surfaces, so that the teeth on one wheel fall into the spaces on the other, a small clearance being allowed to provide for inaccuracy in construction.

The importance of forming the teeth so that the velocity ratio is constant is very great. It is not only that an irregular motion is injurious to the machinery driven, but since the inertia of the heavy moving parts resists alteration of velocity, the driven wheel will alternately fall back and overtake the driving wheel, the gearing working with noise and vibration. This action is called 'back lash.'

The greater the clearance and the less the care taken in forming the teeth, and the greater the speed of the pitch line, the more violent is the back lash. The limiting speeds of pitch line for different classes of heavy gearing are about as follows :—

| | Speed of pitch line in feet per second |
|---|---|
| Ordinary cast-iron gearing . . . . . | 30 |
| Helical or mortice wheels . . . . . | 40 |
| Machine-cut wheels . . . . . . . . | 50 |

Large moulded spur-wheels and pinions with iron teeth have sometimes a pitch-line speed of 42 feet per second, and

when they are exceptionally good and well fitted run smoothly and noiselessly.

219. *Material of gearing.*—Usually toothed wheels are made of cast iron. If great strength is required they may be steel castings, but the large contraction of steel in solidifying gives rise to some difficulty.

To obtain the necessary accuracy of pitch and form cast steel wheels must be machined. Gearing subjected to shock is sometimes made of gunmetal or phosphor bronze. In the case of gearing for high-speed petrol motors for automobiles, it is necessary to secure the greatest lightness and extraordinary power of resisting wear. In such cases a very great advance has been made by the use of high-grade special steels such as nickel chrome steel. In some cases of high-speed gearing, as, for instance, the gearing for reducing the speed of electric motors, to obtain absence of noise, pinions made of raw hide or compressed paper are used. The blocks of hide or paper are provided with metal shrouds and bushes. The teeth are cut through both block and shrouds. Raw hide is damaged by water or oil, but otherwise is very satisfactory.

*Pattern and machine-moulded wheels.*—Formerly all toothed wheels were moulded from a complete wood pattern, and as this was expensive to make, it was stored and used over and over again. Such patterns almost always become distorted from shrinkage, and then the wheels cast from them work badly. About 1860 Messrs. Jackson, of Manchester, introduced a wheel-moulding machine for spur and bevil gearing. A pattern of two or three teeth only is constructed and fixed to a radial arm, which can be rotated accurately through any required fraction of the circumference of the wheel. The whole toothed rim is thus moulded in successive portions. The spaces between the arms are then cored out by dry sand cores moulded in core boxes. Machine-moulded wheels are superior to pattern-moulded wheels, if the wheel-moulding machine is kept in good order and not allowed to become inaccurate by wear. In spur wheels moulded from wood patterns a slight taper or draught is given to the teeth to facilitate withdrawing the pattern from the sand, and the wheels should be placed in gear with the draught of the teeth in opposite directions. Wheels moulded from metal patterns or machine moulded do not require this taper.

About 1880 Messrs. Jackson introduced double helical

machine-moulded gearing, which is silent in action and suitable for heavy duty.

In the case of heavy spur flywheels with rims of large section, it is not a good plan to cast the teeth on the rim. The teeth warp in cooling, owing to the contraction of the large mass of metal in the rim, and are often spongy and weak at the roots. The toothed rim should be moulded by machine and cast as a solid ring or with parting plates. The ends of the segments so formed may then be planed and fitted with distance pieces to insure accuracy of pitch. Both the outer circumference of the flywheel and the inner circumference of the ring should be turned to fit. (See Longridge, 'Report of Boiler Insurance Association,' 1883, p. 18.)

*Machine-cut wheels.*—For a long time small wheels have been cast with a blank rim and the tooth spaces cut out by a milling tool or cutting tool in a wheel-cutting machine. Very much greater accuracy of pitch is secured than when the teeth are moulded, and if care is taken a better tooth form also. Recently the production of accurate machine-cut wheels of all sizes, up to the largest, has been greatly extended. Sometimes milling cutters of the form of the tooth space are used. At other times automatic machines are employed to cut accurate cycloidal or involute templets of the tooth form, and these are then employed as guides of a milling tool or cutting tool in a wheel-cutting machine. Cut wheels are much superior to moulded wheels, and can be made with almost no 'back lash' or side clearance.

In the latest form of wheel-cutting engine '*hobs*' are used, the longitudinal section of which corresponds to the form of an accurate rack. Hob and wheel are geared together and rotated at suitable speeds, and the teeth are all begun and finished simultaneously. Wheel-cutting machines have been constructed capable of machining spur wheels of iron or steel up to 30 ft. diameter, 36 ins. width of face, and 9 ins. pitch. In large wheels the wheels are cast with teeth of extra thickness to allow for machining.

*Machine-cut bevil wheels.*—Bevil wheels are sometimes cut with rotating milling cutters. These cutters, of a thickness less than the space between two teeth, have their cutting faces of the curves suitable for a tooth at the middle of its length.

Strictly accurate forms of bevil teeth cannot be obtained by rotating milling cutters, but only by a planing action, the

direction in which the tool point moves always passing through the apex of the pitch cone.

*Mortice wheels.*—In the case of mill gearing, when wheels are run at high velocities, the teeth of one of each pair are of wood, and are termed 'cogs.' These cogs are morticed into an iron rim, and shaped by hand. The iron wheel, which works with a mortice wheel, is usually 'pitched and trimmed;' that is, the rough surface of the teeth is chipped away, and the teeth are filed perfectly smooth. This insures greater accuracy in the form of the teeth, and prevents the destruction of the wood cogs by the rough surface of the casting. With machine-moulded wheels, it is only necessary to clear off the sand from the surfaces of the teeth and to file them smooth. No chipping is necessary.

As the wood cogs are of a weaker material than iron, they are usually of greater thickness on the pitch line than the iron teeth working with them.

220. *Relation between the number of teeth and the diameter of the wheel. Circular and diametral pitch.*—The distance measured *along the pitch line* from the centre of one tooth to the centre of the next tooth is the *pitch* or *circular pitch*, or it is a length equal to the circumference of the pitch circle divided by the number of teeth in the wheel. A length equal to the diameter divided by the number of teeth is called the *diametral pitch*.[1]

Let $p$ be the circular pitch, $s$ the diametral pitch, $d$ the diameter of pitch line, $\mathrm{T}$ the number of teeth. Then, since $p\,\mathrm{T}$ must be the circumference of the pitch line,

$$\left.\begin{aligned} d &= \frac{p}{\pi}\,\mathrm{T} \doteq 0{\cdot}3183\,p\,\mathrm{T} \\ \mathrm{T} &= \frac{\pi}{p}\,d = 3{\cdot}1416\,d/p \end{aligned}\right\} \quad . \quad . \quad . \quad (2)$$

$$\left.\begin{aligned} &\text{Circular pitch} &&= p = \pi d/\mathrm{T} = \pi s \\ &\text{Diametral pitch} &&= s = d/\mathrm{T} = p/\pi = 0{\cdot}3183\,p \\ &\text{Pitch number} &&= 1/s = \pi/p \end{aligned}\right\} \quad . \quad . \quad (2a)$$

[1] The definition given is the only logical definition of diametral pitch. Curiously it is sometimes defined to be $\mathrm{T}/d$ or $1/s$, which is the number of teeth per inch diameter, as in Leconte, *Mechanics of Machinery*. McCord, *Kinematics of Mechanical Movements*, defines diametral pitch as above. Messrs. Brown and Sharpe, *Treatise on Gearing*, call $s$ the diameter pitch and $1/s$ the diametral pitch, which is confusing. It is better to call $s$ the diametral pitch and $1/s$ the pitch number.

Values of $p/\pi$ and $\pi/p$ are given in the following table to facilitate calculation. It will be seen that the relations (2a) in terms of the diametral pitch are simpler than the relations (2) in terms of the circular pitch. Further, the diameters of wheels will be rational numbers if a series of simple values are chosen for the pitch numbers instead of for the circular pitches. Hence the diametral pitch is more convenient than the circular pitch in forming an arbitrary system of sizes of wheels.

*Table giving Diameter or Number of Teeth of Wheels in Terms of the Pitch*

| Circular pitch in inches $p$ | Pitch number $1/s = \pi/p$ | Diametral pitch $s = p/\pi$ |
|---|---|---|
| $\frac{1}{2}$ | 6·2832 | 0·1592 |
| $\frac{5}{8}$ | 5·0266 | 0·1989 |
| $\frac{3}{4}$ | 4·1888 | 0·2387 |
| $\frac{7}{8}$ | 3·5904 | 0·2786 |
| 1 | 3·1416 | 0·3183 |
| $1\frac{1}{8}$ | 2·7926 | 0·3581 |
| $1\frac{1}{4}$ | 2·5132 | 0·3979 |
| $1\frac{3}{8}$ | 2·2848 | 0·4377 |
| $1\frac{1}{2}$ | 2·0944 | 0·4775 |
| $1\frac{3}{4}$ | 1·7952 | 0·5570 |
| 2 | 1·5708 | 0·6366 |
| $2\frac{1}{4}$ | 1·3963 | 0·7162 |
| $2\frac{1}{2}$ | 1·2566 | 0·7958 |
| $2\frac{3}{4}$ | 1·1424 | 0·8754 |
| 3 | 1·0472 | 0·9549 |
| $3\frac{1}{4}$ | 0·9668 | 1·0345 |
| $3\frac{1}{2}$ | 0·8976 | 1·1141 |
| $3\frac{3}{4}$ | 0·8377 | 1·1936 |
| 4 | 0·7854 | 1·2732 |
| $4\frac{1}{4}$ | 0·7392 | 1·3528 |
| $4\frac{1}{2}$ | 0·6981 | 1·4324 |
| $4\frac{3}{4}$ | 0·6615 | 1·5120 |
| 5 | 0·6283 | 1·5916 |
| $5\frac{1}{2}$ | 0·5711 | 1·7507 |
| 6 | 0·5236 | 1·9099 |
| $6\frac{1}{2}$ | 0·4833 | 2·0691 |
| 7 | 0·4488 | 2·2283 |
| $7\frac{1}{2}$ | 0·4188 | 2·3875 |
| 8 | 0·3927 | 2·5465 |
| 9 | 0·3491 | 2·8647 |
| 10 | 0·3142 | 3·1829 |
| 11 | 0·2856 | 3·5014 |
| 12 | 0·2618 | 3·8200 |

*To lay off the pitch on the pitch line.*—The following construction, due to Rankine, is convenient, when the wheel is so large that it is impossible to find the exact pitch, by stepping round the pitch line. Let the circle, A, fig. 263, be the pitch line. At any point, $a$, draw the tangent $a\ b$. Make $a\ b =$ the pitch. Take $a\ c = \frac{1}{4}\ a\ b$. With centre $c$, and radius $c\ b$, draw the arc $b\ d$. Then the arc $a\ d$ is $= a\ b$, and is the pitch laid off on the pitch line. When the wheel has many teeth the arc $a\ d$ sensibly

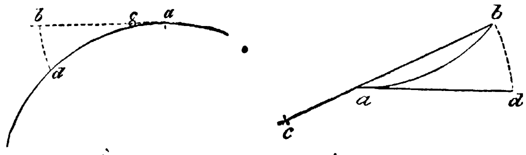



Fig. 263

coincides with its chord, but, if it has few teeth, there is an appreciable error in taking the chord $a\ d$ equal to the pitch.

The corresponding approximate construction for rectifying a circular arc is shown in B, fig. 263. Let $a\ b$ be a circular arc, $a\ d$ its tangent. Draw the chord $b\ a$ and produce it. Take $a\ c = \frac{1}{2}\ a\ b$. With centre $c$ describe the arc $b\ d$. Then $a\ d =$ arc $a\ b$ very nearly.

221. *Parts and proportions of teeth.*—Fig. 264 shows the general form of wheel teeth drawn for convenience on a straight pitch line. $f\ e$ is the pitch (circular pitch) divided into $f\ b$,

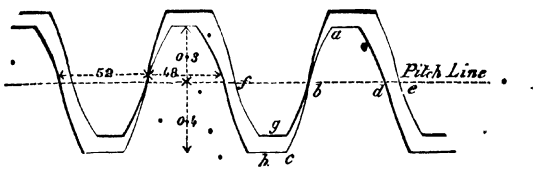



Fig. 264

the tooth thickness, and $b\ e$ the tooth space. $d\ e$ is the side clearance, which may be very small in machine-cut wheels, and must be larger the less the accuracy of the wheels. $b\ a$ is the face, and $b\ c$ the flank of the tooth. $g\ k$ is the bottom clearance.

In working, the faces of the teeth of one wheel come into contact with the flanks of the teeth of the other wheel. Also, while the point of contact is approaching the line of centres, the flank of the driving acts on the face of the driven tooth;

while the point of contact is receding from the line of centres, the face of the driving acts on the flank of the driven tooth. The arc of the pitch circle, through which the wheel turns during contact, is called the 'arc of contact,' or 'arc of action,' and the portions into which it is divided by the pitch point are the arcs of approach and recess. The length of the arc of approach depends on the length of face of the driven tooth, because contact begins at the point of the driven tooth and travels to the pitch point; the arc of recess on the length of the face of the driving tooth, because contact travels from the pitch point to the point of the driving tooth. The arc of action must be at least equal to the pitch or one pair of teeth will go out of action before the next pair are engaged.

Fig. 264 shows ordinary proportions of wheel teeth expressed in simple numbers. It is, however, the opinion of some engineers that these proportions make the teeth of too great a height. The height of the tooth determines the length of the arc of action, and the only essential condition is that the arc of action should be greater than the pitch. Other things being equal, the less the height of a tooth the stronger it is, which is advantageous. The following proportions have been used in different cases—those giving the total height of teeth 0·7 to 0·8 have been most generally used. Some engineers are decidedly in favour of shorter teeth and the proportions giving the total height of tooth 0·45 $p$ have been found quite successful. With involute teeth and the ordinary obliquity of 15° these proportions give a just sufficient arc of contact with wheels of 20 teeth. The arc of action can be found in any case by methods given below.

### Various Proportions of Teeth

| | | | | |
|---|---|---|---|---|
| Height above pitch-line | 0·2 $p$ | 0·26$p$ | 0·30$p$ | 0·36$p$ |
| Depth below ,, | 0·25$p$ | 0·34$p$ | 0·40$p$ | 0·44$p$ |
| Total height of tooth | 0·45$p$ | 0·60$p$ | 0·70$p$ | 0·80$p$ |

The following proportions agree well with the most usual practice for large gearing:

### Machine-moulded Wheels

| | |
|---|---|
| **Thickness of tooth** | 0·485$p$ − 0·03 |
| **Width of space** | 0·515$p$ + 0·03 |
| **Height above pitch-line** | 0·25$p$ to 0·35$p$ |
| **Depth below ,,** | 0·35$p$ + 0·08 to 0·4$p$ + 0·08 |
| **Total height of tooth** | 0·6$p$ + 0·08 to 0·75$p$ + 0·08 |

MACHINE-CUT WHEELS

(*Messrs. Brown and Sharpe*)

| | |
|---|---|
| Thickness of tooth and width of space | $0·5p$ or $1·571s$ |
| Height above pitch line | $0·318p$ or $s$ |
| Depth below ,, | $0·368p$ or $1·157s$ |
| Bottom clearance | $p/20$ or $0·157s$ |
| Total height of tooth | $0·686p$ or $2·157s$ |

SHORT INVOLUTE TEETH

| | |
|---|---|
| Angle of obliquity | 20° |
| Thickness of tooth and width of space | $0·5p$ or $1·571s$ |
| Height above pitch-line | $0·25p$ or $0·785s$ |
| Depth below ,, | $0·30p$ or $0·942s$ |
| Working height | $0·5p$ or $1·571s$ |
| Total height of tooth | $0·55p$ or $1·728s$ |
| Clearance | $0·05p$ or $0·157s$ |

For mortice wheels the wood cog may be thicker than the iron teeth working with it. Then the following proportions are good :

| | |
|---|---|
| Thickness of iron tooth and space of mortice teeth | $= 0·4p$ |
| ,, ,, wood cog and space of iron teeth | $= 0·6p$ |

The following table gives Brown and Sharpe's proportions for machine-cut wheels, and is inserted chiefly to show how a convenient series of pitch numbers may be chosen :

| Pitch number | Diametral pitch | Circular pitch | Thickness of tooth | Height above pitch line | Depth below pitch line | Total height of tooth |
|---|---|---|---|---|---|---|
| $\frac{1}{s}$ | $s$ | $p$ | $0·5p$ | | $s+f$ | $2s+f$ |
| $\frac{1}{2}$ | 2 | 6·283 | 3·142 | 2·000 | 2·314 | 4·314 |
| $\frac{3}{4}$ | 1·333 | 4·189 | 2·094 | 1·333 | 1·543 | 2·876 |
| 1 | 1 | 3·142 | 1·571 | 1·000 | 1·157 | 2·157 |
| $1\frac{1}{4}$ | 0·8 | 2·513 | 1·257 | ·800 | ·926 | 1·726 |
| $1\frac{1}{2}$ | 0·667 | 2·094 | 1·047 | ·667 | ·771 | 1·438 |
| $1\frac{3}{4}$ | 0·571 | 1·795 | ·898 | ·571 | ·661 | 1·233 |
| 2 | 0·5 | 1·571 | ·785 | ·500 | ·578 | 1·078 |
| $2\frac{1}{4}$ | 0·444 | 1·396 | ·698 | ·444 | ·514 | ·959 |
| $2\frac{1}{2}$ | 0·4 | 1·257 | ·628 | ·400 | ·463 | ·863 |
| $2\frac{3}{4}$ | 0·364 | 1·142 | ·571 | ·364 | ·421 | ·784 |
| 3 | 0·333 | 1·047 | ·524 | ·333 | ·386 | ·719 |
| $3\frac{1}{2}$ | 0·286 | ·898 | ·449 | ·286 | ·331 | ·616 |
| 4 | 0·25 | ·785 | ·393 | ·250 | ·289 | ·539 |
| 5 | 0·2 | ·628 | ·314 | ·200 | ·231 | ·431 |
| 6 | 0·167 | ·524 | ·262 | ·167 | ·193 | ·360 |
| 7 | 0·143 | ·449 | ·224 | ·143 | ·165 | ·308 |
| 8 | 0·125 | ·393 | ·196 | ·125 | ·145 | ·270 |

The diameter of the wheel is the number of teeth multiplied by the diametral pitch or divided by the pitch number. Thus a wheel of 75 teeth, 0·4 diametral pitch and $2\frac{1}{2}$ pitch number, will be $75 \times 0·4 = 75 \div 2\frac{1}{2} = 30$ inches in diameter. The introduction of very accurate milling cutters by Brown and Sharpe, with involute teeth, has practically standardised cut gears of small and moderately large sizes.

222. *Width of face of wheels.*—In gearing for cranes and similar cases where the running is intermittent and where the accuracy of adjustment and rigidity of the shafts is not of the highest order, the whole pressure between two teeth may come on a corner of a tooth and in that case the strength is independent of the width of face. In such cases the width of face may be $1\frac{1}{2}$ times to twice the pitch.

In continuous running gearing the question of durability is of as much importance as strength, and widening the face spreads the wear over a greater area and increases the life of a wheel. But even in this case for cast gears there are limits beyond which the advantage is counterbalanced by the increasing difficulty of securing such accuracy that the load is distributed over the whole width of the teeth. For cut gears which are of greater accuracy the width of face may be greater than for cast gears.

When the parallelism and rigidity of the shafts cannot be completely trusted it is not useful to make the width of face of cast gears more than one and a-half times to twice the pitch. In large spur flywheels which are solidly supported and carefully adjusted the width of face is three to four times the pitch. In cut gears greater widths are sometimes adopted. Thus in the comparatively small gearing used for electric motors, where the speed is high and smoothness of action is of great importance, the width of face is sometimes six or even eight times the pitch. But it is doubtful if such exaggerated proportions are desirable. The following rule gives the usual normal width of face of cut gears. Let $p$ be the circular, $s$ the diametral pitch, and $b$ the width of face. Then,—

$$b = 8s + \tfrac{1}{4} = 2·55\,p + \tfrac{1}{4} \quad . \quad . \quad . \quad . \quad (3)$$

| | | | | | |
|---|---|---|---|---|---|
| Pitch number = | 8 | 6 | 4 | 2 | 1 |
| Diametral pitch = $s$ = | ·125 | ·167 | ·25 | ·5 | 1 |
| Circular pitch = $p$ = | ·393 | ·524 | ·785 | 1·571 | 3·142 |
| Width of face = $b$ = | $1\frac{1}{4}$ | $1\frac{5}{8}$ | $2\frac{1}{4}$ | $4\frac{1}{4}$ | $8\frac{1}{4}$ |
| $b/p$ = | 3·2 | 3·1 | 2·9 | 2·7 | 2·6 |

*Least number of teeth.*—In hand gearing and gearing for rough work pinions with 11 or 12 teeth are sometimes used. Generally a pinion should not have less than 15 teeth. For transmission gearing running continuously the smallest pinion should have not less than 24 or 30 teeth.

### THEORY OF WHEEL TEETH. CONDITIONS DETERMINING THE FORM OF TEETH

223. *General definitions.*—A pair of toothed wheels is equivalent in action to a pair of smooth rollers. The intersections of the surfaces of these rollers with a plane normal to the axes are the pitch lines of the wheels, touching at the *pitch point p*, on the line of centres $c_1 c_2$. The teeth profiles, fig. 265,

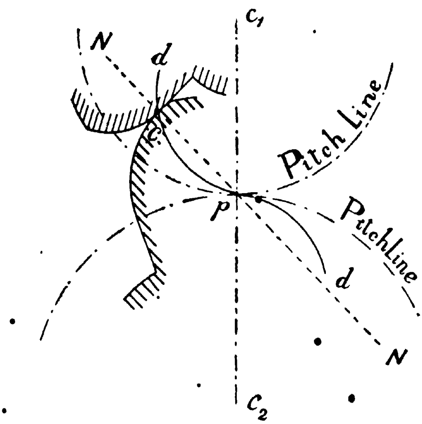



Fig. 265

are constructed with reference to these pitch lines. Let $c$ be the point of contact of two teeth. Then for uniform velocity ratio, the normal N N to the teeth at $c$ must pass through $p$ in all positions of the teeth during contact. The point of contact $c$ describes during the action of the teeth a curve *d d* in space which is termed the *path of contact.* From a purely theoretical point of view any path of contact can be chosen and corresponding tooth forms found. But only certain tooth forms are convenient and practicable.

The arcs of the pitch lines which roll in contact during the action of a pair of teeth are termed *arcs of contact* or *arcs of*

*action*, and the pitch point divides these into arcs of approach and arcs of recess.

224. *Arcs of approach and recess. Arc of action.*—Let fig. 266 show the teeth of two wheels in the positions at which contact begins and ends, $a_1, a_2$, being the first and last points of contact. During the first part of the action of two teeth, the flank of the driver acts on the face of the driven tooth, and the point of contact moves towards the pitch point $p$. During the remainder of the action the face of the driving tooth is acting on the flank of the driven tooth, and the point of contact is travelling away from $p$. Contact begins, therefore, at the point $a_1$ of the driven

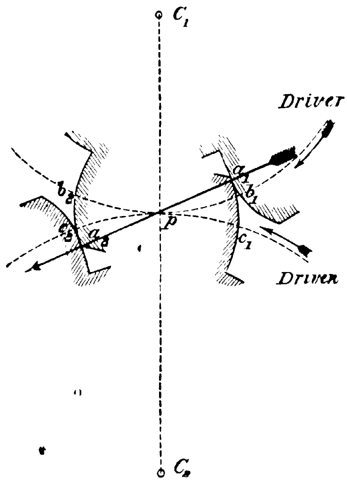



Fig. 266

tooth, and ends at the point $a_2$ of the driving tooth. If the tooth contours intersect the pitch lines in $b_1$, $c_1$, $b_2$, $c_2$, then $b_1$ and $c_1$ come together at $p$ and recede to $b_2$, $c_2$; the arcs $b_1\,p$, $c_1\,p$ are called the arcs of approach, and the arcs $p\,b_2$, $p\,c_2$ the arcs of recess. The arcs $b_1\,p\,b_2$, $c_1\,p\,c_2$ are the arcs of action. The arc of approach depends on the length of face of the driven tooth; the arc of recess on the length of face of the driving tooth.

In order that a pair of teeth may not go out of gear before another pair of teeth has come into gear, the arcs of action $b_1\,p\,b_2$ and $c_1\,p\,c_2$ must be at least equal to the circular pitch.

Let $R_1$, $R_2$ be the radii of the pitch lines. Then the angles

through which the wheels turn during approach are (in circular measure) $p\ b_1/R_1$, and $p\ c_1/R_2$. During recess they are $p\ b_2/R_2$ and $p\ c_2/R_1$. Also it may be noted that $p\ b_1 = p\ b_2$ and $p\ c_1 = p\ c_2$.

In fig. 267, which repeats the positions of the teeth shown in the previous figure, join the points of beginning and ending

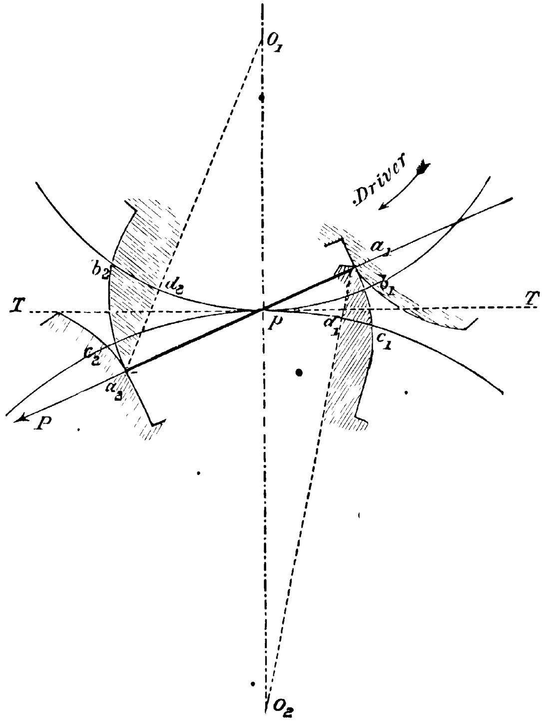



Fig. 267

contact $a_1\ a_2$ to the centres of the wheels. Then, since the tooth thickness cannot be more than half the pitch, if $c_1\ d_1$ is one-fourth of the pitch, the tooth of the lower wheel is pointed, and any greater pitch is impossible. Similarly, if $b_2\ d_2$ is greater than one-fourth of the pitch the tooth of the upper wheel is impossible.

225. *Obliquity of action.*—Neglecting the friction of the teeth, the pressure P between them must be in the direction of

the normal at the point of contact, which in properly formed teeth always passes through $p$. The angle which this line makes with the tangent T T to the pitch circles is called the *angle of obliquity of action*. In some forms of teeth this angle has a constant value during the contact of a pair of teeth. Its value is then commonly $15\frac{1}{2}°$. With other forms of teeth this angle varies, its greatest values $a_1 p$ T, and $a_2 p$ T, occurring at the moment of beginning or ending contact (fig. 267). These values are then usually limited to 30°. Professor Kennedy[1] has pointed out that, if friction is taken into account, P is more inclined to the tangent during approach and less during recess, and that this is probably the reason for the statement that the action of the wheels is less smooth and the friction greater during approach than during recess. For ordinary gearing, however, this difference may be neglected. Its importance has probably been overrated.

In wheels for clockwork, where the friction is specially injurious, in certain cases the wheels are designed so that the driving-teeth have no flanks and the driven teeth no faces. Then contact is entirely confined to the period of recess. The arcs of recess must then be at least equal to the pitch.

226. *Condition of continuous contact of a pair of teeth.*—Let A and B be two spur wheels rotating at any moment with angular velocities $+\omega_1$, and $-\omega_2$. Nothing will be changed in the relative motion of the two wheels, if a rotation $-\omega_1$ is impressed on each. Then A's angular velocity of rotation will be $\omega_1 - \omega_1 = 0$, that is it will be at rest. The centre of B will rotate about A with the velocity $-\omega_1$, and B will rotate about its own centre with the velocity $-(\omega_1 + \omega_2)$. Hence, the motion of B will be the same as if it were at the moment rotating about an axis placed at a point dividing the line of centres in the ratio $\omega_1/\omega_2$, or it will roll on the pitch line of A. Now let a tooth be fixed to B. In order that that tooth may remain continuously in contact with a tooth on A, the form of the latter must be the envelope of the successive positions of the tooth on B, as it moves round A. The form of the tooth on B is not arbitrary. Only certain forms give to the envelope shapes which are practically realisable as wheel teeth. The tooth surfaces of a pair of wheels form, in the terminology of Reuleaux, an unclosed higher pair of elements.

In ordinary wheels with circular pitch lines, any tooth of one

[1] *Mechanics of Machinery*, p. 605.

wheel may have to gear with any tooth of the other wheel. Hence all the teeth must be formed so as to come into gear at the same point and to remain in action while the wheel turns through the same angle. All the teeth of each wheel should therefore be of the same form.[1]

*Constancy of velocity ratio.*—The most important condition is great accuracy and uniformity of pitch. Next to this it is desirable that, as far as requirements of strength permit, the pitch should be small and the teeth numerous. But it is important that the ratio should be constant during the action of each pair of teeth or the wheels are noisy, and this is secured by adopting forms for the profiles such that the common normal at the point of contact passes through the pitch point. Further, as the pairs of teeth come into action successively they must be equally spaced round the pitch circle. The distance from the profile of one tooth to that of the next, measured along the pitch circle, is termed the circular pitch. The pitch must be constant in each wheel and the same for a pair of wheels which mate. Since there cannot be a fraction of a tooth, the circular pitch must be an aliquot part of the circumference of the pitch-line of each wheel.

*Influence of the form of the tooth on its strength.*—It will be seen presently that the teeth tend to break across at the root. The teeth are stronger the shorter they are, and the thicker they are at the root. But they cannot be shortened without reducing the arc of contact, and their length should be such as to insure a sufficient, but not excessive, arc of contact. The thickness at the root depends on the form selected for the teeth. Involute teeth are generally stronger than cycloidal teeth. With cycloidal teeth, the teeth are stronger the smaller the diameter of the describing circle used for the flanks.

*Conditions of durability.*—The rolling of the teeth over each other so as to spread the contact over a considerable area of tooth surface is advantageous. But the sliding of one tooth on the other is disadvantageous. The amount of sliding is the difference of the length of the faces of the teeth and the flanks which work with them. By increasing the height of the tooth the amount of sliding is increased. So that in general, to spread the wear over a larger surface, it is preferable to widen the face

[1] This is, of course, not true of wheels with non-circular pitch-lines, when the same pairs of teeth always gear together.

of the wheel instead of making the height of the teeth greater. Wheels run better after working for some time, no doubt because the more prominent projections are worn off and a better bearing secured. But there is no reason to think that wear improves the form of badly shaped teeth—rather the contrary.

The tooth face is longer than the part of the tooth flank which works with it. Consequently the wear is more distributed on the face and more concentrated on the flank. In important gearing it has often been held to be important that two pairs of teeth should be in action at the same time so as to diminish the pressure and decrease the wear on each pair of teeth. This involves that the arc of action should be at least twice the pitch, and making the teeth of considerable height and relatively weak. Further, even when so designed, it is doubtful, looking to unavoidable inaccuracies of workmanship, whether the pressure is really well distributed to the two pairs of teeth supposed to be in contact. There are obvious advantages in shortening the teeth; short teeth are stronger, work more smoothly and are more easily made approximately accurate in form. Hence many engineers, especially mill engineers in the north of England, are content with an arc of action of 1¼ times the pitch and a height of tooth above the pitch-line equal to one-fifth to one-quarter of the pitch. With such proportions the whole pressure transmitted must be carried by a single pair of teeth. The frictional loss of work is somewhat diminished by shortening the teeth.

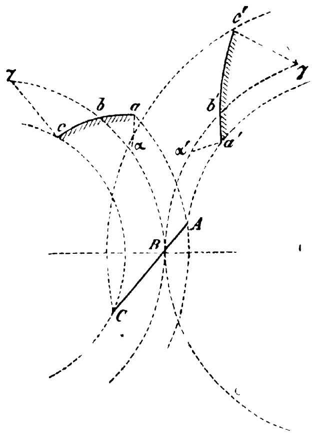



Fig. 268

227. *Given the form of tooth of one wheel, to find the proper form of the tooth of another wheel to gear with it.*—Let *a b c*, fig. 268, be the given tooth, *b* B, *b′* B the pitch lines of the wheels, B the pitch point or point of contact of the pitch lines. From any points, *a*, *c*, draw normals, *a* $\alpha$, *c* $\gamma$, to the curve of the given tooth, cutting the pitch line in $\alpha$, $\gamma$. Then the points *a b c* should be

points of contact, when $\alpha$, $b$, $\gamma$, are at the pitch point. From B set off $BA = a\,\alpha$, $BC = c\,\gamma$. Then A is the point where $a$ is in contact; B the point where $b$ is in contact, and C the point where $c$ is in contact, and some line, passing through A B C, is the path of contact. Through A, C, draw circles A $a'$, C $c'$. Set off arc B $\alpha'$ = arc B $\alpha$; also $\alpha'\,a' = \alpha\,a$. Then $a'$ is a point in the tooth of the second wheel, which will come in contact with $a$ at A, and will have a common normal, passing through the pitch point. Set off arc B $b'$ = arc B $b$; then $b'$ will come in contact with $b$ at B. Also, set off arc B $\gamma'$ = arc B $\gamma$, and $\gamma'\,c' = \gamma\,c$; then $c'$ will come in contact with $c$ at C. A curve $a'\,b'\,c'$

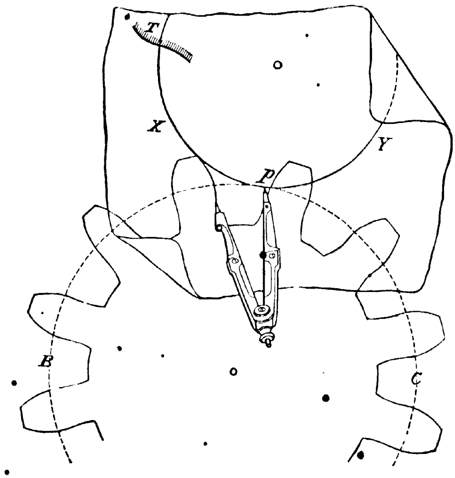



Fig. 269

through the points so found will be the required tooth. In certain cases the construction becomes impossible, and the given tooth is of unsuitable form. Forms should be avoided which make a tooth entirely concave.

*Mr. Last's method.*—A very ingenious method of solving the problem of finding the form of a tooth to work with a given tooth is due to Mr. Last.[1] Let fig. 269 represent a rubbing taken from an existing wheel. Choose on this the most probable pitch circle, B C. On a piece of tracing-paper is described the pitch circle X Y of the wheel which is to gear with the given

[1] *Proc. Inst. Civil Engineers*, vol. lxxxix. p. 341.

wheel. Placing the tracing-paper so that the pitch lines touch at $p$, the needle point of a pair of bow compasses is placed at $p$ and the bows opened till they will describe a circle touching the outline of the given adjacent tooth. A small arc is struck on the tracing-paper. Without removing the bow compass point, the tracing-paper is rotated a little till the circle X Y cuts the circle B C at a point near $p$. The compass point is removed to this point and the tracing-paper is then a little further rotated till the two circles apparently touch. The whole operation does not sensibly differ from rolling the circle X Y on B C. From this new centre a new small arc is struck touching the outline of the tooth. The operation is repeated as often as necessary. Thus the whole outline, shown at T, of the new tooth is obtained, the flank by arcs touching the face of the old tooth, the face by arcs touching the flank of the old tooth.

228. *Usual forms of teeth.*—If the pitch point of a pair of wheels is fixed and the path of contact of the teeth assumed, then the forms of the teeth for constant velocity ratio are determined. Hitherto two systems of wheel teeth have been generally used. If the path of contact consists of two circular arcs touching at the pitch point, then the tooth profiles are cycloidal curves. If the path of contact is a straight line passing through the pitch point, the tooth profiles are involutes of circles concentric with the wheels and having the path of contact as a common tangent. At first there was a strong preference for cycloidal wheels; of late, especially for machine cut wheels, the involute form has been very generally used, partly because a standard series of milling cutters for such wheels have been placed on the market by Brown & Sharpe, which are widely used for machine cut wheels. In involute teeth two surfaces convex to each other are in contact. In cycloidal wheels a convex surface is in contact with a concave surface. Prof. R. H Smith has pointed out that in the latter case there is greater closeness of fit, and that this has the advantage that unguent is less liable to be squeezed out and that friction and wear are diminished.

## Cycloidal Teeth

229. If a circle rolls on a straight line, a tracing point on its circumference describes a *cycloid*. If the circle rolls outside another circle the tracing point describes an *epicycloid*. If the

circle rolls inside another circle, the tracing point describes a *hypocycloid.* If for any of these curves a line is drawn, from the tracing point at any moment, to the touching point of the rolling circle and base line (or circle) at that moment, that line is the normal to the curve.

*Cycloidal teeth with external contact.*—Let fig. 270, represent the pitch circles, root circles and addendum circles of two wheels with centres at $c_1$, $c_2$, the lower wheel being the driver. Also the chosen rolling circles with centres at $o$ $o'$. Cycloidal tooth

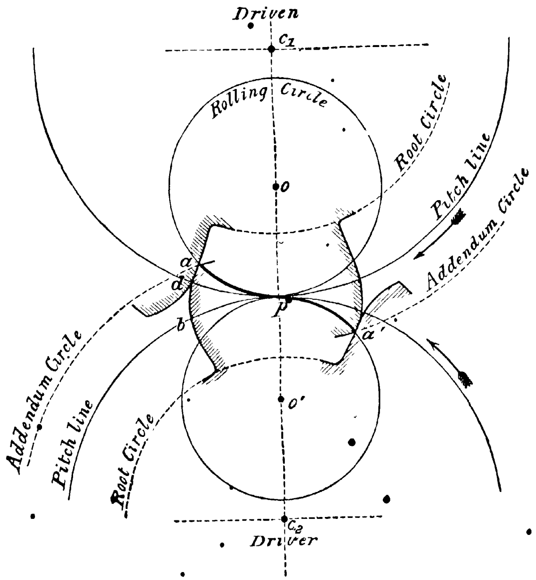



Fig. 270

profiles are drawn by tracing points on the rolling circles, as the two pitch lines roll together and also the two rolling circles, in contact at the pitch point $p$. But in sliding contact the contact radius of the driver must increase, and that of the driven decrease, during action. Hence, contact must begin at the point $a'$ of the driven tooth, and end at the point $a$ of the driving tooth. During approach of contact to $p$, the action will be between the flank of the driving and face of the driven tooth, and during recess from $p$ between the face

of driving and flank of driven tooth. As the tooth profiles are traced out by tracing points on the rolling circles, the points $a$, $a'$, must be at the intersections of the rolling and addendum circles. During approach the tracing point travels over the arc $a'\,p$, and during recess over the arc $p\,a$, so that $a'\,p\,a$ is the whole path of contact. Consider the action during recess, the tracing point being initially at $p$. As the circles roll together at $p$, the tracing point moves in space over the arc $p\,a$. At the same time it traces out on the plane of the lower wheel revolving about $c_2$, the epicycloid $b\,a$, and on the plane of the upper wheel revolving about $c_1$, the hypocycloid $d\,a$. These two curves will at

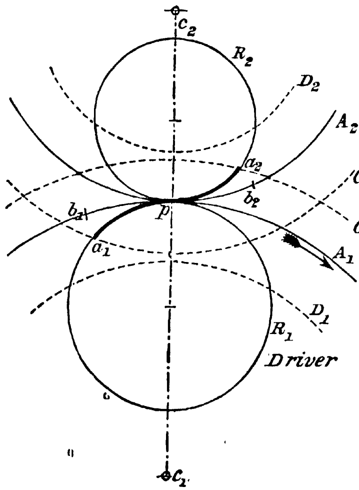


Fig. 271

any moment be in contact at the tracing point and will have a common normal—namely, the line drawn from the tracing point to $p$. The condition of uniform velocity ratio is therefore satisfied. Since $b$ and $d$ are points initially in contact at $p$, the arcs $p\,a$, $p\,d$, $p\,b$, which have rolled together, are all equal. The difference of length of $a\,d$ and $a\,b$ is the amount of sliding during recess. There is a similar action during approach. The tracing point moves in space over the arc $a'\,p$ and traces out an epicycloid on the plane of the upper and a hypocycloid on the plane of the lower wheel. These two pairs of curves are therefore suitable for the tooth profiles.

The obliquity of action of a pair of cycloidal teeth when contact begins is the angle which $a'\,p$ makes with the tangent at $p$ to the pitch lines. It decreases as the point of contact moves along the arc $a'\,p$ and becomes zero when the teeth are in contact at $p$. It then increases again to the angle which $a\,p$ makes

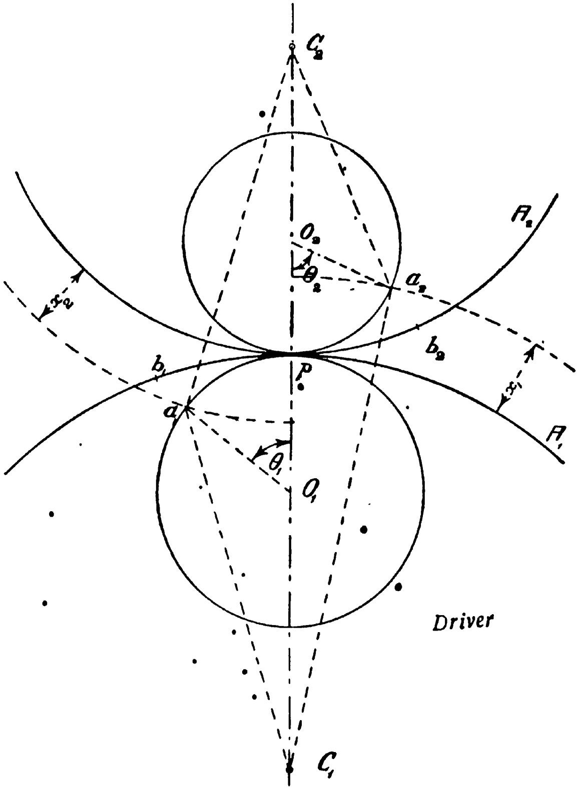



Fig. 272

with the tangent. The maximum obliquity in cycloidal gearing is about 30°, but in most pairs of wheels is less than this.

230. *To determine the lengths of cycloidal teeth for given arcs of approach and recess.*—Let $A_1$, $A_2$ (fig. 271) be the given pitch

lines, $R_1$, $R_2$ the given rolling circles. Set off $b_1\,p$, $p\,b_2$, the given arcs of approach and recess. Step off arc $p\,a_1 = p\,b_1$, and arc $p\,a_2 = p\,b_2$. Then, $a_1\,p\,a_2$ is the required path of contact. Contact begins at $a_1$ and ends at $a_2$. Circles $C_1$ $C_2$, through $a_2\,a_1$ are the required addendum circles. The root circles $D_1$ $D_2$ must be taken so that the points of the teeth of each wheel clear the other wheel, or allow the given amount of bottom clearance.

*Length of arc of contact in terms of addenda of teeth.*—Let fig. 272 represent the pitch lines, addendum circles, and chosen rolling circles of a pair of cycloidal wheels. $C$, $C_2$ are the centres of the pitch lines of radii $R_1$, $R_2$; $O_1$ $O_2$ are the centres of the rolling circles of radii $r_1$, $r_2$; and $x_1$, $x_2$ are the addenda of the teeth. Contact begins at $a_1$ and ends at $a_2$, and the arcs $p\,b_1 = p\,a_1$ and $p\,b_2 = p\,a_2$ are the arcs of approach and recess. Join $C_1\,a_1$, $C_2\,a_1$, $a_1\,O_1$. Then in the triangle $C_2\,O_1\,a_1$, $C_2\,O_1 = R_2 + r_1$, $C_2 a_1 = R_2 + x_2$ and $a_1\,O_1 = r_1$. Let $s = \frac{1}{2}(C_2\,O_1 + C_2\,a_1 + a_1\,O_1) = R_2 + r_1 + \frac{1}{2}x_2$. If $C_2\,O_1\,a_1 = \theta_1$,

$$\tan \tfrac{1}{2}\theta_1 = \sqrt{\left\{\frac{(s - R_2 - r_1)\,(s - r_1)}{s\,(s - R_2 - x_2)}\right\}}$$

$$= \sqrt{\left\{\frac{\frac{1}{2}x_2\,(R_2 + \frac{1}{2}x_2)}{(R_2 + r_1 + \frac{1}{2}x_2)\,(r_1 - \frac{1}{2}x_2)}\right\}}$$

This determines $\theta_1$. Then the arc of approach $= p\,a_1 = p\,b_1$ is

$$r_1\,\theta_1$$

where $\theta_1$ is in circular measure. Similarly, joining $C_2\,a_2$, $C_1\,a_2$, $a_2\,O_2$, $C_1\,a_2 = R_1 + x_1$; $C_1\,O_2 = R_1 + r_2$; $a_2\,O_2 = r_2$; $s = R_1 + r_2 + \frac{1}{2}x_1$.

If $C_1\,O_2\,a_2 = \theta_2$

$$\tan \tfrac{1}{2}\theta_2 = \sqrt{\left\{\frac{(s - R_1 - r_2)\,(s - r_2)}{s\,(s - R_1 - x_1)}\right\}}$$

$$= \sqrt{\left\{\frac{\frac{1}{2}x_1\,(R_1 + \frac{1}{2}x_1)}{(R_1 + r_2 + \frac{1}{2}x_1)\,(r_2 - \frac{1}{2}x_1)}\right\}}$$

which determines $\theta_2$, and the arc of recess $= p\,a_2 = p\,b_2 =$

$$r_2\,\theta_2$$

And the whole arc of contact on either pitch line is

$$r_1\,\theta_1 + r_2\,\theta_2.$$

This must be greater than the circular pitch in any case and

varies as stated above in different cases from about 1¼ to 2 times the pitch.

231. *Internal cycloidal teeth.*—Let $A_1$, $A_2$ (fig. 273) be the pitch lines, and $p$ the pitch point; $a_1\ c_1$ the tooth belonging to $A_1$, and $a_2\ c_2$ the tooth belonging to $A_2$. The flank $p\ c_1$ works on the face $p\ a_2$; both are epicycloids described by the rolling circle R, rolling outside $A_1$ and $A_2$. The face $p\ a_1$ and the flank $p\ c_2$ work together, and are hypocycloids described by R′ rolling inside $A_1$ and $A_2$. Through $a_1$ draw an arc $a_1\ x$ concentric with $A_1$, and through $a_2$ an arc $a_2\ y$ concentric with $A_2$, cutting the rolling circles in $x$ and $y$, then $x\ p\ y$ is the path of contact. As before, lengths equal to the arcs $x\ p$, $p\ y$ set off along the pitch lines will be the arcs of approach and recess.

The action of internal gear is smoother than that of external gear.

*Rack and pinion.*—A rack is a portion of a wheel of infinite radius. The pitch line is straight and tangent to the pinion pitch line. Both faces and flanks of the rack teeth are cycloids.

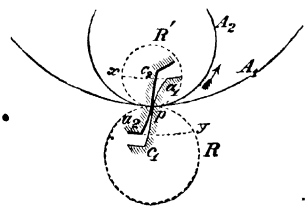


Fig. 273

232. *Choice of the diameter of the rolling circle.*—When the diameter of the rolling circle is equal to the radius of the circle inside which it is rolled, the flanks of the teeth are radial straight lines, and the teeth are rather thinner at the roots than at the pitch line. If the rolling circle is larger they become still weaker. With a smaller rolling circle they are stronger.

Pairs of wheels, each having radial flanks, were at one time largely used, and have about the smallest practicable obliquity of action. In that case the flanks of the teeth of one wheel and the faces of the teeth of the other are described with a rolling circle of diameter equal to the radius of the former wheel.

When a set of wheels have to be constructed any two of which will work together, the same rolling circle must be used for both faces and flanks of all the wheels of the set. It is usual then to take the diameter of the rolling circle equal to the radius of the smallest wheel of the set or at most 1¼ times that radius. If taken larger, the teeth of the smaller wheels are too weak. Let

$p$ be the pitch, T the number of teeth, and $d$ the diameter of a rolling circle equal to the radius of the wheel. Then,—

$$d = \frac{\text{T}p}{2\pi}$$

| No. of teeth T= | Diam. of rolling circle $d$= |
|---|---|
| 11 | 1·751 $p$ |
| 12 | 1·910 $p$ |
| 13 | 2·068 $p$ |
| 14 | 2·228 $p$ |
| 15 | 2·387 $p$ |
| 16 | 2·546 $p$ |
| 20 | 3·183 $p$ |
| 25 | 3·981 $p$ |

Fig. 274 shows the influence of the size of the rolling circle on the form of the teeth. Let R be the radius of the wheel, $r$ the

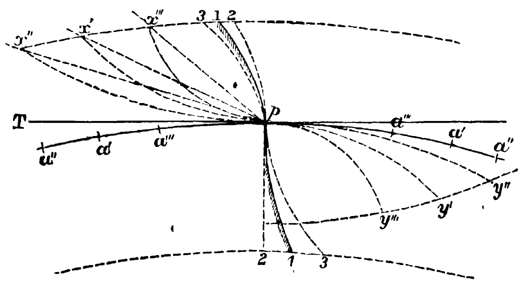



Fig. 274

radius of the describing circle. The tooth curve 1 $p$ 1 is described with rolling circles of radius $r = \frac{1}{4}$ R, and $x'\,y'$ is the corresponding path of contact. The tooth curve 2 $p$ 2 is described with $r = \frac{1}{2}$ R, the flank of the tooth is radial, and $x''\,p\,y''$ is the path of contact. The tooth 3 $p$ 3 is described with $r = \frac{1}{8}$ R, and $x'''\,p\,y'''$ is the path of contact. It will be seen that the smaller the rolling circle, the stronger the tooth is at the root. On the other hand, the smaller the rolling circle, the shorter is the path of contact and the greater the maximum obliquity of action for a given length of tooth.

For a pitch of 2 inches and the height of tooth shown, the arcs of contact are 1·22, 1·5, and 0·97 times the pitch. The

corresponding maximum angles of obliquity are $x' \, p \, \text{T} = 24°$; $x'' \, p \, \text{T} = 15°$ and $x''' \, p \, \text{T} = 38°$. In order that the obliquity of action should not be too great when a sufficient arc of action is secured, it is usual to have at least twelve to fifteen teeth in a cycloidal wheel. By lengthening the teeth beyond the usual proportion, and reducing the arc of action to $1\frac{1}{4}$ pitch, wheels with a smaller number of teeth may be made in special cases; but then the obliquity of action is greater than is desirable, at least in wheels constantly running. For important transmissions it is desirable that no wheel should have less than from twenty-four to thirty-six teeth.

233. *Gee's patent gearing.*[1]—Messrs. Jackson, of Manchester, introduced a peculiar form of tooth, which is 35 per cent. stronger than the usual form. In this gearing the driving faces of the teeth (fig. 275) are of the usual form. The other faces have much more obliquity than ordinary teeth. Gearing of this

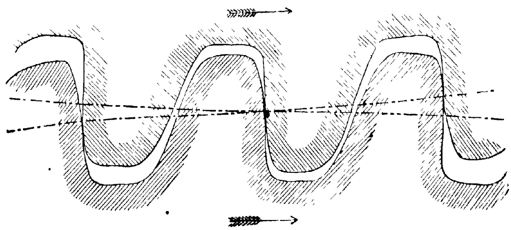


Fig. 275

kind should therefore be used to drive in one direction only. The back faces may be cycloidal curves, described with very small describing circles or involute curves of considerable obliquity. The advantage of this gearing in strength is so considerable that it is surprising it has not been more generally used.

*Methods of drawing cycloidal teeth.*—The curves of the teeth may be found by rolling a templet of the size of the rolling circle inside and outside templets of the size of the pitch circle. A pencil held in contact with the rolling templet describes the required curve. The curves may also be obtained by the ordinary rules for describing cycloidal curves. When they have been drawn, it is usually convenient to replace the cycloidal curves

[1] These teeth were described originally in Willis's *Mechanism*, second edition, p. 142. Willis proposed that the backs of the teeth should be involutes of considerable obliquity.

by circular arcs, sensibly coinciding with them, and which can be used by the pattern-maker more conveniently than the true curves. In proceeding thus, two sources of error are introduced. It is not easy to draw small cycloidal arcs very exactly, and in fitting circular arcs to them a new source of error is introduced. To obviate these objections, it was proposed by Professor Willis to find directly the centres of circular arcs which would approximate to the cycloidal arcs. The method of Professor Willis, however, does not give a very good approximation, the teeth being too thin at the points and too thick at the roots.

The following method, founded on Rankine's rules for rectifying circular arcs, gives a much nearer approximation to the

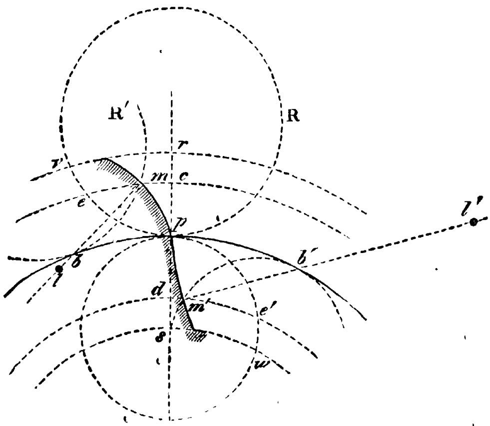



Fig. 276

true curves. The method is also much easier in practice than that of drawing first the true curves. The method is based on this principle. For each cycloidal arc a circular curve is found, which coincides with it at the pitch line, and at $\frac{2}{3}$ its length from the pitch line, and which has at the latter point a common normal with it.

In fig. 276 the strongly marked circle $b\ p\ b'$ is the pitch circle, and $p$ the pitch point. The complete dotted circles are the rolling circles. The height of the tooth outside the pitch line is $p\ r$, and its depth within it is $p\ s$, so that the circles through $r$ and $s$ are the addendum and root circles. The arcs $v\ p\ w$

mark the path of contact. As the rolling circle R rolls to the position R′, the tracing-point moves from $p$ to $m$, marking out the epicycloid $p\ m$, which forms the face of the tooth.

*Method* 1.—Take $p\ c = \frac{2}{3}\ p\ r$, and draw the arc $c\ e$ concentric with the pitch line. Step off arc $p\ b$ = arc $p\ e$. Take the chord $p\ e$ in the compasses, and with centre $b$ mark off $b\ m = p\ e$; then $m$ is a point of the true epicycloid, and $m\ b$ is the normal to the curve at $m$. It is then easy to find in $m\ b$, by trial, a centre $l$ for a circular arc, which will pass through $m$ and $p$.

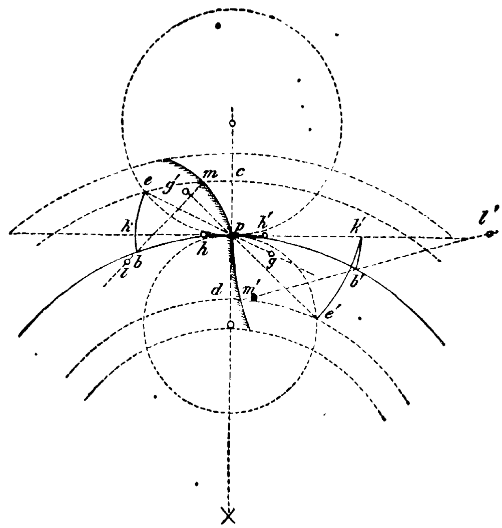



Fig. 277

That circular arc will be the required approximation to the epicycloid.

For the flank of the tooth, make $p\ d = \frac{2}{3}\ p\ s$, and draw the arc $d\ e'$. Step off with the compasses the arc $p\ b'$ = arc $p\ e'$. With centre $b'$ and radius = chord $p\ e'$, cut $d\ e'$ in $m'$. Then $m'$ is a point in the hypocycloid, and $m'\ b'$ is the normal to the curve at $m'$. Find, by trial, a centre $l'$ on $m'\ b'$, for a circular arc passing through $m'$ and $p$. That arc is the required approximation to the hypocycloid.

*Method* 2.—The following method (fig. 277) is the same as the last, except that all the points are found by construction

instead of by trial. Take as in the last method, $p\ c = \frac{2}{3}$ of the height of the tooth, outside the pitch line, and $p\ d = \frac{2}{3}$ the depth within the pitch line. Draw the arcs $c\ e$, $d\ e'$ concentric with the pitch line. Through the pitch point $p$, draw a tangent to the pitch circle. Join $e\ p$, produce it, and make $p\ g = \frac{1}{2}\ p\ e$. With centre $g$ and radius $g\ e$, describe an arc $e\ k$, cutting the tangent at the pitch point in $k$. Then, $p\ k =$ arc $p\ e$. In $p\ k$ take $p\ h = \frac{1}{4}\ p\ k$. From centre $h$, with radius $h\ k$, describe an arc $k\ b$, cutting the pitch line in $b$. Then, arc $p\ b =$ arc $p\ e$. With centre $b$ and radius = chord $p\ e$ cut $c\ e$ in $m$. Join $m\ b$, and in $m\ b$ find a centre $l$ of a circular arc, passing through $m$

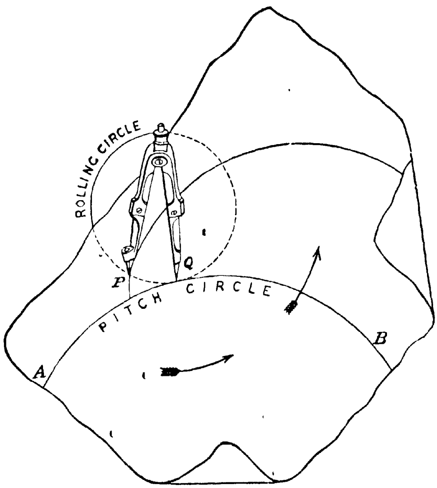



Fig. 278

and $p$. That point $l$ may be found by joining $m\ p$, and drawing a line, bisecting $m\ p$ at right angles. The line so drawn will intersect $m\ b$, produced in $l$. Then the arc $p\ m$, drawn with centre $l$ and radius $l\ m$, is the required approximation to the epicycloid. The same construction gives the flank of the tooth, and the same description is applicable if accented letters are substituted for unaccented letters.

*Method* 3.—*Mr. Last's method of drawing cycloidal curves.*— The same method already described for approximating to a tooth to work with a given tooth may be applied to drawing

cycloidal curves. Let the rolling circle be drawn on a sheet of drawing-paper, and the pitch line on a sheet of tracing-paper (fig. 278). Let the circles first touch at P. Place there the compass point and shift the tracing-paper till the pitch line cuts the rolling circle in a near point Q. Remove the compass point to Q and shift the tracing-paper till the circles touch at Q. Then with Q P as radius, describe a small arc. Next shift the tracing-paper till the pitch line again cuts the rolling circle in a near point and repeat the process. The new radius is to be taken so as to continue the arc already drawn. In the figure an epi-

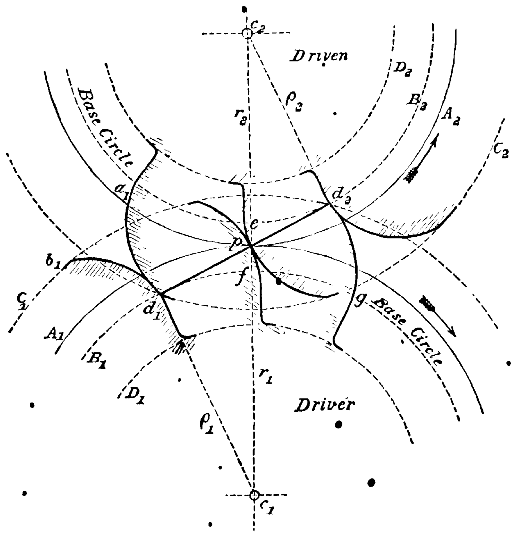



Fig. 279

cycloid is drawn; a hypocycloid or cycloid is described with equal facility. The process with care is very accurate.

### Involute Teeth

234. When the path of contact is a straight line, inclined to the line of centres, the form of the teeth is an involute of a base circle, concentric with the pitch circle, and having the path of contact for a tangent. The pressure between the teeth is in the direction of their common normal very nearly, and this normal coincides in involute teeth with the path of contact.

*Involute teeth with external contact.*—Let $B_1$ $B_2$ be two circles centred at $c_1$ $c_2$, fig. 279. Suppose a string wrapped round and fastened to both circles and lying between them in the direction $d_1$ $d_2$ of their common tangent, which cuts the line of centres at $p$. If the upper wheel turns it will drive the lower wheel. A tracing-point attached to the string will move in space along the straight line $d_1$ $d_2$. At the same time it will mark out on the plane of the upper wheel revolving about $c_2$ an involute $d_1$ $a_1$, and on the plane of the lower wheel revolving about $c_1$ an involute $d_1$ $b_1$. At any moment these two curves will be in contact at the then position of the tracing-point which is moving along $d_1$ $d_2$ and will have a common normal—namely, the line $d_1$ $d_2$ which cuts the line of centres in the constant point $p$. These curves are suitable, therefore, for the teeth of wheels. Circles $A_1$, $A_2$ drawn through $p$ will be the pitch circles. In the case of the string, which acts by pulling, the upper wheel is the driver, but in the case of the teeth, which act by pushing, the lower wheel is the driver. Contact begins at $d_1$ and ends at $d_2$, and $d_1$ $d_2$ is the greatest possible length of the path of contact. Circles $C_1$, $C_2$ through $d_1$, $d_2$ will be the addendum circles if the full possible length of the path of contact is utilised.

The triangles, $c_2$ $p$ $d_2$ and $c_1$ $p$ $d_1$, are similar. Hence, if $r_1$ $r_2$ are the radii of the pitch lines and $\rho_1$ $\rho_2$ the radii of the base circles,

$$\frac{r_1}{r_2}=\frac{\rho_1}{\rho_2}=\frac{pd_1}{pd_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Most commonly the path of contact $d_1$ $d_2$ makes an angle of $14\frac{1}{2}$ or $15\frac{1}{2}°$ with the tangent to the pitch lines, this being the constant angle of obliquity of action of the teeth. In some rather shortened involute teeth used in automobiles, an angle of obliquity of 20° has been adopted. If $\theta$ is the angle of obliquity, the radius of the base circle is

$$\rho = r\cos\theta$$

| | | | |
|---|---|---|---|
| $\theta =$ | $14\frac{1}{2}°$ | $15\frac{1}{2}°$ | $20°$ |
| $\rho/r =$ | ·9681 | ·9636 | ·9397 |

If we move the centres $c_1$ $c_2$ further apart, the obliquity of action is increased, but we obtain for the teeth the same involute curves. Hence in the new position the profiles of the teeth are the same. With involute wheels, if the distance

between the axes of the wheels alters by wear of journals or otherwise, the teeth still act correctly. But the obliquity of action and length of the path of contact is changed. Further, any pair of involute wheels of the same pitch gear correctly together.

The figure shows the tooth profiles at the pitch point, and at beginning and ending contact. Hence arcs of the pitch lines, measured from $p$ to the tooth contour of the wheel at beginning or ending contact, are the arcs of approach and recess for the length of tooth shown.

With the length of path of contact shown the point $d_1$ of the tooth of the upper wheel enters some way within the base circle of the lower wheel and the point $d_2$ of the tooth of the lower

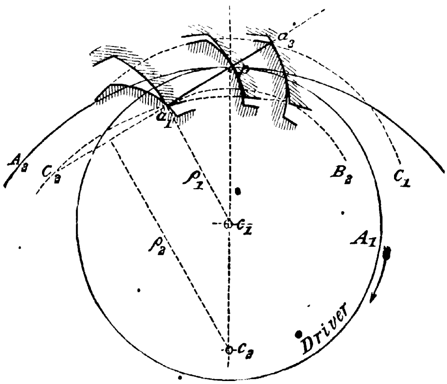


Fig. 280

wheel within the base circle of the upper wheel. Consequently the root circles $D_1$ $D_2$ must be taken inside the base circles so as to clear sufficiently the addendum circles $C_1$ $C_2$.

The flanks of the teeth within the base circles are not working parts of the teeth, hence within the base circles the flanks of each tooth may be straight parallel lines, ending with a fillet at the root circle, or may have any form which does not interfere with the faces of the other teeth. If the arc of action is too large the teeth may be shortened.

The obliquity of action is constant for all positions of the teeth, for $d_1$ $d_2$ is always the normal at the point of contact.

235. *Involute wheels with internal contact.*—Let $c_1 c_2$ (fig. 280)

be the centres of the wheels; $A_1$, $A_2$ the pitch lines. Draw $a_1\,p\,a_2$ inclined at the given angle of obliquity. Perpendiculars on $a_1\,p\,a_2$, from $c_1\,c_2$, are the radii $\rho_1\,\rho_2$ of the base circles. If the smaller wheel is the driver, contact begins at $a_1$, and circle $C_2$, through $a_1$, is the addendum circle of the larger wheel. Choose a length for the addendum of the teeth of the smaller wheel and draw the addendum circle $C_1$. Then contact ends at $a_2$.

236. *Mixed involute and cycloidal teeth.*—It is sometimes necessary to increase the addendum of the teeth in pinions with small numbers of teeth in order to obtain a sufficient arc

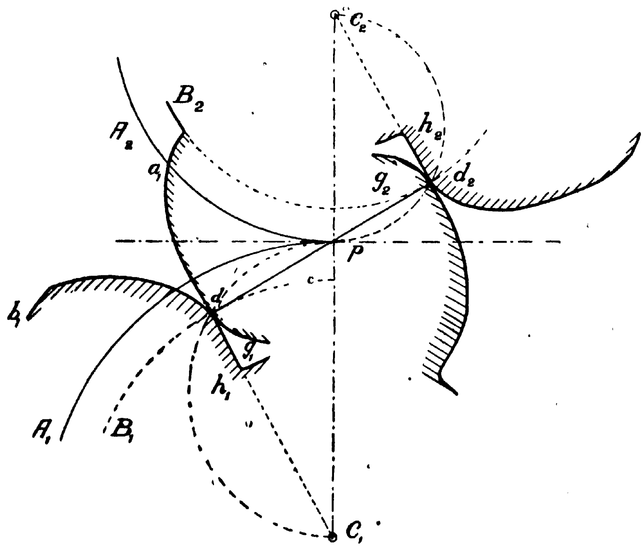



Fig. 281

of action. This may be done by making the tooth profile partly involute and partly cycloidal curves. Let $A_1\,A_2$, fig. 281, be pitch circles, $B_1\,B_2$ base circles, and $d_1\,d_2$ the maximum path of contact for purely involute teeth. Let $d_1\,b_1$, $d_1\,a_1$ be involute curves in contact at $d_1$, on the lower base circle. The working part of the flank of the lower wheel cannot extend as an involute curve within the base circle, and consequently the face of the tooth of the upper wheel, if prolonged as an involute, will not have a corresponding involute flank to work with. But the

teeth and path of contact may be prolonged by adding cycloidal curves, $d_1 g_1$, and $d_1 h_1$. On $C_1 p$ describe a circle which will pass through $d_1$, since $p d_1 C_1$ is a right angle. A tracing point $d_1$ on this circle rolling inside $A_1$, will describe a hypocycloid $d_1 h_1$, which will be a straight radial line on the plane of the lower wheel. Simultaneously by rolling outside $A_2$, it will describe an epicycloid $d_1 g_1$, on the plane of the upper wheel. These curves will work together and have a

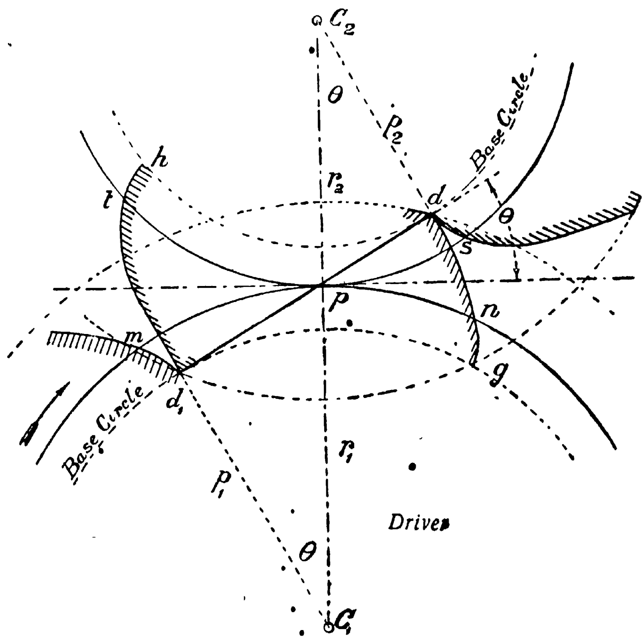



Fig. 282

common normal at the point of contact passing through $p$. Similarly the flank of the tooth of the upper wheel and face of that of the lower wheel may be prolonged by cycloidal curves $d_2 h_2$, $d_2 g_2$ described by a tracing point on a rolling circle having $C_2 p$ for diameter. The whole path of contact of such teeth will consist of the straight line $d_1 d_2$ and arcs of the two rolling circles. The proportions in the figure have been much exaggerated for clearness.

237. *Least number of teeth in involute wheels.*—In fig. 282, $d_1\, p\, d_2$ tangent to the base circles is the maximum possible length of the path of contact, if the tooth curves are involutes. But $d_1\, d_2 = (r_1 + r_2) \sin \theta$, where $\theta$ is the angle of obliquity of the teeth. Also $d_1\, d_2 = \text{arc } d_1\, g = \text{arc } d_2\, h$, the arcs of the base circles turned through as the contact of the teeth moves from $d_1$ to $d_2$, along the path of contact and from $g$ to $d_2$ along the tooth profile. The corresponding arcs of the pitch lines are $m\, n = s\, t = (r_1 + r_2) \sin \theta \times r_1/\rho_1 = (r_1 + r_2) \sin \theta \times r_2/\rho_2 = (r_1 + r_2) \tan \theta$. This is the arc of action of either wheel. But the arc of action must be at least equal to the pitch or one pair of teeth would go out of action before the next pair engaged. Let the arc of action be $x$ times the pitch. Then in wheels working intermittently $x$ should be at least $= 1\frac{1}{4}$ and in continuously driving wheels is generally greater. Let $T_1$, $T_2$ be the numbers of teeth in the wheels. Then the pitch is—

$$p = 2\,\pi\, r_1/T_1 = 2\,\pi\, r_2/T_2$$

$$(r_1 + r_2) \tan \theta = \frac{2\,\pi\, r_1\, x}{T_1} = \frac{2\,\pi\, r_2\, x}{T_1}$$

Suppose the wheels are equal, so that $T_1 = T_2 = T$ and $r_1 = r_2 = r$, Then the least possible number of teeth is the nearest integer greater than

$$T = \pi\, x/\tan \theta \quad . \quad . \quad . \quad . \quad . \quad (5)$$

It can be seen that the addendum of the teeth is slightly greater than $r \sin \theta \tan \theta$, when the whole possible path of contact is used. The ratio of addendum to pitch is $(T \sin \theta \tan \theta)/2\pi$ approximately. The following table gives values of the smallest number of teeth and addendum for various values of $\theta$ and $x$.

| $\theta =$ | 15° | | | 20° | | |
|---|---|---|---|---|---|---|
| $x =$ | 1 | $1\frac{1}{4}$ | 2 | 1 | $1\frac{1}{4}$ | 2 |
| Least number of teeth | 12 | 15 | 24 | 9 | 11 | 18 |
| Addendum/pitch | ·13 | ·27 | ·27 | ·18 | ·22 | ·36 |

Here the ratio addendum/pitch is only approximately calculated, but it is sufficient to show that in wheels with the minimum number of teeth this ratio is less than that most commonly adopted. If, for instance, this ratio is to be 0·318 $p$, the pitch

must be reduced and the number of teeth increased for the given value of $r$. Thus for 15° obliquity the least number of teeth would be 29.

238. *Approximate method of describing the involute.*—The involute is not difficult to describe, but the following method gives a very accurate circular approximation. Let $e_1\ e_2$ (fig. 283) be the working height of the teeth, or the distance between circles $C_1\ B_1$, in fig. 279, measured along the line of centres. Take $e_1\ g = \frac{2}{3}\ e_1\ e_2$. Draw a tangent $g\ h$ to the base circle. Take $h\ k = \frac{1}{4}\ h\ g$. Then, a circle $m\ n$ struck from $k$ with radius $k\ g$, will be the required approximation to the involute. It will coincide with the involute at $n$ and $g$, and will have the same normal at $g$. The part of the tooth below the base circle which is not a working part of the tooth may be a tangent to the involute,

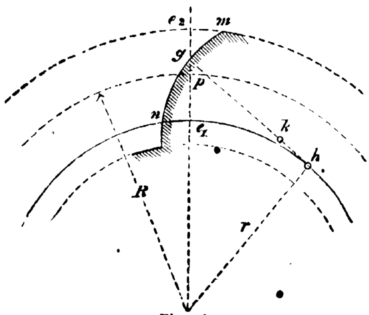


Fig. 283

and there is room for a fillet at the root without interfering with the addendum of the tooth of the other wheel.

239. *To determine the length of involute teeth for given arcs of approach and recess.*—Let $A_1\ A_2$ be the pitch lines (fig. 284). Draw $a_1\ p\ a_2$ at the required obliquity. Perpendiculars $\rho_1\ \rho_2$ on $a_1\ p\ a_2$ are the radii of the base circles $B_1\ B_2$. Let the line of centres $c_1\ c_2$ cut the base circles in the points $n\ o$. Now lay off on the pitch lines the required arcs of approach and recess $b_1\ p$ and $p\ b_2$. Draw radii from $b_1$, $b_2$ to $c_1$, $c_2$, the centres of the wheels, cutting off the corresponding arcs $n\ e_1$, $o\ e_2$ on the base circles. Now laying off on $a_1 p\ a_2$, $p\ d_1 =$ arc $n\ e_1$ and $p\ d_2 =$ arc $o\ e_2$, $d_1\ p\ d_2$ is the required path of contact. Contact begins at $d_1$ and ends at $d_2$, and circles $C_1\ C_2$ through $d_2$ and $d_1$ are the addendum circles. The root circles $D_1 D_2$ must be drawn for each

wheel to allow room for the points of the teeth of the other wheel.

Take a radius through $d_1$, then $d_1\,h$ is the working height of the tooth of the lower wheel. Take $d_1\,k = \frac{2}{3}\,d_1\,h$, and from $k$ draw a tangent to the base circle. Take $m\,k = \frac{3}{4}$ of this tangent, then $m$ is the centre of a circular arc $f\,k\,g$ approximating closely to the involute. The part of the tooth between $g$ and the root circle $D_1$, may be radial. A corresponding construction gives the form of tooth for the other wheel.

240. *Brown and Sharpe's construction for approximate involute teeth.*—For wheels with 30 teeth or more. Draw the pitch circle,

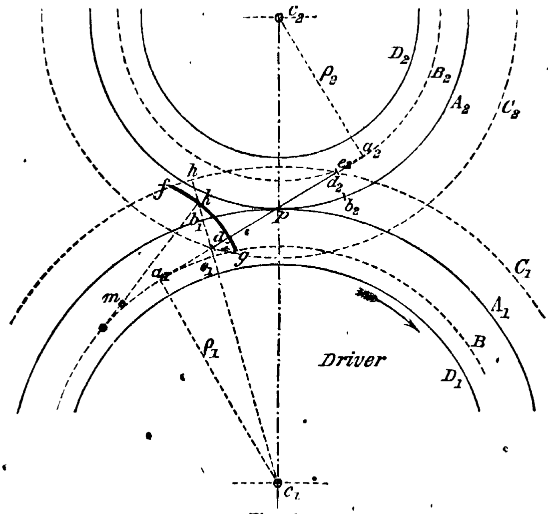



Fig. 284

addendum circle, and root circle and let C be the centre and C $p$ the radius of the pitch circle. Then $p$ is the pitch point. On C $p$ describe a semicircle. From $p$ set off as a chord of the semicircle $pa = \frac{1}{4}$ C $p$. With centre C and radius C $a$ draw the base circle; $p\,a$ is the path of contact and makes an angle of $14\frac{1}{2}°$ with the tangent to the pitch circle at $p$. C $a$ = 0·968 C $p$. A circle struck from $a$ with radius $a\,p$ gives the tooth profile. The profile is completed by a fillet circle touching the profile and the root circle, and having a radius equal to $\frac{1}{8}$ of the space, $m\,n$, between the teeth at the addendum circle. The other

profiles are easily drawn by spacing out the pitch circle with distances equal to the tooth thickness and space, and drawing circles through these points with radius $p\ a$ and centres on the base circle.

For wheels with less than 30 teeth the construction is modified. Draw the pitch circle, addendum circle, and root circle, fig. 286,

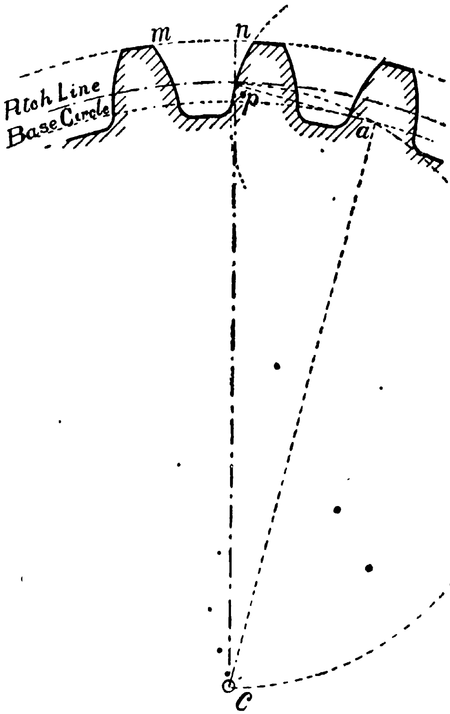



Fig. 285

and let as before $p$ be the pitch point. Describe a semicircle on c $p$. Take $p\ a = \frac{1}{4}$ c $p$ cutting the semicircle in $a$. Through $a$ draw the base circle. With centre $a$ and radius $a\ p$ draw the tooth profile from the base circle to the addendum circle. So far the construction is the same as before. Within the base circle it is different. With centre c describe a circle with radius

equal to half the tooth space at the base circle. Continue the tooth profile within the base circle by a straight line tangent to this circle, for a distance equal to $\frac{1}{3}$ *s* for wheels of 12 or 13 teeth; $\frac{1}{5}$ *s* for wheels of 15 to 16 teeth, and $\frac{1}{8}$ *s* for wheels of 17 to 20 teeth. For wheels of 20 to 30 teeth the straight part is omitted. Then continue the profile by an arc of a circle struck from *e*, the middle of the thickness of the next tooth on the pitch line. Complete the profile by a fillet of radius $\frac{1}{8}$ *m n*, the width of space on the addendum circle, touching the profile and the root circle.

241. *Rack with involute teeth.*—The rack tooth profile is a straight line normal to the path of contact. Describe the semi-

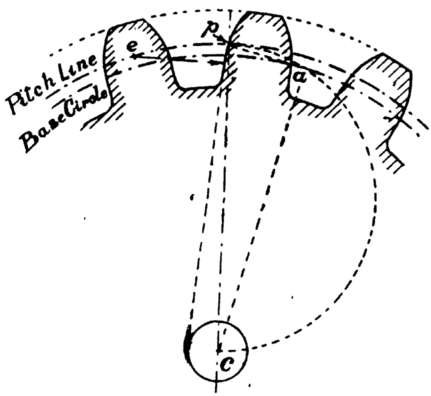



Fig. 286

circle on the radius c *p* of the wheel. Take $p\,a = \frac{1}{4}$ c *p*. Then *p a* is the path of contact and the tooth profile is normal to *p a*. It is straight from the addendum circle to a depth below the pitch line equal to the addendum and is finished by a fillet curve below.

*General considerations.*—Involute teeth have two remarkable properties. All involute wheels, whose teeth have the same pitch and the same obliquity of the line of contact, work well together. A pair of involute wheels may be drawn a little further apart without the accuracy of action of the teeth being impaired, though the arc of contact is diminished. Involute wheels cannot be made with very long teeth, because then the obliquity of the

line of contact must be great. Hence, the centres cannot be moved much further apart than their normal distance, without

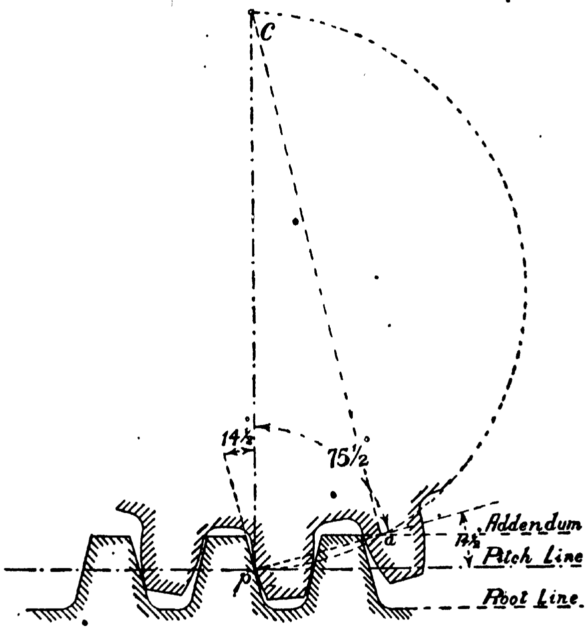



Fig. 287

too much reducing the arc. of contact. But this property of involute wheels is a valuable one, as it neutralises the injurious

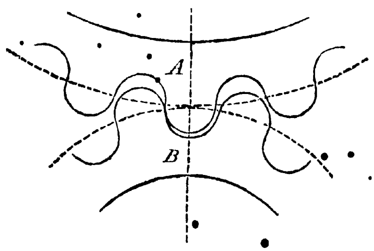



Fig. 288

effect of wear of the supports of the wheels. It is probably a practical advantage that the profile of an involute tooth is a

single continuous curve, with no change of curvature at the pitch line.

242. *Knuckle gearing* (fig. 288) is an imperfect form of gearing

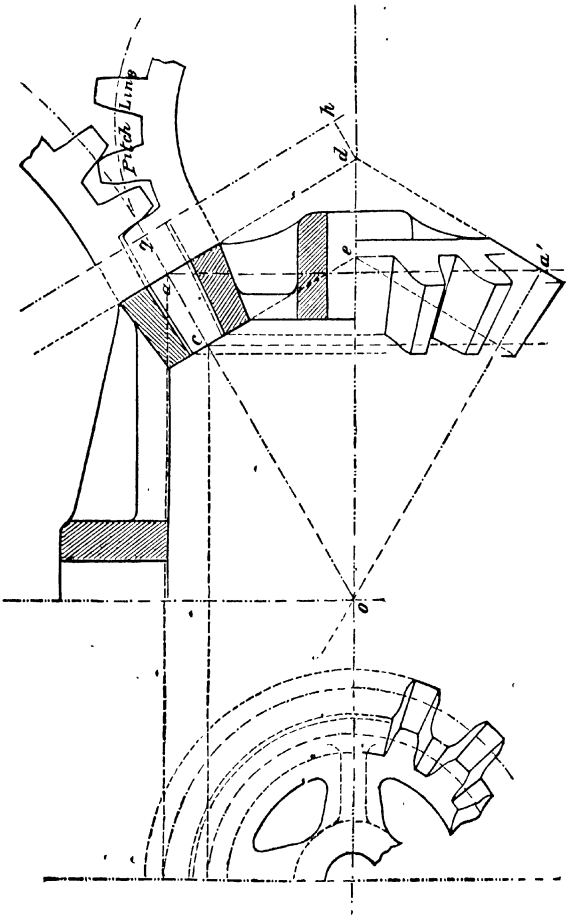



Fig. 289

for cranes and slow-moving machinery. The action of the teeth is, however, very imperfect except for an arc of contact smaller than the pitch. There must be, therefore, a change of

velocity ratio as the teeth come into and go out of contact. They are, however, shorter, and therefore stronger than normal teeth.

## TEETH OF BEVIL WHEELS

243. The teeth of bevil wheels may be cycloidal or involute, and are described in the same way as the teeth of spur wheels, upon a development of the conical surfaces which limit their length. Let fig. 289 represent the section of a bevil wheel rim. O $a$ is the intersection of the conical pitch surface with the plane of the paper, O $d$ the axis of the wheel. Let $a$ $c$ be the width of face of the wheel. Draw $a$ $d$, $c$ $e$, perpendicular to O $a$, cutting the axis of the wheel in $e$ and $d$. Then the teeth are limited in length by the conical surfaces, whose intersections with the paper are $e$ $c$, $d$ $a$ and which have O $d$ as axis.

Project $d$ $a$ to $h$ $g$ for convenience. Then a circle drawn with $h$ $g$ as radius is the virtual pitch line of the ends of the teeth, and the teeth are described on that circle as if it were the actual pitch line.

If $R_1$ $R_2$ are the radii of two bevil wheels on shafts at right angles, and $r_1$ $r_2$ the corresponding radii of the virtual pitch lines,

$$\frac{r_1}{R_1} = \frac{\sqrt{(R_1^2 + R_2^2)}}{R_2}$$

$$\frac{r_2}{R_2} = \frac{\sqrt{(R_1^2 + R_2^2)}}{R_1}$$

## SUMMARY OF CURVES FOR TOOTHED WHEELS

244. $R_1$, $R_2$ = the radii of the wheels ; $r$ = radius of rolling circle ; $p$ the pitch of the wheels ; T the number of teeth in the smallest wheel of the set ; $\rho_1$ $\rho_2$ radii of base circles of involutes.

### EXTERNAL CONTACT

*Cycloidal Curves*

Case I.—*Pair of wheels with radial flanks* (fig. 290).
Face of $R_1$, epicycloid, $r = \frac{1}{2} R_2$.
Flank of $R_1$, radial line, $r = \frac{1}{2} R_1$.
Face of $R_2$, epicycloid, $r = \frac{1}{2} R_1$.
Flank of $R_2$, radial line, $r = \frac{1}{2} R_2$.

Case II.—*Set of wheels of which any two are to work together* (fig. 291).

Faces of $R_1$ and $R_2$, epicycloids.

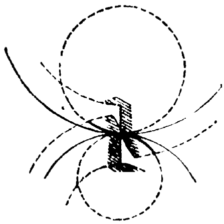


Fig. 290

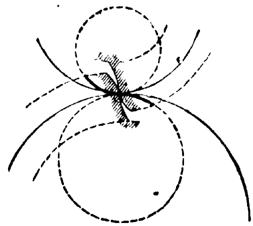


Fig. 291

Flanks of $R_1$ and $R_2$, hypocycloids.

Radius of rolling circle for all the cycloidal curves $r = \frac{1}{2}\frac{pT}{2\pi}$.

Case III.—*Pair of wheels, contact during recess only* (fig. 292).
Flank of $r_1$, radial line.
Face of $R_2$, epicycloid, $r = \frac{1}{2} R_1$.
$R_2$ is the driver.

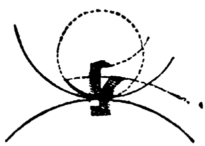


Fig. 292

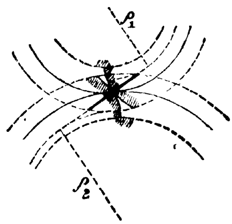



Fig. 293

*Involute Curves*

Case IV.—*Set of wheels, any two of which work together.*

The curve of each tooth (fig. 293) is an involute, the base circles being chosen so that

$$\frac{\rho_1}{\rho_2} = \frac{R_1}{R_2}.$$

The parts of the teeth beyond the region of contact may be radial.

INTERNAL CONTACT

*Cycloidal Curves*

Case V.—*Set of wheels, any two of which work together* (fig. 294.)

Face of $R_1$ and flank of $R_2$, epicycloids.
Flank of $R_1$ and face of $R_2$, hypocycloids.
Radius of rolling circle for all the curves $r = \frac{1}{2} \cdot \frac{p\,T}{2\,\pi}$.

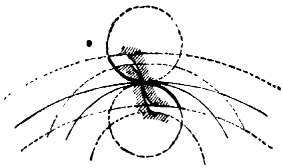


Fig. 294

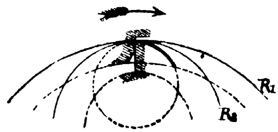



Fig. 295

Case VI.—*Two wheels having contact only during recess.*
Face of $R_1$, epicycloid, $r = \frac{1}{2} R_2$.
Flank of $R_2$, radial line.
$R_1$ is the driver (fig. 295).

*Involute Curves*

Case VII.—*Set of wheels, any two of which gear together.*
The curve of each tooth is an involute (fig. 296), the base circles being chosen so that

$$\frac{\rho_1}{\rho_2} = \frac{R_1}{R_2}.$$

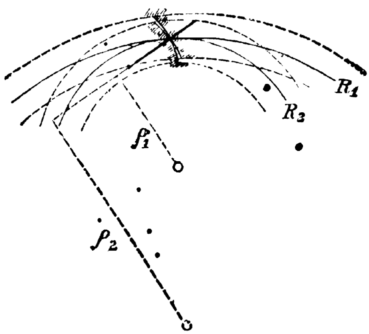



Fig. 296

Beyond the region of contact the tooth of $R_2$ may be tangential to the curve.

PINION AND RACK

Case VIII.—*Set of wheels to work with one rack* (fig. 297).
Face of pinion tooth, epicycloid.

Flank of pinion tooth, hypocycloid.
Face and flank of rack tooth, cycloids.

*Radius of rolling circle for all the curves* $r = \frac{1}{2}\frac{p\,T}{2\,\pi}$.

*Case IX.—Single wheel to work with rack (fig. 298).*

Face of pinion tooth an epicycloid described with $r = \infty$, consequently an involute.

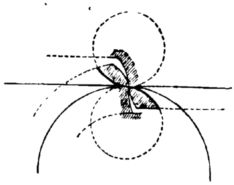


Fig. 297

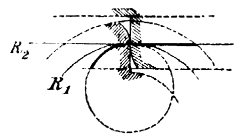



Fig. 298

Flank of pinion tooth, hypocycloid, $r = \frac{1}{2}\,R_1$.

Face of rack tooth, cycloid, $r = \frac{1}{2}\,R_1$.

Flank of rack tooth, hypocycloid described with $r = \infty$, and therefore a straight line perpendicular to the pitch line.

*Involute Curves*

Pinion tooth an involute, with tangential prolongation beyond the region of contact (fig. 299).

Rack tooth, a straight line perpendicular to the path of contact.

In all the figures, the pitch lines are thin full lines, the rolling

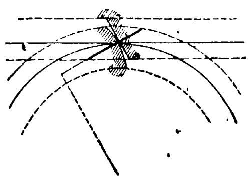


Fig. 299

or base circles dotted lines, the path of contact a thick full line.

## Strength of Wheel Teeth

245. In determining the strength of wheel teeth, it is not usually necessary to take into account their curved form. It is sufficiently accurate to treat the tooth as a rectangular

cantilever (fig. 300), of thickness $3b$, uniform and equal to the thickness of the actual tooth at the pitch line. Usually two pairs of teeth are simultaneously in contact. The pressure transmitted is therefore shared by two pairs of teeth. The wheels cannot be made accurately enough to insure an equal distribution of the pressure to both pairs. Hence, if P is the whole pressure transmitted, the greatest pressure on one pair of teeth is $n$ P, where $n$ is a fraction lying between $\frac{1}{2}$ and 1. The teeth are in contact at a line which, in spur wheels, is parallel to the axis of rotation. The line of contact varies in position during the action of the teeth, and either at the beginning or end of contact coincides with the extreme edge of the tooth. Ordinarily, in teeth which have worn a little by mutual friction, the pressure will be distributed with approximate uniformity along the edge of the tooth, and will tend to break the tooth across at its root along its whole breadth. Another contingency less favourable to the strength of the tooth is possible. From inaccurate form in the teeth or inaccurate fixing of the wheels, or deflection of the shafts, the pressure may be restricted to a small portion of the edge of the tooth. In that case the whole pressure P will come on a single tooth and may come on a corner of one tooth.

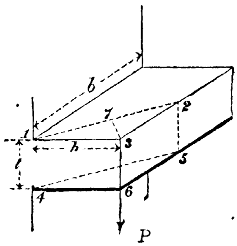


Fig. 300

*Relation of load on teeth, velocity of pitch line, and power transmitted.*—Let P be the whole pressure of one wheel on the other, estimated in the direction of motion; H the number of horses' power transmitted by the wheel; N the number of revolutions of the wheel per minute; $d$ its diameter in inches; V the velocity of the pitch line in ft. per sec.

$$V = \frac{\pi d N}{12 \times 60} = \cdot 00436\, d\, N$$

$$P = \frac{550\, H}{V} = 126{,}040\, \frac{H}{d\, N} \qquad (6)$$

Let $p$ be the pitch of the wheels in inches, T the number of teeth. Then the velocity of the pitch line in feet per second is

$$V = \frac{p\, T\, N}{12 \times 60};$$

consequently the pressure in lbs. on all the teeth in action at any moment be expressed in the form

$$P = \frac{550 \times 12 \times 60 \times H}{p\,T\,N};$$
$$= 396{,}000 \frac{H}{p\,T\,N} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

246. *Case I. Strength of wheel teeth when the pressure may come on a corner of one tooth.*—In crane gearing, hand-worked gearing, and similar cases where permanence of the initial adjustments cannot be relied on, or where, from looseness of bearings or flexibility of shafts, perfect parallelism of the tooth surfaces cannot be secured, and where, generally the speed is slow, the strength should be determined on the assumption that the whole load, P, may come on a corner of one tooth.

Let the height of the tooth 13, fig. 300, $= h$; its thickness $14 = t$; the width of face $= b$. Then, if the pressure P is applied at a corner, it tends to break off a triangular prism, bounded by a plane 1 2 5 4, which passes through the root of the tooth. Draw 37 perpendicular to that plane, and let the angle $2\,1\,3 = \theta$; then,

$$\overline{37} = \overline{13} \sin\theta = h \sin\theta.$$
$$\overline{12} = \overline{13} \sec.\theta = h \sec.\theta.$$

The bending moment of P, with respect to the section 1 2 5 4, is $P\,h \sin\theta$. The moment of resistance of that section to bending is $\frac{1}{6} f\,h\,t^2 \sec.\theta$. Equating the bending moment and moment of resistance, we get for the greatest stress due to bending,

$$f = \frac{3\,P}{t^2} \sin 2\,\theta.$$

The stress on the diagonal section of the tooth will be a maximum when $\theta = 45^\circ$ and $\sin 2\,\theta = 1$. Then,

$$f = \frac{3\,P}{t^2}$$

Let $t = 0{\cdot}45\,p$, then

$$\left.\begin{aligned} f &= 14{\cdot}85\,P/p^2 \\ P &= {\cdot}067 f\,p^2 \\ p &= 3{\cdot}85 \sqrt{(P/f)} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The formula fails, of course, if the width of face $b < h$, but such

a proportion does not occur in actual wheels. It is not desirable in gearing of this kind to make $b$ more than $1\frac{1}{2}$ to $2\ p$. In obtaining this formula some assumptions are made and the calculation is a rather rough one. For different wheels strained to the same working stress, $p$ is proportional to $\sqrt{\text{P}}$, and hence

$$\left.\begin{aligned} p &= \text{K} \sqrt{\text{P}} \\ \text{P} &= \text{C} p^2 \end{aligned}\right\} \qquad \ldots \ldots \ldots \quad (9)$$

where K and C are constants to be determined by experience. These constants vary considerably in different cases. In slowly moving gearing, of cast iron, not subjected to much vibration or shock, K = 0·045 and C = 500; in average cases, K = 0·05 and C = 400; in wheels subjected to excessive vibration and shock, as in gearing which drives some machine tools, K = 0·055 and C = 330.

*Safe pressure* P *at pitch line of Cast Iron Wheels. Crane Gearing.*

| Pitch in Inches | Safe pressure on teeth in lbs. | | |
|---|---|---|---|
| | Little shock C=500 | Average cases C=400 | Excessive shock C=330 |
| $\frac{3}{4}$ | 281 | 225 | 185 |
| 1 | 500 | 400 | 330 |
| $1\frac{1}{4}$ | 780 | 624 | 515 |
| $1\frac{1}{2}$ | 1,125 | 900 | 742 |
| $1\frac{3}{4}$ | 1,530 | 1,224 | 1,009 |
| 2 | 2,000 | 1,600 | 1,320 |
| $2\frac{1}{4}$ | 2,530 | 2,024 | 1,670 |
| $2\frac{1}{2}$ | 3,125 | 2,500 | 2,062 |
| $2\frac{3}{4}$ | 3,780 | 3,024 | 2,495 |
| 3 | 4,500 | 3,600 | 2,970 |
| $3\frac{1}{4}$ | 5,300 | 4,240 | 3,496 |
| $3\frac{1}{2}$ | 6,120 | 4,900 | 4,043 |
| $3\frac{3}{4}$ | 7,050 | 5,640 | 4,653 |
| 4 | 8,000 | 6,400 | 5,280 |

The values of the working stress $f$ in the three cases given above are about 7,300, 5,900, and 4,900 lbs. per sq. in.

Cast-steel gearing will carry about double, forged steel about three times, and phosphor bronze about one and a-half times the loads given in the Table.

247. Case II.—*Strength of wheel teeth in carefully fitted gearing, when the pressure is assumed to be distributed along the whole*

*width of the tooth* (fig. 301).—Let P be the total pressure and $n$ P the pressure on one pair of teeth, as before, and let $p$ be the pitch, $t$ the thickness of tooth, $h$ the height of tooth, and $b$ the width of face. If the pressure is distributed along the edge of the tooth, the bending moment at its root is $n$ P $h$. The moment of resistance of the section 1 2 3 4 of the tooth is $\frac{1}{6} f\, b\, t^2$ where $f$ is the working stress. Equating these,

$$P = \tfrac{1}{6} \frac{b\, t^2}{n\, h} f.$$

Let $t = 0{\cdot}45\ p$ for cast gearing, $0{\cdot}5$ for cut gearing, and let $h = 0{\cdot}7\ p$ for normal and $0{\cdot}45\ p$ for shortened wheel teeth.

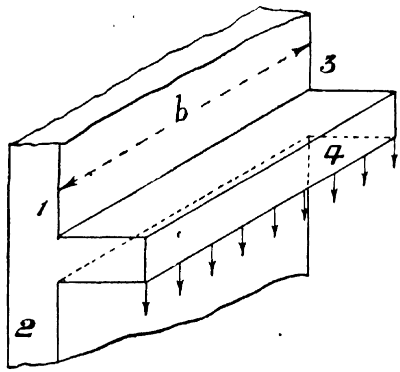



Fig 301

These proportions rather underestimate the root thickness in wheels of 24 teeth and upwards, and somewhat overestimate it for low-numbered pinions. In this last case it is desirable to increase a little the calculated pitch. Also let $n = 1$, which is on the side of safety, the load being probably distributed more or less to two pairs of teeth. Then

$$\left.\begin{aligned} P &= c\, b\, p\, f \\ p &= P/(c\, b\, f) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

*Values of c*

| | Cast Gearing. | Cut Gearing. |
|---|---|---|
| Teeth of normal height | ·048 | ·060 |
| Shortened teeth | ·075 | ·092 |

The stress on the teeth is a constantly and very frequently

changing stress. Allowing something for shock, the values of the safe working stress $f$ may be taken as follows :—

| | lbs. per sq. in. |
|---|---|
| Cast iron . . . . . . | $f =$ 4,200 |
| Steel casting . . . . . . | 6,000 |
| Forged steel . . . . . | 10,000 |
| Phosphor bronze . . . . . | 6,500 |

Putting the equation 10 above in the form—

$$P = k\,b\,p \text{ or } p = P/kb \quad . \quad . \quad . \quad . \quad (11)$$

Where $k$ is the safe load per inch width of face on a wheel of one inch pitch, the values of $k$ are as follows :—

*Values of constant k.*

| — | Cast Gearing | | Cut Gearing | |
|---|---|---|---|---|
| | Teeth of normal height | Shortened teeth | Teeth of normal height | Shortened teeth |
| Cast iron . . . | 202 | 315 | 250 | 386 |
| Steel casting . . | 288 | 450 | 360 | 550 |
| Forged steel . . | — | — | 600 | 920 |
| Phosphor bronze . | 312 | 480 | — | — |

For gearing of other materials there is less experience to determine the value of the constant. For a raw hide pinion $k =$ about 85. For fairly fast gearing on electric motors, $k = 250$ for forged steel and 500 for chrome steel for teeth of normal height.

The tables on p. 402 are useful in settling a preliminary approximation to the pitch of cast-iron wheels.

248. *Ratio of width of face to pitch.*—In the formula for P there are two dimensions $b$ and $p$ to be determined. The ratio $b/p$, which is $1\frac{1}{2}$ to 2, in wheels where strength only has to be considered, may be increased in transmission gearing with the advantage that by increasing the width of face a smaller pitch suffices, the number of teeth for a given diameter is greater, and the wheels work more smoothly. Hence $b/p$ is usually greater in high-speed than in low-speed gearing. But as the width of face is made larger the assumption of a uniform distribution of stress along the edge becomes somewhat less trustworthy. For that reason when $b/p$ is large a lower working stress and therefore a lower value of $k$ should be taken. There are practical limits to the width of face which is convenient. In cast gearing

$b/p$ does not generally exceed 4; in cut gears somewhat greater widths are permitted. In small high-speed cut gearing, where

*Safe pressure* P *at pitch line of cast-iron wheels, transmission gearing.*

$k = 200$

| Pitch in inches | Pressure on teeth in lbs when $b/p$ = | | | | | |
|---|---|---|---|---|---|---|
| | 2 | 2¼ | 2½ | 3 | 3½ | 4 |
| 1 | 400 | 450 | 500 | 606 | 700 | 800 |
| 1¼ | 624 | 702 | 780 | 930 | 1,092 | 1,248 |
| 1½ | 900 | 1,012 | 1,150 | 1,400 | 1,550 | 1,800 |
| 1¾ | 1,224 | 1,377 | 1,530 | 1,836 | 2,142 | 2,448 |
| 2 | 1,600 | 1,800 | 2,000 | 2,400 | 2,800 | 3,200 |
| 2¼ | 2,024 | 2,277 | 2,530 | 3,036 | 3,542 | 4,048 |
| 2½ | 2,500 | 2,812 | 3,125 | 3,750 | 4,375 | 5,000 |
| 2¾ | 3,024 | 3,402 | 3,780 | 4,536 | 5,292 | 6,048 |
| 3 | 3,600 | 4,050 | 4,500 | 5,400 | 6,300 | 7,200 |
| 3¼ | 4,224 | 4,752 | 5,280 | 6,336 | 7,392 | 8,448 |
| 3½ | 4,900 | 5,512 | 6,125 | 7,350 | 8,575 | 9,800 |
| 3¾ | 5,624 | 6,327 | 7,030 | 8,436 | 9,842 | 11,248 |
| 4 | 6,400 | 7,200 | 8,000 | 9,600 | 11,200 | 12,800 |
| 4½ | 8,100 | 9,112 | 10,125 | 12,150 | 14,175 | 16,200 |
| 5 | 10,000 | 11,250 | 12,500 | 15,000 | 17,500 | 20,000 |
| 5½ | 12,100 | 13,612 | 15,125 | 18,150 | 21,175 | 24,200 |
| 6 | 14,400 | 16,200 | 18,000 | 21,600 | 25,200 | 28,800 |

$k = 400$

| Pitch in inches | Pressure on teeth in lbs when $b/p$ | | | | | |
|---|---|---|---|---|---|---|
| | 2 | 2¼ | 2½ | 3 | 3½ | 4 |
| 1 | 800 | 900 | 1,000 | 1,200 | 1,400 | 1,600 |
| 1¼ | 1,248 | 1,404 | 1,560 | 1,872 | 2,184 | 2,496 |
| 1½ | 1,800 | 2,025 | 2,250 | 2,700 | 3,150 | 3,600 |
| 1¾ | 2,448 | 2,754 | 3,060 | 3,672 | 4,284 | 4,896 |
| 2 | 3,200 | 3,600 | 4,000 | 4,800 | 5,600 | 6,400 |
| 2¼ | 4,048 | 4,048 | 5,060 | 6,072 | 7,084 | 8,096 |
| 2½ | 5,000 | 5,625 | 6,250 | 7,500 | 8,750 | 10,000 |
| 2¾ | 6,048 | 6,804 | 7,569 | 9,072 | 10,584 | 12,096 |
| 3 | 7,200 | 8,100 | 9,000 | 10,800 | 12,600 | 14,400 |
| 3¼ | 8,480 | 9,540 | 10,560 | 12,720 | 14,840 | 16,960 |
| 3½ | 9,800 | 10,980 | 12,250 | 14,640 | 17,080 | 19,520 |
| 3¾ | 11,280 | 12,690 | 14,060 | 16,920 | 19,740 | 22,560 |
| 4 | 12,800 | 14,400 | 16,000 | 19,200 | 22,400 | 25,600 |
| 4½ | 16,240 | 18,270 | 20,250 | 24,360 | 28,420 | 32,480 |
| 5 | 20,000 | 22,500 | 25,000 | 30,000 | 35,000 | 40,000 |
| 5½ | 24,240 | 27,270 | 30,250 | 36,360 | 42,420 | 48,480 |
| 6 | 28,800 | 32,460 | 36,000 | 43,200 | 50,400 | 57,600 |

smoothness of action is very important, $b/p$ is sometimes 6 to 8, though these proportions seem excessive.

In constantly running gearing, durability is as important as strength. To obtain durability it is desirable that values of $k$ should be taken smaller as the speed of the wheel is greater. For cast-iron mill gearing with normal teeth a rule of the following form may be adopted.—

$$k = 325 - 14\sqrt{\text{N}} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where N is the revolutions per minute of the smaller wheel of the pair. This value of $k$ should be used only when it is smaller than that in the table above.

*Values of k for cast-iron mill gearing*

| | | | | | | |
|---|---|---|---|---|---|---|
| N = | 50 | 100 | 150 | 200 | 250 | 300 |
| $k$ = | 226 | 185 | 154 | 128 | 104 | 83 |

The following table gives data of some actual spur flywheels and pinions of large size.

| Horse-power transmitted | Revolutions of pinion per minute | No. of teeth in pinion | Pitch, inches | Width of face, inches | Pressure on teeth per inch width | Value of $k$ |
|---|---|---|---|---|---|---|
| 700 | 133 | 42 | 4·5 | 14 | 722 | 161 |
| 1,000 | 94 | 46 | 5·6 | 18 | 833 | 149 |
| 1,100 | 113 | 47 | 5·0 | 17 | 883 | 177 |
| 1,130 | 99 | 43 | 5·8 | 18 | 923 | 160 |
| 1,200 | 93 | 52 | 5·0 | 19 | 942 | 189 |
| 1,300 | 80 | 71 | 4·5 | 18 | 1,070 | 239 |

249. *Lewis Formula.*—Mr. Wilfred Lewis ('Proc. Philadelphia Engineering Club,' 1893) has made an attempt to form an equation taking account of the actual tooth thickness at the root. Involute teeth were drawn out to a large scale and the position and thickness of the dangerous section ascertained. Mr. Lewis's investigation led to the following formulæ for involute wheels with 15° and 20° obliquity of action and T teeth in the smaller wheel :—

$$\theta = 15^\circ. \quad \text{P} = p\,b\,f\left(0{\cdot}124 - \frac{0{\cdot}684}{\text{T}}\right)$$

$$\theta = 20^\circ. \quad \text{P} = p\,b\,f\left(0{\cdot}154 - \frac{0{\cdot}912}{\text{T}}\right)$$

The value of the quantity in brackets is as follows :—

| | | | | |
|---|---|---|---|---|
| T = | 15 | 24 | 30 | 36 |
| $\theta = 15^\circ$ | ·078 | ·096 | ·101 | ·105 |
| $\theta = 20^\circ$ | ·093 | ·116 | ·124 | ·129 |

This quantity in brackets corresponds to $c$ in equation 10, p. 400, and the values are markedly greater than those found by assuming a tooth of simple rectangular form. This is not surprising, as involute teeth, especially of large obliquity, are very strong. But the value to be taken for $f$ is left uncertain.

250. *Strength of mortice teeth.*—In the case of hornbeam teeth, used in important gearing to secure silent running, $t = 0{\cdot}6\ p$; and $f = 1{,}060$ to $1{,}600$. Hence in this case $k = 95$ to $144$.

251. *Other convenient forms of the equation for the strength of wheel teeth.*—Very often the ratio $b/p$ is decided initially, or assumed by comparison with some similar case. Then

$$\left.\begin{aligned} P &= k(b/p)\ p^2 \\ p &= \sqrt{\frac{1}{k}}\ \sqrt{\{(p/b)\ P\}} \end{aligned}\right\} \qquad (13)$$

*Values of* $\sqrt{(1/k)}$

| — | Cast gearing | | Cut gearing | |
|---|---|---|---|---|
| | Normal teeth | Shortened teeth | Normal teeth | Shortened teeth |
| Cast iron . . . . | ·070 | ·056 | ·063 | ·051 |
| Steel casting . . . | ·059 | ·047 | ·053 | ·043 |
| Forged steel . . . . | — | — | ·041 | ·033 |

For values of $k$ determined with reference to the speed by eq. 12 for cast-iron mill gearing—

| | | | | | | |
|---|---|---|---|---|---|---|
| $N =$ | 50 | 100 | 150 | 200 | 250 | 300 |
| $\sqrt{(1/k)} =$ | ·067 | ·074 | ·081 | ·088 | ·098 | ·109 |

Let $H$ be the horse-power to be transmitted, $V$ the velocity of the pitch line in ft. per sec., $N$ the revolutions per min., $d$ the diameter in ins., then, eq. 6, § 245—

$$P = (550\ H)\ /V = (126040\ H)\ /\ (d\ N).$$

Replacing $P$ in terms of $p$—

$$H = (k\ b\ p\ d\ N)\ /\ (126040)$$

If $b/p$ is assumed—

$$p = \sqrt{\frac{126040}{k}}\ \sqrt{\frac{H}{d\ N\ (b/p)}} \qquad (14)$$

*Values of* $\sqrt{\{(126040)/k\}}$

| | Cast gearing | | Cut gearing | |
|---|---|---|---|---|
| | Normal teeth | Shortened teeth | Normal teeth | Shortened teeth |
| Cast iron . . . . | 25·0 | 20·0 | 22·4 | 18·1 |
| Steel casting . . . | 21·7 | 16·8 | 18·7 | 15·2 |
| Forged steel . . . | — | — | 14·5 | 11·7 |

For cast-iron mill gearing when $k$ is taken with reference to the speed of the wheel, eq. 12—

$$\begin{array}{lcccccc} \text{N} = & 50 & 100 & 150 & 200 & 250 & 300 \\ \sqrt{\{(126040)/k\}} = & 23·6 & 26·1 & 28·6 & 31·4 & 34·9 & 39·0 \end{array}$$

There is yet another form of the equation which has been serviceable in designing ordinary transmissive gearing. Assuming $b/p = 2\frac{1}{2}$ and putting T for the number of teeth in the wheel, then the number of teeth necessary for strength in ordinary cast-iron gearing is—

$$\text{T} = (791\,\text{H})/(p^3\,\text{N}) \quad . \quad . \quad . \quad . \quad . \quad (15)$$

From this the table on p. 406 is calculated. It gives the least number of teeth suitable for strength when H/N is known and the pitch $p$ is assumed. If the numbers in the table are increased by one-fifth, they will be numbers suitable for mortice wheels.

252. *Limiting velocity of toothed wheels.*—If the wheels are run at a sufficiently high velocity, the wheel rim bursts in consequence of the centrifugal tension. Toothed wheels are in this respect materially in a worse position than pulleys, because the teeth add considerably to the weight of the rim without adding to the section which resists bursting. No increase of the pitch or section of the rim renders the wheels safe, because the increase of weight increases the centrifugal tension in the same ratio as the increase of section. For very high velocity, wheels must be made of a stronger material than cast iron.

From the equation previously given (§ 247), we have for the load on the teeth for ordinary cast gearing

$$\text{P} = 0·048\, b\, p\, f$$

where $f$ is the safe stress allowed for the breaking across of the teeth. The actual section of the rim is about $0·5\, b\, p$. But

*Table giving least Numbers of Teeth for a Wheel of given Pitch and Speed, from equation* 15.

| Pitch in inches $p$ | $p^2$ | Least number of teeth for $\frac{HP}{N}$ = | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.012 | .025 | .05 | .075 | .1 | .15 | .2 | .25 | .3 | .35 | .4 | .45 | .5 | .6 | .7 | .8 | .9 | 1.0 | 1.25 | 1.5 | 1.75 | 2.0 | 2.25 | 2.5 | 2.75 | 3.0 | 3.5 | 4.0 | 4.5 | 5.0 | 6.0 | 7.0 | 8.0 |
| 1 | 1.00 | 10 | 20 | 39 | 60 | 79 | 119 | 158 | 200 | 236 | 276 | 316 | 356 | 394 | | | | | | | | | | | | | | | | | | | | |
| $1\frac{1}{4}$ | 3.37 | | | 12 | 17 | 24 | 35 | 47 | 59 | 70 | 82 | 94 | 106 | 114 | 141 | 164 | 187 | 211 | 234 | 293 | 351 | | | | | | | | | | | | | |
| $1\frac{1}{2}$ | 5.36 | | | | 11 | 15 | 22 | 30 | 37 | 44 | 52 | 59 | 67 | 74 | 89 | 103 | 118 | 135 | 148 | 184 | 221 | 258 | 295 | | | | | | | | | | | |
| 2 | 8.00 | .. | . | .. | . | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 59 | 69 | 79 | 89 | 99 | 124 | 148 | 173 | 198 | 223 | | | | | | | | | | |
| $2\frac{1}{4}$ | 11.4 | | | | | | 11 | 14 | 18 | 21 | 25 | 28 | 31 | 35 | 42 | 49 | 50 | 63 | 70 | 87 | 104 | 123 | 138 | 156 | 173 | 101 | 208 | 243 | | | | | | |
| $2\frac{1}{2}$ | 15.6 | | | | | | | 10 | 13 | 16 | 18 | 21 | 23 | 25 | 31 | 36 | 41 | 46 | 51 | 63 | 76 | 89 | 103 | 114 | 127 | 130 | 153 | 177 | 202 | | | | | |
| $2\frac{3}{4}$ | 20.8 | | | | | | | | 10 | 12 | 14 | 16 | 17 | 19 | 23 | 20 | 31 | 34 | 38 | 43 | 57 | 67 | 76 | 86 | 95 | 105 | 114 | 133 | 152 | 171 | 183 | 228 | 266 | |
| 3 | 27.0 | .. | .. | .. | .. | . | .. | .. | .. | | 11 | 12 | 13 | 15 | 18 | 21 | 24 | 27 | 30 | 37 | 44 | 52 | 59 | 66 | 73 | 81 | 88 | 103 | 117 | 132 | 146 | 176 | 205 | 234 |
| $3\frac{1}{4}$ | 34.3 | | | | | | | | | | | | | 12 | 14 | 16 | 10 | 21 | 23 | 29 | 35 | 41 | 47 | 52 | 58 | 64 | 69 | 81 | 93 | 104 | 115 | 138 | 161 | 185 |
| $3\frac{1}{2}$ | 42.9 | | | | | | | | | | | | | | 11 | 13 | 15 | 16 | 19 | 23 | 28 | 33 | 37 | 42 | 40 | 51 | 55 | 65 | 74 | 83 | 92 | 110 | 129 | 147 |
| $3\frac{3}{4}$ | 52.7 | | | | | | | | | | | | | . | | 11 | 12 | 14 | 15 | 19 | 23 | 26 | 30 | 34 | 37 | 41 | 45 | 53 | 60 | 68 | 75 | 89 | 105 | 120 |
| 4 | 64.0 | . | | .. | . | . | | . | . | .. | .. | | | | | . | .. | 11 | 13 | 16 | 19 | 22 | 25 | 28 | 31 | 34 | 37 | 43 | 50 | 56 | 62 | 74 | 87 | 99 |
| $4\frac{1}{4}$ | 76.8 | | | | | | | | | | | | | | | | | | | 13 | 16 | 18 | 21 | 24 | 26 | 29 | 31 | 36 | 41 | 46 | 51 | 62 | 72 | 82 |
| $4\frac{1}{2}$ | 91.1 | | | | | | | | | | | | | | | | | | | | 13 | 16 | 18 | 20 | 22 | 24 | 26 | 31 | 35 | 39 | 44 | 52 | 61 | 70 |
| $4\frac{3}{4}$ | 107.2 | | | | | | | | | | | | | | | | | | | | | 13 | 15 | 17 | 19 | 21 | 22 | 26 | 30 | 34 | 37 | 45 | 52 | 59 |
| 5 | 125.0 | .. | .. | .. | .. | .. | .. | .. | .. | | .. | .. | .. | .. | | | .. | | .. | | | .. | 13 | 15 | 16 | 18 | 19 | 22 | 26 | 29 | 32 | 38 | 44 | 51 |
| $5\frac{1}{2}$ | 166.4 | | | | | | | | | | | | | | | | | | | | | | | 11 | 12 | 13 | 14 | 17 | 19 | 22 | 24 | 29 | 34 | 38 |
| 6 | 216.0 | | | | | | | | | | | | | | | | | | | | | | | | | 10 | 11 | 13 | 15 | 17 | 19 | 22 | 26 | 30 |

RULE.—To find the least number of teeth in either wheel of a pair, which will ensure sufficient strength, divide the horse-power transmitted by the revolutions of the wheel per minute; under the nearest number to the quotient so obtained, and opposite the pitch selected, is the required least number of teeth.

if the teeth were thrown into the rim the section would be about $0 \cdot 85\ b\ p$. Hence the weight of the rim per foot of length (12 cubic inches of iron weighing $3 \cdot 36$ lbs.) is $0 \cdot 85 \times 3 \cdot 36\ b\ p = 2 \cdot 86\ b\ p$ lbs.

Each foot of rim has a radial centrifugal force of $\frac{2 \cdot 86\ b\ p}{g} \cdot \frac{v^2}{R}$ lbs. where $v$ is the velocity of the rim in feet per second, and R its radius in feet. The resultant centrifugal force of half the rim is

$$\frac{2 \cdot 86\, b\, p}{g} \cdot \frac{v^2}{R} \times 2\, R = \frac{5 \cdot 72\, b\, p\, v^2}{g} \text{ lbs.}$$

This is balanced by the stress on two radial sections of the rim. Hence the stress due to rotation is

$$\frac{5 \cdot 72\, b\, p\, v^2}{b\, p\, g} = \frac{5 \cdot 72\, v^2}{g} \text{ lbs. per sq. in.}$$

The stress in the rim due to the pressure on the teeth will be on the average $\frac{1}{2}$ P; the load being transmitted half to the arm in advance and half to the arm behind the teeth in contact. But as the proportion transmitted each way will depend on the relative nearness of the arms, it seems probable that the maximum stress due to the load may amount to twice the mean value, or $2\ P/bp$ lbs. per sq. in. Putting in the value of P above, this becomes

$$0 \cdot 096\, f$$

Consequently the whole stress per sq. in. in the rim is

$$f_2 = 0 \cdot 096\, f + \frac{5 \cdot 72\, v^2}{g}.$$

For wheels run at high speed, we may take $f = 4{,}000$ lbs. per sq. in. The safe limit of tensional resistance for cast iron is about 3,000 lbs. per sq. in.; but looking to the fact that there are initial stresses in wheels due to contraction in cooling, and bending stresses due to the oblique action of the teeth, which have been neglected, it does not appear safe to take $f_2$ at more than 2,000 lbs. per sq. in. Then the limiting safe velocity is

$$v = \sqrt{\left\{\frac{g}{5 \cdot 72}(2000 - 0 \cdot 096 \times 4000)\right\}}$$
$$= 96 \text{ feet per sec. nearly.}$$

It is doubtful if even this calculation allows quite margin enough in the case of fast-running heavy wheels.

253. *Strength of bevil wheels.*—In stating the size of bevil wheels, the pitch at the outer circumference of the wheel is always given, but in estimating their strength the pitch at the mean circumference of the rim should be taken.

If $d_o$ $d_i$ are the diameters at the outside and inside circumferences the mean diameter is $\frac{1}{2}(d_o + d_i)$. If $p$ is the pitch at the outer circumference then the mean pitch is

$$p_m = p_o \{(d_o + d_i)/2\,d_o\}$$

254. *Shrouded wheels.*—The teeth of wheels are sometimes united at the ends by annular rings cast with the wheel, and the wheel is then said to be shrouded. The shrouding may extend the whole depth of the teeth of the pinion of a pair of wheels. In that case the shrouding has the effect of neutralising the weakness of the pinion teeth, which in very small wheels are of a weak form. With the pinion shrouded, it is stronger than the wheel, but it wears more rapidly than the wheel, so that the shrouding may be regarded as a provision against the failure of the pinion in consequence of wear.

255. *Friction of toothed gearing.*—Professor R. H. Smith ('Trans. Soc. of Engineers,' 1908) has found for the work wasted in friction by a pair of wheels, the numbers of teeth in which are N and $n$, the expression

$$F = 1{\cdot}06\,\mu\,\frac{N + n}{Nn}$$

There $\mu$ is the coefficient of friction, and therefore the efficiency of the pair is

$$\eta = \frac{1}{1 + 1{\cdot}06\,\mu\,\dfrac{N + n}{Nn}}$$

This is exclusive of journal friction.

Some experiments were made by Mr. W. Lewis for Messrs. Sellers & Co. on a pair of spur wheels of 39 and 12 teeth, $1\frac{1}{2}$ in. pitch, on shafts with journals $2\frac{5}{16}$ in. in diameter. The efficiency, including journal friction, was 0·90 at 3 revolutions of pinion or 4·5 feet per minute velocity of pitch line, and ·986 at 200 revolutions of pinion or 300 feet per minute velocity of pitch line. The pressure on the teeth varied from 430 to 2,500 lbs.

## Construction and Proportions of Wheels

256. *Rim of wheel.*—In iron wheels the teeth are cast on, and in mortice wheels they are tenoned into, a continuous rim.

Fig. 302 shows the section of a spur-wheel rim, and fig. 303 that of a bevil-wheel rim. The unit for the proportional figures is the pitch. The proportional figures for the teeth are approxi-

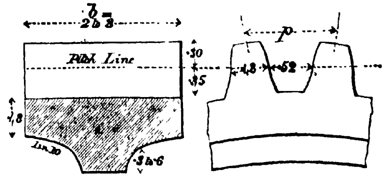


Fig. 302

mate only, more exact proportions having been already given in § 221.

Fig. 304 shows the section of a mortice spur-wheel rim, the

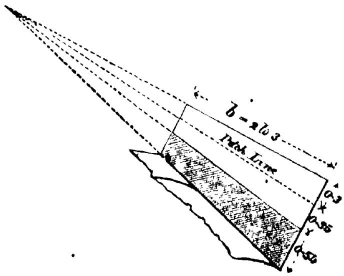


Fig. 303

end elevations indicating two ways of forming the tenons. The mortice teeth are either fixed by wood keys, or by round iron pins driven in behind the rim of the wheel. Both methods are shown

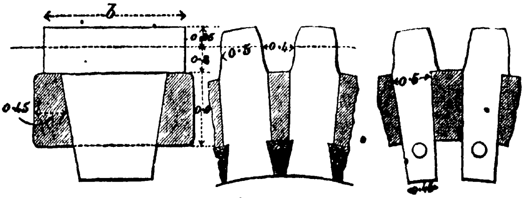


Fig. 304

in fig. 304. In fig. 305 the cogs are fixed by bolts, iron plates about 2 ft. long being fitted to the inside of the rim of the wheel. Fig. 306 shows a mortice bevil wheel. The radiating lines in

the figures of bevil wheels meet at the intersection of the shafts on which the wheels are placed.

257. *Arms of wheels.*—The arms of wheels are most com-

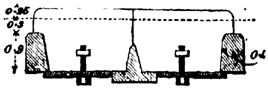


Fig. 305

monly cross-shaped in section for spur wheels, and T-shaped for bevil wheels (fig. 307). For machine-moulded wheels, the arms are often I-shaped, the spaces between the arms being cored out in

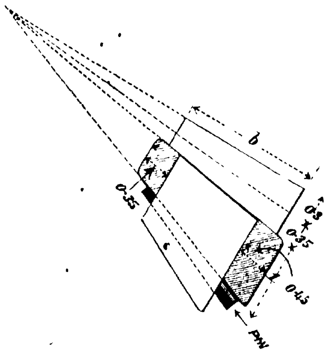


Fig. 306

casting with loam cores (fig. 308). The number of arms in wheels is fixed very arbitrarily. Usually there are four arms for wheels not exceeding four feet diameter; six arms for wheels of from

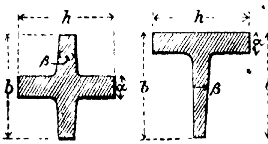



Fig. 307

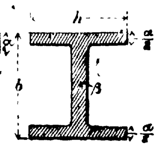



Fig. 308

four feet to eight feet; and eight arms for wheels from eight feet to sixteen feet diameter.

The arms are subjected to bending, and when the arms and rim are cast in one piece, they are fixed at both ends. If the arms are attached to the rim by bolts, they are free at the rim,

and fixed at the nave. It will be assumed that the arms are equally loaded, and that they may in all cases be treated as if they were fixed at one end and free at the other. This will give a slight excess of strength when the arms are cast in one with the rim, but such arms are at the same time weakened by contraction in cooling.

Let $\nu$ be the number of arms,
- R the radius of the pitch line in inches,
- P the effort at the pitch line in lbs.,
- $y$ the distance from pitch line to section of arm near the nave,
- Z the modulus of the section of the arm, § 49,
- $f_b$ the working stress in bending in lbs. per sq. in.

The bending moment on each arm is $\mathrm{P}\,y/\nu$

$$\mathrm{P}\,y/\nu = f_b\,\mathrm{Z}.$$

The strength of the arm is almost wholly due to that part which is parallel to the plane of rotation. The ribs or feathers at right angles to this add little to the resistance to bending. They are necessary to give lateral strength and rigidity, and to resist accidental straining actions at right angles to the plane of rotation. Let $h$ be the width, and $a$ the thickness of the arm, exclusive of the feathers. Then $\mathrm{Z} = \frac{1}{6}\,a\,h^2$ and $a\,h^2 = (6\,\mathrm{P}\,y)/(\nu f_b)$.

In proceeding to design the arm either of the three following methods may be followed.

Given the ratio $\frac{h}{a}$, and the limiting stress on the arm.

$$\text{If } \frac{h}{a} = 5,\ h = \sqrt[3]{\frac{30}{f_b}}\sqrt[3]{\frac{\mathrm{P}\,y}{\nu}}$$

The limiting stress must be taken at a low value, partly to allow for unequal distribution of load on the arms, and partly because of the initial stresses due to contraction in cooling. If $f = 3{,}000$ lbs. per sq. in.,

$$h = \frac{0{\cdot}2154}{\sqrt[3]{\nu}}\sqrt[3]{\mathrm{PR}}$$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\nu =$ | 3 | 4 | 6 | 8 | 10 | 12 |
| $\frac{0{\cdot}2154}{\sqrt[3]{\nu}} =$ | ·149 | ·136 | ·119 | ·108 | ·100 | ·094 |

Towards the rim the arm is usually tapered, the amount of

taper being $\frac{1}{4}$ in. per foot of length on each side. The thickness of the arm $a$ is constant.

The width of the cross feathers (marked $b$ in fig. 307) may be $b$ to $1\frac{1}{4}\, b$ at the centre, and $\frac{3}{4}\, b$ to $\frac{15}{16}\, b$ at the rim, where $b =$ width of face of wheel. The thickness of the feathers may be $\beta = 0{\cdot}3\, p$. The feathers must be slightly tapered at right angles to their length, so as to draw easily from the sand.

258. *Nave of the wheel.*—Let $d$ be the diameter of the bore of the nave, and let $d_0$ be the diameter of a shaft capable of transmitting the twisting moment P R. That is, let $d_0 = 0{\cdot}8 \sqrt[3]{\text{P R}}$. The thickness of the nave may be $0{\cdot}2\ (d_0 + 0{\cdot}5\ d)$.

It is more common in practice to roughly proportion the nave to the diameter of the shaft on which the wheel is fixed. If $d$ is the diameter of the wrought-iron shaft, then the boss for the wheel should be $1{\cdot}17\ d$ in diameter. The nave thickness may be $0{\cdot}35\ d$ and the nave length $1\frac{1}{2}\ d$ or more.

Large wheels may be fixed by four keys. Then the eye of the wheel has a diameter $1\frac{1}{4}$ times that of the shaft at the place where it is fixed. Heavy wheels have the nave split to prevent fracture of the arms from contraction in casting. The nave is then gripped by two strong wought-iron rings or hoops, fitted over the nave on each side and shrunk on.

259. *Weight of toothed gearing.*—Let $p$ be the pitch, $b$ the breadth of face, and $n$ the number of teeth of a wheel. Then, its weight in lbs. is, approximately,

$$\text{W} = k\, n\, b\, p^2$$

where $k = 0{\cdot}38$ for spur wheels and $0{\cdot}325$ for bevil wheels. The weight of a pair of wheels is independent of the radii, and depends directly on the H.P. transmitted and the numbers of revolutions of the wheels. The weight of a train of wheels is smaller when the number of pairs of wheels is as small as possible, and when all the pairs, except the quickest-running pair, have the greatest practicable velocity ratio.

Mr. D. K. Clark gives the following formula for the weight of cast-iron spur wheels per inch of breadth in lbs. :

$$\begin{aligned} \text{W} &= (5{\cdot}6 + 9\,p)\ (d + 0{\cdot}1\ d^2) \text{ Spur wheels,} \\ &= (4 + 6{\cdot}3\,p)\ (d + 0{\cdot}1\ d^2) \text{ Bevil wheels,} \end{aligned}$$

where $d$ is the diameter in ft. and $p$ the pitch in ins.

DIRECTOR OF INDUSTRIES BENGAL

# CHAPTER XIII

## HELICAL AND SCREW GEARING

260. Helical gearing is a modification of ordinary toothed gearing in which the velocity ratio is inversely as the radii of the pitch surfaces. Screw gearing is gearing in which the velocity ratio is independent of the radii of the pitch surfaces. It has the great advantage that high velocity ratio can be obtained with comparatively small wheels. Its defect is that the friction and wear is greater than with toothed gearing.

In helical gearing the pitch surfaces may be cones or cylinders as in bevil or spur gearing. In screw gearing the pitch surfaces are cylinders. In both helical and screw gearing the teeth intersect the pitch surfaces in helical lines. A screw wheel may have one or any number of teeth. A one-toothed wheel corresponds to a one-threaded screw; a many-toothed wheel to a many-threaded screw. In screw gearing the axes may be at any angle.

### Helical Gearing

261. *Toothed gearing with helical teeth.*—Let an ordinary spur wheel be cut into $n$ slices of equal thickness by planes perpendicular to the wheel axis, and let the slices be arranged so that each slice is rotated 1-$n$th of the pitch relatively to the adjoining one. Such a wheel is termed a *stepped spur wheel* (fig. 309). In practice such a wheel is cast in one block. A pair of such wheels will work together, the steps coming into gear successively, and there is simultaneous line contact, nearer to or further from the pitch surface on each of the steps in gear. On one or other of the steps there is contact near the pitch surface. The teeth may be so formed, though this is not usual, that contact only occurs near the pitch surface, but then there is contact only on one step at a time. Stepped wheels transmit motion very uniformly, and have been used in driving the

tables of planing machines and in other cases where great regularity of motion is important.

If the number of slices is infinite, the wheels become *helical wheels*, in which the straight teeth, parallel to the wheel axis of ordinary spur wheels, are twisted into helices (fig. 310). The section of the teeth on planes normal to the axes of rotation are identical with cycloidal or involute teeth of ordinary gearing. But two teeth in contact do not touch along a line parallel to the axes, but along a kind of helix. There is simultaneous contact at different points of the face from the crest to the root. The resultant pressure between two teeth is no longer in the plane of rotation, but at right angles to the helix, and it has a lateral component parallel to the wheel axis, tending to shift the wheel along the shaft. Helical wheels transmit motion very uniformly, and they are increasingly used (for instance in milling machines) where constancy of velocity ratio and noiseless running is important. The front of a tooth intersects the pitch surface in a helix, the pitch of which is found thus: let $d$ be the diameter, $b$ the width of face, and $p$ the circular pitch, measured along the pitch line in the plane of rotation; then the axial pitch

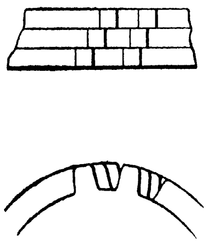


Fig. 309

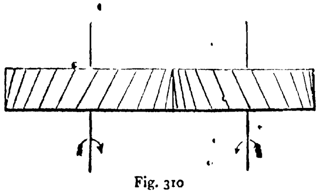


Fig. 310

of the helix in which the tooth surface intersects the pitch surface is $\pi\, d\, b/p$.

Let fig. 311 represent a plane development of the pitch surface, the sections of the teeth being shaded. The slope of the teeth relatively to the axis of rotation is $\beta$, and the width of face is $b$. The distance $s = b \tan \beta$ may be called the *skew* of the teeth. Often $\beta = 35°$; and $s = 0{\cdot}7\, b$. The length of tooth is $l = b \sec \beta$.

Let P be the pressure on the teeth in the plane of rotation, which can be calculated from the power transmitted by eq. 6, § 245. The normal pressure on the teeth is $P_n = P \sec \beta$, and the lateral pressure tending to shift the wheel along the shaft is $P_l = P \tan \beta$. For $\beta = 35°$, $P_n = 1{\cdot}22\ P$, and $P_l = 0{\cdot}7\ P$.

262. *Strength of helical teeth.*—Let $p$ be the circular pitch, $l$ the length of the tooth, $t$ the thickness of the tooth near the root, $h$ the height above this point at which the resultant pressure acts. Then, if the whole pressure acts on one tooth, the bending moment is $P_n\ h$. For a working stress $f$

$$P_n h = \tfrac{1}{6} l t^2 f.$$

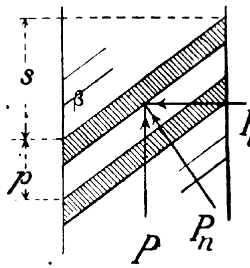


Fig. 311

Values of $P_n$ and $l$ are given above. The thickness of a tooth in the plane of rotation may be taken at $0{\cdot}45\ p$, but in the direction of $P_n$ it is only $0{\cdot}45\ p \cos \beta$. As to the height at which the pressure acts, the whole height of a tooth may be taken at $0{\cdot}6$ to $0{\cdot}7\ p$, and in straight teeth the pressure in one position of the teeth acts at this distance from the root. But with helical teeth, if of exact form, there is simultaneous contact at various heights from 0 to $0{\cdot}6$ or $0{\cdot}7\ p$. The average height, allowing for some inexactness of form, may be taken at $0{\cdot}4\ p$. Inserting these values—

$$0{\cdot}4\ P\ p \sec \beta = \tfrac{1}{6}\ b \sec \beta\ (0{\cdot}45\ p \cos \beta)^2 f$$
$$P = 0{\cdot}0833\ b\ p\ f \cos^2 \beta$$

and if $\beta = 35°$,

$$P = 0{\cdot}056\ b\ p f.$$

The values of $f$ may be those already given for straight teeth.

If the equation is written $P = c\ p\ b$, then $c =$ about 300 for cast iron, 500 for steel castings.

By combining two helical wheels constructed with teeth twisted opposite ways (right- and left-handed helices), the endway thrust due to one is balanced by that due to the other (fig. 312). Such wheels are termed double helical wheels. It is possible in wheel moulding machines to mould wheels of this kind, and even to produce bevil wheels with twisted teeth. Double helical spur gearing can also be machine cut. Double

helical gearing is stronger than ordinary gearing and works more smoothly, especially when it is necessary to use low numbered pinions. But it requires to be very accurately constructed, or the pressure is not uniformly distributed along the surfaces of the teeth. Ordinary proportions are given in fig. 313.

There appear, however, to be some difficulties in using double helical gearing. For their correct action the middle plane of the two wheels must coincide. If it does not all the load is thrown on one half of the tooth. But to secure this permanently, it seems almost necessary that one of the wheels should have some small lateral play so that it can accommodate itself to the other wheel. It is desirable that a sliding coupling should be placed on each side of one of a pair of helical wheels, so that a lateral shifting of the wheel is possible.

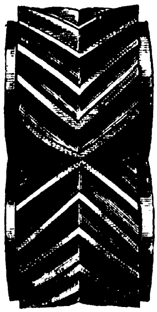


Fig. 312

### Screw Gearing

263. *Screw gearing when the axes are not parallel.*—When the axes are not parallel the pitch cylinders touch at a single point, which may be termed the pitch point. Draw through that point a tangent to the pitch surfaces. If helices are traced on the pitch cylinders touching that tangent, they define the fronts of teeth which will drive each other.

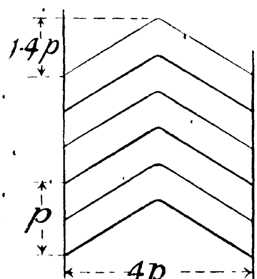



Fig 313

Let fig. 314 represent two cylindrical pitch surfaces, A and B, in contact at the pitch point O; *a b c d* a plane touching the pitch surfaces at O. Let *e f* be a line on the plane passing through O. If the plane is wrapped on A, *e f* will mark out a helix, and if wrapped on B another helix. A series of equidistant parallel lines on *a b c d* will mark out two sets of helices on A and B. Draw on the plane O *m*, O *n*, perpendicular to the axes of the cylinders; O *h*, O *k*, parallel to the axes; O *g*, perpendicular to *e f*. In order that the helices should be equidistant round A, O *n* must be a submultiple of A's circumference and O *m*

must be a submultiple of B's circumference, but the number of helices on the two pitch surfaces will in general be different. Now suppose the cylinders to rotate and the plane to slide so that $e^1f^1$ on the plane comes to $ef$, and at the same time the helices, of which $e^1f^1$ is the development, come to touch at O; only points on the pitch surfaces which are in planes through O normal to the axes can come to O, and therefore a point on

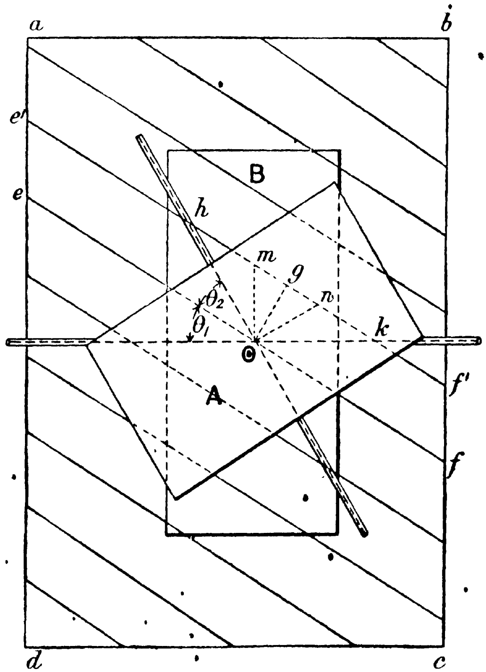



Fig. 314

A's helix, corresponding to $n$ on the development, and a point on B's helix, corresponding to $m$, come together at O, when $e^1f^1$ comes to $ef$. Hence, A rotates through an arc of length O$n$ and B through an arc of length O$m$ simultaneously. It is convenient to restrict the term *pitch* to the distance between corresponding points on one helix. Then the distance between corresponding points of adjacent helices may be termed the

*divided pitch.* O g is the divided *normal pitch*, and this, from the way in which the helices are generated by wrapping $a\,b\,c\,d$ on the pitch cylinders, is necessarily the same for both sets of helices; O $n$ measured round A's circumference, and O $m$ round B's circumference, are the divided *circumferential pitches*, and O $h$ measured along A's axis and O $k$ along B's axis are the divided *axial pitches*. There are two limiting cases: if the axes become parallel ($\theta_1 + \theta_2 = 0$), O $m$ and O $n$ coincide, and the circumferential divided pitches of the two sets of helices are the same; this corresponds to the case of the helical wheels already described. If the axes are at right angles ($\theta_1 + \theta_2 = 90°$), O $m$ coincides with O $h$ and O $n$ with O $k$, then the circumferential divided pitch of A is equal to the axial divided pitch of B and *vice versa*.

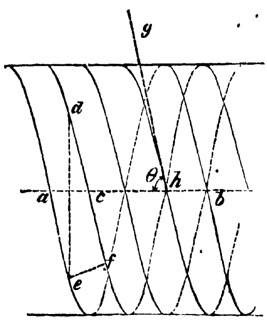


Fig. 315

The motion assumed above, which brings $n$ on A and $m$ on B together at O, can be realised, if helical ribs or threads are formed on the pitch surfaces, so that contact continues on one pair of helices till it begins on the adjacent pair. The action will be in some respects the same as between two spur-wheel teeth, but there will be a sliding parallel to $ef$, the magnitude of which is given by $m\,n$. The common tangent $ef$ to the pitch surfaces and the helices or threads at the pitch point O is termed the *line of contact*. The angles $\theta_1, \theta_2$ it makes with the axes are the inclinations of the threads.

Fig. 315 shows a helical wheel with four helices ($\nu = 4$). Then $ef = n$ is the divided normal pitch; $ed = c$ is the divided circumferential pitch; and $ac = a$ is the divided axial pitch. If $r$ is the radius of pitch surface,

$$\tan\theta = 2\pi r/\nu a$$

$$a : c : n :: 2\pi r \cot\theta : 2\pi r : 2\pi r\cos\theta$$

$$:: \cot\theta \quad : \quad 1 \quad : \quad \cos\theta$$

264. *Velocity ratio in screw gearing.*—Fig. 316 shows a pair of helical wheels for axes inclined at an angle $i = \theta_1 + \theta_2$. Let

$r_1, r_2$ be the radii; $n$ the common divided normal pitch; $c_1, c_2$ the divided circumferential pitches; $a_1$, $a_2$ the divided axial pitches; $\nu_1$, $\nu_2$ the numbers of threads on each wheel; $\alpha_1$, $\alpha_2$ their angular velocities. Then,

$$\nu_1 = 2\pi r_1/c_1$$
$$\nu_2 = 2\pi r_2/c_2$$

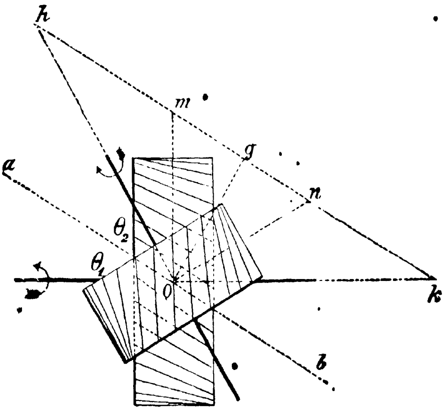



Fig. 316

Also the arc $c_1$ of one pitch surface passes the pitch point in the same time as the arc $c_2$ of the other. Hence

$$\frac{\alpha_1 r_1}{\alpha_2 r_2} = \frac{c_1}{c_2} = \frac{2\pi r_1 \nu_2}{2\pi r_2 \nu_1}$$

$$\frac{\alpha_1}{\alpha_2} = \frac{\nu_2}{\nu_1}$$

That is, the angular velocities and rotations per minute are inversely as the number of threads in the wheels. But for the two wheels to work together the normal pitch must be the same for the two. Using the proportion given above,

$$n = c_1 \cos\theta_1 = c_2 \cos\theta_2$$

$$\left.\begin{aligned} \therefore \cos\theta_1 &= \frac{c_2 \sin i}{\sqrt{(c_1^2 - 2c_1c_2\cos i + c_2^2)}} \\ \cos\theta_2 &= \frac{c_1 \sin i}{\sqrt{(c_1^2 - 2c_1c_2\cos i + c_2^2)}} \end{aligned}\right\}$$

The axial pitches are given by the equations,

$$a_1 = c_1 \cot \theta_1$$
$$a_2 = c_2 \cot \theta_2$$

In fig. 316, set off from the pitch point o, the lines o $m$, o $n$ perpendicular to the axes, in the directions the wheels are moving at the point o. Take o $m$, o $n$ equal to the surface velocities $a_1 r_1$ and $a_2 r_2$ of the wheels or to the circumferential pitches; join $m\,n$ and produce it to meet the axes; then $a$ o $b$ parallel to $h\,k$ is the line of contact, making angles $\theta_1$ $\theta_2$ with the axes. Draw o $g$ perpendicular to $h\,k$. Then o $g$ is the common component of the surface velocities, and $m\,n$ the velocity of transverse sliding of the teeth

$$c_1 : c_2 : a_1 : a_2 : n$$
$$:: \text{o}m : \text{o}n : \text{o}k : \text{o}h : \text{o}g$$

265. *Screw gearing when the shafts are at right angles. Worm and wheel.*—If $i = 90°$, then $\cos \theta_2 = \sin \theta_1$, and the axial divided pitch of one wheel is equal to the circumferential pitch of the other.[1]

The most common form of screw gearing is that in which the shafts are at right angles, and a wheel of one thread, or sometimes of two or three threads, works with a wheel of many threads. Then the former is termed a *worm*, and the latter a *worm wheel*. With this arrangement, a high velocity ratio is obtained with a pair of small wheels. If $N_1$ $N_2$ are the numbers of revolutions of the worm and wheel, $a_1$ $a_2$ their angular velocities, and $\nu_1$ $\nu_2$ the number of threads on each,

$$\frac{N_1}{N_2} = \frac{a_1}{a_2} = \frac{\nu_2}{\nu_1}$$

Thus, if the worm has one thread and the wheel twenty-five, the velocity ratio is twenty-five.[2] Spur wheels for that velocity ratio would have to be about ten times larger in diameter. The disadvantage of screw gearing of this kind is that the friction and wear is excessive, hence it is not generally

[1] For simplicity in what follows $p$ may be written for either the axial divided pitch of the worm or the circumferential divided pitch of the worm wheel, and this is practically called *the pitch* of the gearing.

[2] Observe that in screw gearing the velocity ratio is independent to a great extent of the radii of the wheels. If the circumferential pitch of the worm wheel is fixed on considerations of strength, that determines the radius of the worm wheel. But then the radius of the worm may be chosen to suit practical convenience.

used for the continuous transmission of power. If the obliquity of the helices exceeds a certain amount, the wheels are no longer reciprocal: that is, one wheel will drive the other, but the second will not drive the first. In that case the motion is prevented by the friction at the point of contact of the teeth. The worm and wheel are commonly so constructed that the worm will drive the wheel, but the wheel will not drive the worm.

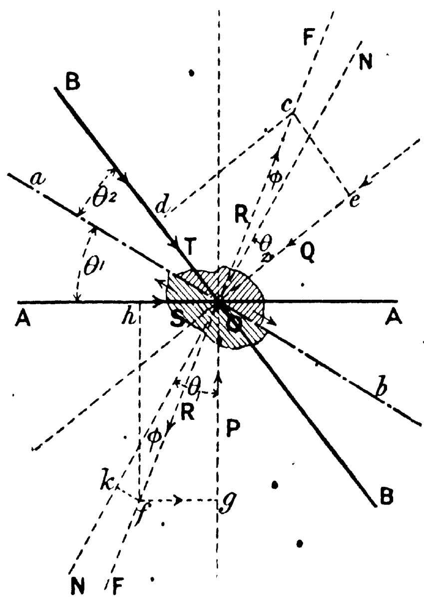



Fig. 317

This is often advantageous, because the gearing remains stationary in any position after being moved.

266. *Ratio of effort and resistance in screw gearing.*—Let fig. 317 represent a small portion of two screw threads in contact at the pitch point O; A A and B B are the axes of the wheels, *a b* the line of contact, N N the normal to the surfaces at O. Let the lower thread, belonging to the wheel the axis of which is A A, be the driver. The lower wheel is in equilibrium under the action of an effort P, in the direction of motion at O, perpen-

dicular to its axis, a thrust S along its axis, and the reaction R of the other wheel. Similarly the upper wheel is acted on by a resistance overcome, Q, in the direction of its motion at O, perpendicular to its axis, a thrust T along its axis, and the reaction R of the other wheel at the point of contact. If there were no friction, R would act along the normal to the surfaces N N, but when friction is considered it acts at the angle of repose with the normal. Draw F F, making an angle $\phi$ with N N such that $\tan \phi = \mu$ the coefficient of friction. Then R acts along F F. Take a length O $c$ to represent R on any scale, and complete the parallelogram O $d$ $c$ $e$ with its sides parallel to Q and T. Then O $e$, O $d$ represent Q and T on the scale on which O $c$ represents R. Next take O$f$ = O $c$ = R, and complete the parallelogram O $h$ $f$ $g$ with sides parallel to P and S. Then O $g$, O $h$ represent P and S on the same scale.

$$P = R \cos (\theta_1 - \phi)$$
$$Q = R \cos (\theta_2 + \phi)$$
$$\frac{P}{Q} = \frac{\cos (\theta_1 - \phi)}{\cos (\theta_2 + \phi)}$$

where $\theta_1$ is the inclination of the axis of the driver and $\theta_2$ that of the follower to the line of contact.

*Efficiency of screw gearing.*—Referring to fig. 316, P acts through a distance O $m = c_1$ while Q acts through O $n = c_2$.

$$\text{Work expended} = P\, c_1$$
$$\text{Work done} = Q\, c_2$$

$$\text{Efficiency} = \eta = \frac{Q\, c_2}{P\, c_1} = \frac{c_2}{c_1} \cdot \frac{\cos (\theta_2 + \phi)}{\cos (\theta_1 - \phi)}$$

or, using the value for $c_1/c_2$ above,

$$\eta = \frac{\cos \theta_1 \cos (\theta_2 + \phi)}{\cos \theta_2 \cos (\theta_1 - \phi)}$$

267. *Pressure on teeth and endway thrust on shafts.*—In designing screw gearing either P or Q will be given. Then the pressure on the teeth is $R = P \sec (\theta_1 - \phi) = Q \sec (\theta_2 + \phi)$ where $\theta_1$, $\theta_2$ are the angles between the shafts and the line of contact. The thrust along the driving shaft is $S = P \tan (\theta_1 - \phi)$ and along the driven shaft $T = Q \tan (\theta_2 - \phi)$. Generally $\phi = 8\frac{1}{2}$ degrees.

*Power transmitted and efficiency of worm gearing.*—When the

shafts are at right angles, $\theta_1 + \theta_2 = 90°$; $\cos\theta_2 = \sin\theta_1$; $\cos(\theta_2 + \phi) = \sin(\theta_1 - \phi)$. In that case,

$$\frac{P}{Q} = \cot(\theta_1 - \phi) = \frac{1 + \mu\tan\theta_1}{\tan\theta_1 - \mu}$$

The efficiency is

$$\eta = \frac{\tan(\theta_1 - \phi)}{\tan\theta_1} = \frac{\tan\theta_1 - \tan\phi}{(1 + \tan\theta_1\tan\phi)\tan\theta_1}.$$
$$= \frac{1 - \mu\cot\theta_1}{1 + \mu\tan\theta_1}$$

But if $c_1$, $a_1$ are the circumferential and axial pitches of the worm, $\tan\theta_1 = c_1/a_1$. In worm gearing the axial pitch $a_1$ of the worm is equal to the circumferential pitch $c_2$ or $p$ of the worm wheel, and if there are $\nu_2$ threads in the worm wheel of radius $r_2$, then $\nu_2 p = 2\pi r_2$. In any screw wheel of $\nu_1$ threads and $r_1$ radius, $\nu_1 c_1 = 2\pi r_1$. Substituting these values

$$\eta = \frac{1 - \mu\dfrac{\nu_1 p}{2\pi r_1}}{1 + \mu\dfrac{2\pi r_1}{\nu_1 p}}.$$

Let $r_1 = kp$ where $k$ varies in practical cases from $1\frac{1}{4}$ to 3 or more. Then, for few threaded worms, neglecting the term $\mu\nu_1$ which is small,

$$\eta = \frac{\nu_1}{2\pi\mu k + \nu_1} \quad \text{very nearly};$$

a very simple expression, applicable to all usual cases of worm gearing.

For worm gearing at slow speeds $\mu = 0{\cdot}15$, and then this expression reduces to

$$\eta = \frac{\nu_1 p}{\nu_1 p + r_1} \quad \text{nearly}.$$

It is clear that the efficiency is greater the less the radius of the worm. If $r_1 = 2p$, $\eta = 0{\cdot}33$ for a one-threaded worm; $\eta = 0{\cdot}50$ for a two-threaded worm; and $\eta = 0{\cdot}60$ for a three-threaded worm.

These results are independent of the number of teeth in the worm wheel. Clearly the efficiency of worm gearing for this coefficient of friction is a low efficiency.

The radius of the pitch surface of the worm is very variable in practice. The least value (which gives the greatest efficiency) is about $r_1 = p$. More commonly, especially if the worm is to be cast and keyed on the shaft, $r_1 = 1\frac{1}{4}$ to $1\frac{1}{2} p$. Sometimes for special reasons $r_1 = 4 p$ to $6 p$.

268. *Settling the proportions of a worm gearing.*—The velocity ratio $N_1/N_2$ being given ($N_1$ being the revolutions per minute of the driver), the ratio of the number of threads in worm and wheel is $\nu_2/\nu_1 = N_1/N_2$.

Choosing the number of threads $\nu_1$ in the worm the number of threads $\nu_2$ (or teeth) in the worm wheel can be found. The circumferential pitch $p = c_2$ of the worm wheel is then settled by the rules for strength, and this is also the axial pitch $a_1$ of the worm. The radius of worm wheel is $r_2 = \nu_2 p/2\pi$, and the radius $r_1$ for the worm can be selected arbitrarily. Then the circumferential pitch of worm is $c_1 = 2\pi r_1/\nu_1$. The line of contact can then be found, for the angle between the worm shaft and the line of contact is $\theta_1 = \cot^{-1} a_1/c_1$, and that between worm-wheel shaft and line of contact is $\theta_2 = 90° - \theta_1$. The pressure on the teeth is $R = P \sec(\theta_1 - \phi)$, and the thrust along worm shaft is $S = P \tan(\theta_1 - \phi)$.

269. *Coefficient of friction and efficiency in worm gearing.*—Probably in slow moving worm gearing, as in gearing worked by hand, $\mu$ is about 0·15, corresponding to a friction angle of $8\frac{1}{2}$ degrees. A very valuable series of experiments on the efficiency of worm gearing was made by Mr. W. Lewis for Messrs. Sellers & Co.[1] The efficiency in given conditions of speed and pressure was found to vary, as might be expected; conditions of smoothness of surface and lubrication necessarily being inconstant. Mr. Lewis has plotted his results and drawn averaging curves. The table on p. 425 gives some of these results taken from the average curves. The value of $\mu$ corresponding to the observed efficiency has been deduced in each case by the equation given above. Circumference of worm one foot nearly.

According to these results, the coefficient of friction diminishes and the efficiency of the gear increases as the speed increases. It is possibly an effect of more perfect lubrication as the speed increases. It should be noted that the friction of the collar or step bearing of the worm shaft is included.

Worm gearing has been used for reducing the speed of

[1] *Trans. Am. Soc. Mec. Eng.* vii. 273.

electric motors by Mr. Reckenzaun, who gives the following particulars of a case of this kind: Steel worm polished, 6 in. in diameter, 6 in. long, three threads, 2 in. pitch. Worm wheel of phosphor bronze, trimmed teeth 15·3 in. diameter, $3\frac{1}{2}$ in. face, 24 teeth. Velocity ratio, 8 to 1. The worm ran in an oil-bath, and the efficiency was 81 to 87 per cent. In this case the coefficient of friction must have been from 0·060 to 0·075, values not discordant with those found by Mr. Lewis.

270. *Limit of thrust on worm gearing.*—If the pressure on the teeth of worm gearing exceeds a certain limit, abrasion occurs. Before this limit is reached there is a pressure at which the friction appears to increase suddenly, probably from the squeezing out of the lubricant and rise of temperature. Data

*Efficiency of Worm Gearing*

| Revs. of worm per min. Approximate velocity of sliding of teeth in feet per minute | Cast threads and teeth | | | | | | Machine cut threads and teeth | |
|---|---|---|---|---|---|---|---|---|
| | Worm two-threaded $\nu_1 = 2$, $p = 1\frac{1}{2}$ $k = 4/3$ | | Worm two-threaded $\nu_1 = 2$, $p = 1\frac{1}{2}$ $k = 4/3$ | | Worm one-threaded $\nu_1 = 1$, $p = 2$ $k = 4/3$ | | Worm one-threaded $\nu_1 = 1$; $p = 1\frac{1}{2}$ $k = 4/3$ | |
| | $\mu =$ | $\eta =$ | $\mu$ | $\eta$ | $\mu =$ | $\eta =$ | $\mu =$ | $\eta =$ |
| 5 | 0·166 | ·59 | 0·230 | ·52 | — | — | 0·146 | ·45 |
| 10 | 0·128 | ·65 | 0·180 | ·57 | 0·124 | ·49 | 0·116 | ·51 |
| 20 | 0·102 | ·70 | 0·152 | ·61 | 0·106 | ·53 | 0·090 | ·57 |
| 40 | 0·084 | ·74 | 0·124 | ·66 | 0·084 | ·59 | 0·070 | ·63 |
| 80 | 0·076 | ·76 | 0·120 | ·70 | 0·060 | ·67 | 0·054 | ·69 |

for determining these limits are scanty, but it is certain the pressure which can be carried diminishes as the velocity of rubbing of worm thread on wheel tooth increases. Some experiments were made by Mr. W. Lewis, for Messrs. Sellers & Co.,[1] with a 4-inch one-threaded cast-iron worm and cast-iron worm wheel, $1\frac{1}{2}$-inch pitch, run at speeds varying from 300 to 900 revolutions per minute, and with a total thrust ranging up to 6,000 lbs. It appeared that abrasion began in less than ten minutes, if P V exceeded 1,000,000 to 1,500,000. P is the worm thrust in lbs., and V the velocity of rubbing in feet per minute (approximately the circumference of pitch line in feet × revolutions per minute). In some of the curves of efficiency given by Mr. Lewis there is a sudden increase of friction when

[1] *Trans. Am. Soc. Mec. Eng.* vii. 286.

P V exceeds about 300,000. In the case of some hardened steel worms running on cast-iron worm wheels, tested by Mr. B. Flint, the thrust ranging up to 1,500 lbs. and the velocity of sliding up to 320 feet per minute, a sudden increase of friction occurred with P V exceeding about 200,000. The limit of thrust which is safe must depend on the number of wheel teeth simultaneously in contact with the worm thread or threads, because the pressure is distributed over three or possibly a greater number of teeth, but this was not noted in the experiments quoted. In the case of a worm of the Hindley form, with no doubt many teeth in contact, a value of P V = 2,000,000 was found not to produce cutting in a test made by Mr. Sprague.

271. *Form of worm-wheel rim.*—Fig. 318 shows the forms adopted for the rims of worm wheels. The simplest form is shown at A, but it is only suitable for those worm wheels which are intermittently driven; contact is confined to a point on the median plane of worm wheel, and the wear is excessive. B and C

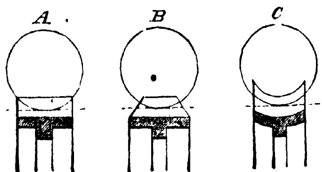



Fig. 318

show forms suitable when line contact is secured across the face of the worm wheel, and especially for those worm wheels in which the teeth are cut by a hob of the form of the worm. Greater wearing surface is obtained in C than in A.

In a special form of worm known as the Hindley worm the axial section of the pitch surface is curved to fit the pitch line of the worm wheel. With this form of worm a large number of teeth may be simultaneously in contact. It has been used in the electric lifts of Mr. Sprague.

272. *Form of worm-wheel teeth and threads of worm. Point and line contact.*—In the cases which most commonly occur, the axes of the worm and wheel are at right angles. Hence, the axial divided pitch of one is equal to the circumferential pitch of the other. For this pitch the common symbol $p$ will be used. In such cases it is convenient to design, at first, the teeth of worm and wheel on a plane passing through the worm

axis and normal to the wheel axis, as shown in fig. 319. For shortness the plane of this section will be called the median plane of the worm wheel. For sections on this plane we may take—

| | |
|---|---|
| Thickness of tooth on pitch line . . | 0·48 $p$ to 0·5 $p$ |
| Height outside pitch line . . . | 0·3 $p$ to 0·4 $p$ |
| Depth below pitch line . . . . . | 0·4 $p$ to 0·5 $p$ |
| Width of face of worm wheel usually . | 1·5 to 2·5 $p$ |
| Length of worm . . . . . . | 3 to 6 $p$ |
| ,, ,, usually . . . . | 4 $p$ |

For worms in which the threads make at least a complete turn, this fixes completely the form of the worm threads, for all

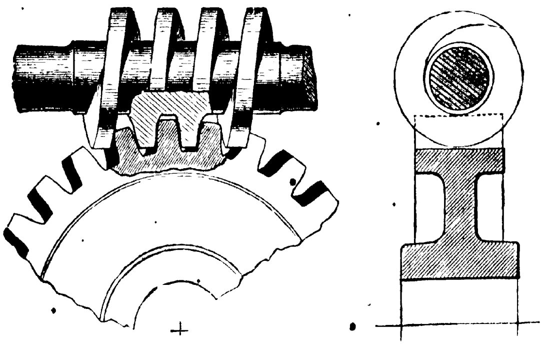


Fig 319

radial sections must be identical, since all in turn come into the median plane of the worm wheel. But the form of the worm-wheel teeth requires further investigation.

Willis pointed out that, if the section of the teeth of the worm wheel by the median plane be made like those of the spur wheel of the same radius and pitch, and the threads of the worm like the teeth of a rack suitable for working with such a spur wheel, the worm and wheel will gear correctly together so far as contact in that plane is concerned. Any of the wheel and rack constructions given in the table above, therefore, may be used in designing the section of worm and wheel on the median plane of the worm wheel.

It has commonly been assumed and acted on that all radial

sections of the worm threads must be of the form so fixed, which is right and necessary; and also that all sections of the worm-wheel teeth on sections parallel to the median plane should be similar, which is unnecessary, and, as will be seen, gives a bad form of worm-wheel teeth. If that proceeding is adopted, the worm-wheel teeth become merely twisted spur-wheel teeth, which only touch the worm teeth at a point in the median plane. The point of contact moves from root to point of the teeth along the intersection of the median plane with the tooth, the rest of the surface of the tooth is never in contact at all, and the whole of the pressure and wear is concentrated at a single line on the face of the tooth. It is probably in part due to this imperfect construction of the worm wheel that screw gearing owes its bad reputation for friction and rapid wear.

Fig. 319 shows a worm and wheel, the teeth of which are drawn in this way. The worm here shown is of wrought iron or malleable cast iron, formed in one piece with its shaft. Usually the worm is of cast iron, and when small may be fixed by a pin passing through both worm and shaft. When larger its rotation on the shaft may be prevented by a key, and its tendency to slide along the shaft by collars, one of which may be fixed and the other a loose collar fixed by a set screw. Sometimes the bearings which support the worm shaft are so arranged as to prevent the endways motion of the worm.

273. *Screw gearing with line contact.*—Given the form of worm tooth a method can be found for drawing sections of the worm-wheel teeth on a series of parallel planes such that in each of those planes there should always be a point of contact with the worm thread. There is then always contact between worm and wheel along a line across the wheel face, and the whole surface of the worm-wheel tooth comes into action during the rotation of the wheel through the arc of contact. Such wheels naturally wear longer, and probably have less friction. They are, however, somewhat difficult to construct by ordinary processes of moulding and casting. Probably they could be moulded without difficulty in a wheel-moulding machine. An old plan occasionally used in making worm wheels for dividing engines, when great accuracy was required, was to make a steel worm like the worm to be used, to cut grooves along it to form cutting edges, then to harden it and to use it as a cutting tool, to cut

the worm-wheel teeth on a worm wheel cast with a blank rim. A worm wheel cut in this way has teeth of exactly the form given by the method mentioned above. There is always line contact between the worm and wheel. More recently Messrs. Browne & Sharpe have extended this method, and steel hobs ready prepared as chasers for worm wheels of almost any usual pitch are obtainable.[1]

*Mr. Briggs's account of the mode of designing screw gearing.*—When originally writing the chapter on worm gearing the author received from the late Mr. Briggs, of Philadelphia, a paper which led him to re-examine the subject. Mr. Briggs's statements are so interesting that they may be repeated here. After reverting to current opinion that the friction and wear of worm gearing is disastrous, Mr. Briggs says:—

'Now, the fact is, that the use of worm gearing for hoists, cranes, boring-bars, lathes, &c., has been growing in favour, and it is found that neither excessive loss of power nor excessive wear of gearing ensues. In regard to friction, it is established that for ordinary ratio of wheel to worm, say not to exceed 60 or 80 to 1, well-fitted worm gear will transmit motion backward through the worm, exhibiting a lower co-efficient of friction than is found in any other description of running machinery.

'It remains to be shown how to lay out a worm gear and worm so that this result will be reached, and to exhibit this the accompanying figures of a worm in position have been prepared. Accept the teeth on the worm to be 0·65 of the pitch, radially, of which 0·60 $p$ is to be the line of contact with the teeth of the wheel (on the radius and also on the plane through the middle of wheel), with 0·05 $p$ for clearance between the roots and points of worm and wheel teeth.

'Let the teeth of the wheel follow the circle of the worm through the arc $2a$, which ought not to exceed 60°, and is shown as 60° in the figures. Let $R$ = outside radius of worm; $R_p$ = radius of pitch line of worm; $p$ = pitch; $F$ = width of face of wheel at the root of the teeth. Then

$$R_p = \tfrac{1}{2}\,\{R + (R - 0{\cdot}6\,p)\cos a\}$$
$$F = 2\,(R + 0{\cdot}05\,p)\sin a.$$

[1] *Treatise on Gearing.* Brown & Sharpe Manufacturing Company. Providence, 1886.

'To simplify the process of drawing worm wheels, it has been usual to make R = 2 $p$ and 2 $\alpha$ = 60°. Then

$$R_p = 1{\cdot}606\,p.$$
$$F = 2{\cdot}05\,p.$$

'It will be found better to limit the number of teeth in the worm wheel to not less than 30; and if any less ratio of speed

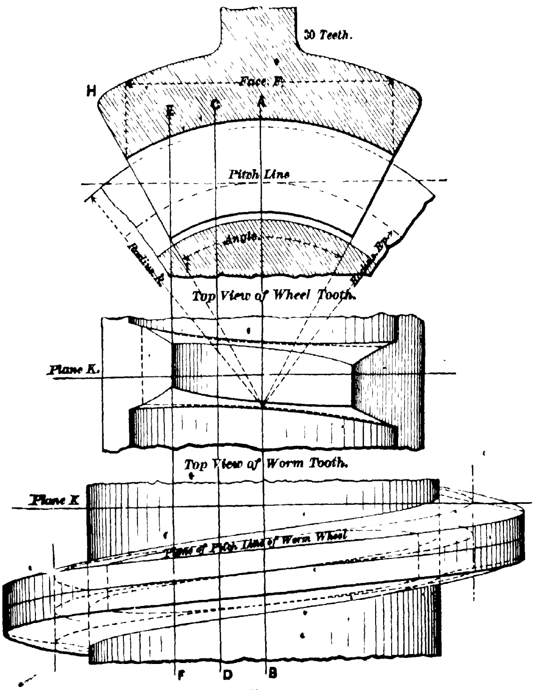



Fig. 320

than 1 in 30 be demanded, to employ double- or treble-threaded screws.

'The figures (fig. 320) consist of (1) a cross-section of worm wheel and worm; (2) a top view of wheel teeth; (3) a top view of worm teeth; fig. 321, (4) a side view development on the line or plane H I, or on the inclined face of the wheel teeth.

and forming a radial section through the worm ; (5, 6, and 7) horizontal sections on the planes or lines A B, C D, and E F, where the plane A B passes through the middle of the wheel and on the axis of the worm, and the planes C D and E F are

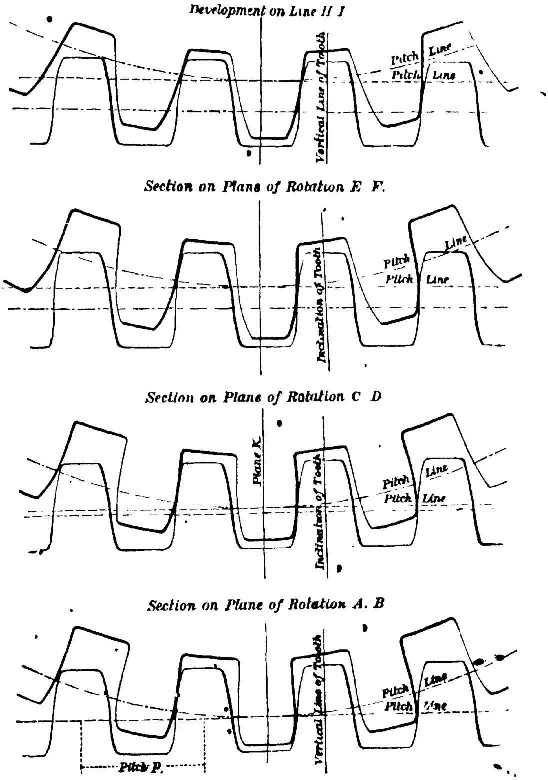


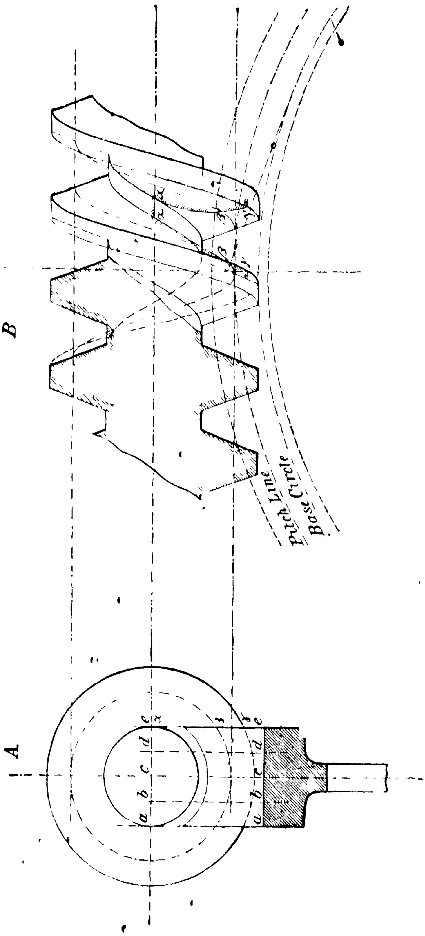

Fig 321

parallel to A B, as shown on cross-section of worm wheel and worm.'

Having assumed a form for the radial section of the worm thread, it is possible to find by projection the sections on A B, C D, E F, fig. 321, and, treating these as rack teeth, to find, by such a method as that in § 227, the proper forms of the worm

wheel teeth at these sections. The method, however, is inconvenient.

274. *Accurate method of designing worm-wheel teeth.*—The




following method is believed to be new (1882). A form of worm thread is assumed, and a number of parallel sections of the worm-wheel tooth are obtained which will work with it. The

case has been chosen so as to show in a marked way the difference between the proper sections and those ordinarily adopted.

To obtain correct forms of teeth, the rules applied to the sections in the plane passing through the worm axis normal to the wheel axis must be applied to all sections on planes parallel to that plane: that is, all the sections of the worm threads and wheel teeth, on planes normal to the wheel axis, must be of forms suitable for a spur wheel and rack of the same pitch. To obtain these sections proceed as follows :—

Draw first the two views, A, B, fig. 322, of the worm and wheel on planes passing through each axis normal to the other axis. With the proportions given above mark off the root and addendum of the teeth, and design the section of the worm threads and wheel teeth on the plane *c c* in accordance with Professor Willis's principle mentioned above. The sections chosen in the present example are shown at *c c* in fig. 323. The wheel teeth are involute teeth, and the worm threads are similar to those of a rack suitable to work with such teeth. They are bounded by straight lines normal to the base circle tangent. The section of the worm teeth is shown again in fig. 322.

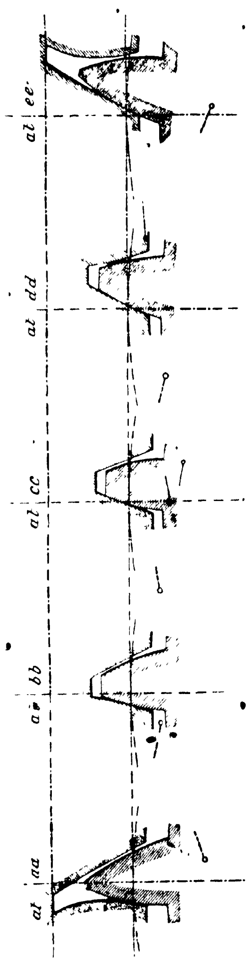


Fig 323

Next in fig. 322, B, draw helices corresponding to any points in the worm-thread section. In the figures three helices are shown corresponding to the point, root, and pitch point of the thread. Then the true section of the worm thread on, for example, the plane *e e*, fig. A, is found in fig. B by projecting the points $\alpha\,\beta\,\gamma$, in fig. A, to the corre-

sponding helices in fig. B. We thus get the points $\alpha\beta\gamma$, $\alpha\beta\gamma$ marking the section in fig. B, and the same section has been transferred to *e e*, fig. 323. The other sections in that figure were obtained in the same way. It remains to find the corre-

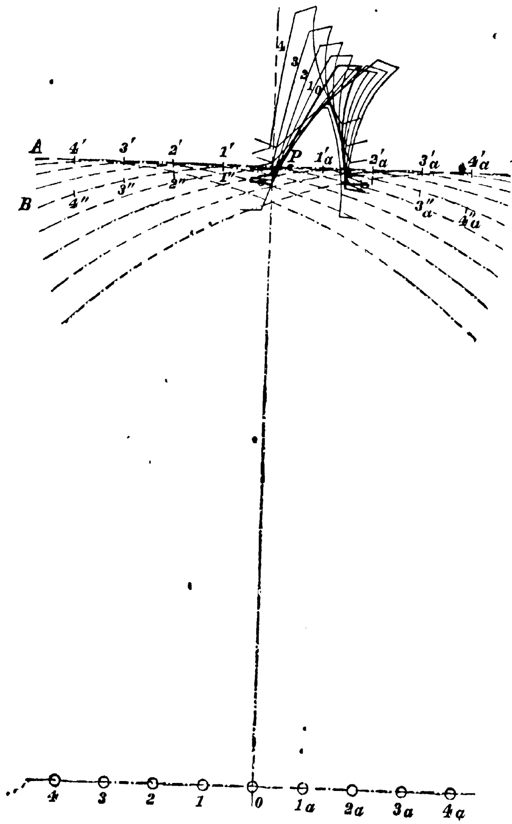



Fig. 324

sponding worm-wheel tooth sections, of which only that for *c c* section is as yet determined.

In fig. 324, let *o* be the centre of the worm wheel, A A the worm pitch line, B B the worm-wheel pitch line, and of the worm thread forms let *ȯ* be the one corresponding to the sec-

tion at *e e* in fig. 323. It will make no difference in the relative motion of the worm and wheel if we suppose the worm-wheel pitch line to roll along the worm pitch line. On a piece of tracing paper mark off the centre *o*, and the pitch line B B. In order to roll this traced pitch line on A A, mark off equidistant positions of the centre of the worm wheel, 0, 1, 2, 3, 4, 1*a*, 2*a*, 3*a*, 4*a*, and also the corresponding touching points P, 1′, 2′, 3′, 4′, . . . . along the worm pitch line. On the traced worm-wheel pitch line take P 1″ = P 1′; 1″ 2″ = 1′ 2′; and so on. The traced pitch line can then be easily placed in the successive positions shown by the dotted arcs touching the worm pitch line. In each of these positions trace off on the tracing paper the worm-thread form 0. We shall thus obtain on the tracing paper the figure of the threads shown at 0, 1, 2, 3, 4. The envelope of these positions is the proper form of the worm-wheel thread. In drawing this, however, the top and side clearance must be left as shown. The side clearance may be 0·04 of the pitch.

In fig. 323 the result of applying this method to four sections other than the central one is shown. It may be remarked that a circular arc can be found by trial for each side of the worm tooth, which agrees accurately enough for any practical purpose with the required envelope of the worm threads. The teeth in fig. 323 are drawn with such circular arcs, the centres of the arcs being marked on the figure. The pattern of the wheel would have to be made by cutting the teeth to templets of the forms found by the construction. The worm wheel in the figure is shown with the front of the teeth concave. But they may be cut off parallel to the rim without altering the correct action of the teeth.

The process shown in fig. 323 seems to have suggested to Mr. Last the tracing-paper method of drawing wheel teeth, described in Chapter XII, § 233. It is easy to see that Mr. Last's mode of rolling the pitch circle described on tracing paper, along a straight line A A, on which has been drawn the worm thread section, facilitates the process of finding the worm wheel tooth section.

275. *Cutting worm wheels by hobs*.—Suppose a steel worm cut in the lathe, the section of the threads on a plane through the worm axis being the same as that of involute or cycloidal rack teeth. Let cutting edges, properly cleared, be formed on

the faces of the thread, and the worm then hardened and tempered. The worm is then termed a *hob*, and can be used in cutting the worm wheel. Very perfect hobs of this kind are now regularly made by some tool makers. The worm wheel blank has the tooth spaces first roughly formed by a rotating milling cutter, the spaces being cut across the blank at the same obliquity as the worm threads at their outside diameter. Then the wheel blank is placed on a loose turn-table, and the hob, used as a milling cutter, is gradually brought into gear with the roughly cut spaces. The hob begins by cutting the points of the teeth, and is fed till the distance between its axis and that of the worm wheel is that required. The most convenient form for the teeth is to make the section of the worm thread straight sided on an axial plane, and the section of the worm wheel teeth on the mid plane normal to the axis the same as involute teeth of a spur wheel. Very commonly the hob in cutting drives the worm wheel, but more accuracy is secured if the worm wheel blank is connected to the axis carrying the hob by gearing, so that the two revolve with the required velocity ratio. Mr. I. H. Gibson has designed and constructed a worm wheel cutting engine, in which an ordinary fly cutter is used to cut the worm wheel.[1]

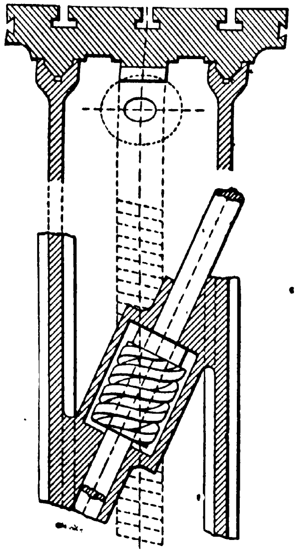


Fig. 325

276. *Worm and rack.*—If the radius of the worm wheel is infinite it becomes a rack. In the general case the rack teeth will be straight and parallel, and may be perpendicular or inclined to the direction of motion of the rack. Fig. 325 shows a worm driving a rack, the axis of the worm being oblique to the direction of motion of the rack. This arrangement has been

[1] 'On the Machine Cutting of Accurate Bevil and Worm Gears.' I. H. Gibson, *Minutes of Proceedings of North East Coast Inst. of Engineers and Shipbuilders*, vol. xiii. 1896-7.

used with great advantage by Sellers for driving a planing machine table. In that case the worm has four threads. The teeth of the rack are straight, and are placed at an angle of 85° to the direction of motion to counterbalance the tendency of the worm to move the table sideways. The thrust of the worm shaft is received during the cutting stroke on a footstep bearing at the end of the shaft, and the smaller thrust during the return stroke is taken against hardened collars on the other side of the worm. The arrangement is very convenient, and durable and smooth in action. Strictly to secure line contact, a hob of the worm form should be used to cut the rack teeth.

277. *Strength of worm gearing.*—The worm is usually at least as strong as the worm wheel, and in deciding the pitch it is accurate enough for practical purposes to consider merely the circumferential pitch of the worm wheel. It is desirable to reduce the pitch of worm gearing as much as possible in order to diminish friction and wear. With gearing of this kind, if the worm is of suitable length, two teeth of the worm wheel are always in contact with the threads of the worm, and may be more, and there is no valid reason why this fact should not be taken into account in considering the strength of the teeth, at least in the case of cut gearing in those cases in which strength is the chief consideration. But generally durability is more important, and then it is better to take the whole load at the pitch line to be acting on one tooth.

Let M be the moment exerted by the worm wheel, in inch lbs., H the horse-power transmitted, V the velocity of pitch line, in feet per second, R the radius of the wheel, in inches, N the revolutions per minute, and $p$ the circumferential pitch of worm wheel. Then, as in § 245, the total driving effort at the pitch line of the worm wheel is

$$P = \frac{550\ H}{V} = 63020 \frac{H}{R\ N} = \frac{M}{R}.$$

The pitch may be determined by the rules for spur gearing. Thus if $p$ is the pitch and $b$ the width of face of the worm wheel,

$$P = c\ b\ p$$

where for strength in gearing used intermittently $c = 250$ to 400 for cast iron. For continuously running gear the load on the teeth should be reduced to increase the durability and $c$ may be 120 to 300. For a steel worm and phosphor bronze wheel well designed and run in an oil bath $c = 500$ to 700.

# CHAPTER XIV

## BELT GEARING

278. The term *belt*, *band*, or *strap* is applied to a flexible connecting piece, which drives a rotating piece termed a *pulley* by its frictional resistance to slipping. The belt always acts by tension.

*Material of belting.*—Belts are most commonly of leather tanned by oak bark. The best part of the hide is cut into strips, which are united into lengths by cementing, lacing, or riveting. Special kinds of leather are used where great strength is required. In some cases so-called raw hide is used. Pure vulcanised india-rubber, or more often india-rubber with interposed plies of strong canvas, is often used, especially in wet places where leather is unsuitable. Gutta-percha has been used, but it stretches permanently too much. Waterproofed cotton woven belting is now a good deal used, and can be made of great width (up to 60 inches). It may be applied in the open air and where exposed to damp.

Ordinary belts are flat belts, and are used on pulleys with flat or slightly rounded rims. But round belts are also used running in pulleys with V-shaped grooves. Rope belting will be treated in another chapter. The smaller round belts are of catgut, or of twisted leather, which is cheap and effective if the tension is small.

*Endless Belt.*—When one shaft is driven from another, a pulley is placed on each shaft, and an endless belt is strained over the two pulleys. The belt may be an open belt (fig. 326) or a crossed belt (fig. 327). In the former case the two shafts rotate in the same direction. In the latter case they rotate in opposite directions.

For special cases where smoothness of motion is important endless woven belts are used. Usually a belt has a joint made by lacing or by some form of metal fastener which can be easily disconnected if the belt requires tightening.

279. *Strength of leather belting.*—The ultimate strength of the leather used for belting is 3,000 to 5,000 lbs. per square inch, or, stating it more conveniently, 750 to 1,500 lbs. per inch width of single belt. At the laced joints the strength is reduced to about 0·3 of this, or, say, 250 to 450 lbs. per inch of width. The following table gives some tests of belting and of different kinds of joint fastening :—

*Strength of Leather Belts and Fastenings*

| — | Tenacity in lbs per inch of width | Authority |
|---|---|---|
| Single Leather Max. | 1,600 | Riehle Bros. |
| ,, ,, Min. | 700 | ,, ,, |
| ,, ,, Mean | 1,280 | ,, ,, |
| ,, ,, Max. | 1,272 | Unwin |
| ,, ,, Min. | 616 | ,, |
| ,, ,, Mean | 964 | ,, |
| Double Belt, copper sewn | 1,110 | ,, |
| Single Belt, ordinary laced joint | 473 | ,, |
| ,, ,, butt laced | 265 | ,, |
| ,, ,, joint scarfed and glued | 544 | ,, |
| ,, ,, grip fastener | 242 | ,, |
| ,, ,, Crowley's fastener | 635 | ,, |
| ,, ,, hose riveted, rivets in two rows | 394 | Watertown |

Experience shows that the working tension (that in the tight side of the belt) should only be a small fraction of the breaking strength. Unless this is so the belt rapidly loses its tension by stretching.

The thickness of leather belts varies from $\frac{5}{32}$ to $\frac{7}{32}$ (0·16 to 0·22) inch in single belting, and from $\frac{5}{16}$ to $\frac{7}{16}$ (0·32 to 0·44) inch in double belting. The smallest diameter of pulleys on which belts will run fully loaded without excessive wear from slipping is about thirty-five times the belt thickness, and greater diameters are preferable.

| | Single belt | Double belt | Treble belt |
|---|---|---|---|
| Thickness of belt, ins. | 0·2 | 0·4 | 0·6 |
| Smallest pulley diameter, ins. | 7 | 14 | 21 |

The safe working stress will be discussed later. It varies usually from 160 to 330 lbs. per sq. in., or about 30 to 65 lbs. per inch width of single belt. That is, from at most one fourth to as little as one-fifteenth of the strength at the joint.

*Extension and coefficient of elasticity of leather.*—The behaviour

of leather is unlike that of metals. For ordinary ranges of stress the leather becomes stiffer as the load increases. Hence if $e$ is the extension per inch of length with a stress $f$ lbs. per sq. in., $e$ increases less fast than the stress. In the ordinary expression for the coefficient of direct elasticity, $E = f/e$, the value of $E$ is not constant, but increases as $f$ is greater. But if an expression of the form

$$C = f^n/e \quad . \quad . \quad . \quad . \quad . \quad (1)$$

is assumed, then for leather both $C$ and $n$ are nearly constant. The value of $n$ is about 0·5, so that for leather $C = \sqrt{f}/e$, and $C$ has values ranging from 700 to 900,* with a mean value for used belting about 850. It is impossible to give very definite values, because the stretch increases with the time. The results here are for rapid loading. The following table, calculated from the test of a piece of belt 92 inches long by Mr. Lewis, shows the variation of $E$ and the approximate constancy of $C$.

| Stress, lbs. per sq. in. | Elongation in inches | Extension per inch of length $e$ | $E = f/e$ | $C = \sqrt{f}/e$ |
|---|---|---|---|---|
| 500 | 2·5 | ·0272 | 18,400 | 824 |
| 300 | 1·9 | ·0207 | 14,500 | 836 |
| 100 | 1·1 | ·0120 | 8,370 | 834 |

280. *Velocity ratio in belt transmission.*—A belt is not used in cases where a very exact velocity ratio is necessary, hence it is generally accurate enough in calculating speeds to regard the belt as inextensible and as of very small thickness. If there is no slipping of the belt on the pulleys, the velocity of the belt and the surface velocities of the pulleys must all be equal. Let $V$ be the velocity of the belt, $d_1$ $d_2$ the diameters of the pulleys, and $N_1$ $N_2$ their revolutions per minute—

$$\left.\begin{array}{l}\pi d_1 N_1 = V \\ \pi d_2 N_2 = V\end{array}\right\} \therefore \frac{d_1}{d_2} = \frac{N_2}{N_1} \quad . \quad . \quad . \quad (2)$$

These equations are in strictness only true when the belt is infinitely thin. When the belt has a thickness $\delta$, the effective diameters of the pulleys are $d_1 + \delta$, and $d_2 + \delta$. Then,

$$\frac{N_2}{N_1} = \frac{d_1 + \delta}{d_2 + \delta}$$

As the belt thickness is small compared with the pulley diameter, $\delta$ may generally be neglected. But another circum-

stance affects the velocity ratio. In consequence of the elasticity of the belt, it is more stretched on the tight than on the slack side and the change of stretch is effected by creep in passing over the pulleys.[1] There is usually in addition some slipping of the belt as a whole. The total slip, according to Mr. Lewis, in ordinary driving may be one per cent. of the belt speed on

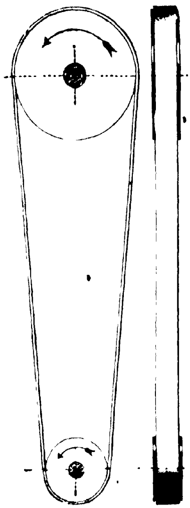


Fig. 326

Fig. 327

each pulley or 2 per cent. altogether. If the slip exceeds 10 per cent. on each pulley the belt generally throws off the pulley.

[1] Let $d_2 N_2$ be the diameter and revolutions of the driving and $d_1 N_1$ of the driven pulley, and let $t_2$ and $t_1$ be the tight and slack tensions of the belt per sq. in. The quantity of belt passing over each pulley in a given time will be the same, but the lengths will be different in consequence of the different stretch of the tight and slack sides. The ratio of the lengths of a given quantity of belt in the tight and slack sides is $(1 + \sqrt{t_2}/C) / (1 + \sqrt{t_1}/C)$. But the belt passes on to the driver in its tight state and on to the driven in its slack state, at which points there can be no creep accommodating its change of length by creep on the pulleys. Hence so far as elastic creep is concerned:

$$\frac{d_2 N_2}{d_1 N_1} = \frac{1 + (\sqrt{t_2}/C)}{1 + (\sqrt{t_1}/C)} = \frac{C + \sqrt{t_2}}{C + \sqrt{t_1}}$$

For $t_2 = 244$, $t_1 = 44$, $C = 850$,

$$d_2 N_2 = 1 \cdot 01\, d_1 N_1$$

$$\frac{d_2}{d_1} = 1 \cdot 01 \frac{N_1}{N_2}$$

281. *Length of belts.*—Let D and $d$ be the diameters of the two pulleys in inches; $c$, their distance apart, from centre to centre; L, the length of the belt. Also, let $D + d = \Sigma$, and $D - d = \Delta$.

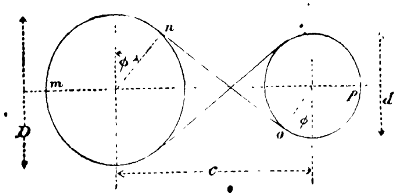


Fig. 328

For a crossed belt (fig. 328) the total length—

$$= L = 2(m\,n + n\,o + o\,p)$$

$$= \left(\frac{\pi}{2} + \phi\right)D + 2\,c\cos\phi + \left(\frac{\pi}{2} + \phi\right)d$$

$$= \left(\frac{\pi}{2} + \phi\right)\Sigma + 2\,c\cos\phi \qquad . \quad . \quad . \quad (3)$$

$$\sin\phi = \frac{D + d}{2\,c} = \frac{\Sigma}{2\,c} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

The length of the belt is obtained thus. Calculate the value of $\sin\phi$. From a table of natural sines and cosines find the nearest values of $\cos\phi$ and $\phi$, the latter being expressed in circular measure. Then eq. (2) gives the belt length. If $\phi$ is found or measured off the drawing in degrees, the circular measure of the angle is obtained by multiplying by 0·0175.

With a crossed belt $\phi$ depends only on $D + d$. Hence, if $\Sigma$ and

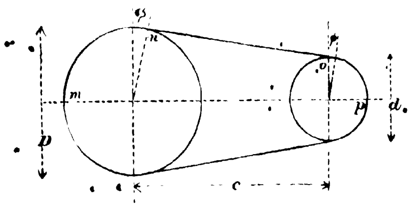


Fig. 329

$c$ are constant for two or more pairs of pulleys, the same belt will run on any pair of pulleys of the set.

When the belt is an open one (fig. 329) the equations are rather less simple—

$$L = 2\,(m\,n + n\,o + o\,p)$$
$$= \left(\frac{\pi}{2} + \phi\right) D + 2\,c\cos\phi + \left(\frac{\pi}{2} - \phi\right) d$$
$$= \frac{\pi}{2}\Sigma + \phi\,\Delta + 2\,c\cos\phi \quad . \quad . \quad . \qquad (4)$$

$$\operatorname{Sin}\phi = \frac{\Delta}{2c}; \quad \cos\phi = \sqrt{\left(1 - \frac{\Delta^2}{4c^2}\right)} \quad . \qquad (4a)$$

For an open belt $\phi$ is generally small, so that,

$$\phi = \sin\phi, \text{ nearly}$$

$$L = \frac{\pi}{2}\Sigma + 2\,c\left\{\frac{\Delta^2}{4c^2} + \sqrt{\left(1 - \frac{\Delta^2}{4c^2}\right)}\right\}$$
$$= \frac{\pi}{2}\Sigma + 2\,c\left\{1 + \frac{1}{8}\frac{\Delta^2}{c^2}\right\} \text{ nearly} \quad . \quad . \quad . \quad (5)$$

Hence, if an open belt runs on a pair of pulleys, the sum and difference of whose diameters are $\Sigma_1$ and $\Delta_1$, and the same belt is also to run on another pair of pulleys, the sum and difference of whose diameters is $\Sigma_2$ and $\Delta_2$, since the length of the belt is the same in the two cases,

$$\frac{\pi}{2}\Sigma_1 + 2\,c\left\{1 + \frac{1}{8}\frac{\Delta_1^2}{c^2}\right\} = \frac{\pi}{2}\Sigma_2 + 2\,c\left\{1 + \frac{1}{8}\frac{\Delta_2^2}{c^2}\right\}$$
$$\Sigma_2 = \Sigma_1 + \frac{\Delta_1^2 - \Delta_2^2}{2\,\pi\,c} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

282. *Speed cones.*—When a shaft running at a constant speed has to drive a machine at several different speeds, sets of pulleys are used which are termed stepped speed cones.

The speed cones (fig. 330) are placed opposite one another, so as to form a series of pairs of pulleys, and by shifting the belt from one pair to another the speed of the machine is altered. In designing these speed cones the ratio of the diameters of each pair depends on the speeds of the shafts, and the sum of the diameters should be so arranged that the same belt will work on any pair of the set without alteration of length.

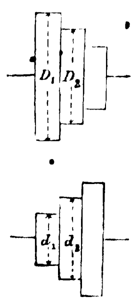


Fig. 330

Let $n$ be the number of rotations of the driving shaft carrying pulleys of diameters $D_1, D_2, D_3$ . . . and $n_1, n_2, n_3$ . . the required rotations of driven shaft. The same belt is to work with equal tightness on any pair of the pulleys. For definiteness let $n_1$ be the fastest speed of the

driven shaft, so that $D_1$ is the largest driving and $d_1$ the smallest driven pulley.

*Case I. Crossed belt.*—The velocity ratio conditions give the equations

$$D_1/d_1 = n_1/n\ ;\ D_2/d_2 = n_2/n\ ;\ \text{and so on} \quad . \quad . \quad (a)$$

The constancy of belt length gives the equations

$$D_1 + d_1 = D_2 + d_2 = \ldots = \Sigma \quad . \quad . \quad . \quad (b)$$

One more equation is wanted. Suppose $\Sigma$ given. Then

$$\left.\begin{array}{l} D_1 = n_1\,\Sigma / (n + n_1)\ ;\ d_1 = n\,\Sigma / (n + n_1) \\ D_2 = n_2\,\Sigma / (n + n_2)\ ;\ d_2 = n\,\Sigma / (n + n_2) \\ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \end{array}\right\} \quad . \quad . \quad (c)$$

More commonly the diameter $D_n$ of the smallest driving pulley, or the diameter $d_1$ of the smallest driven pulley, will be given. Then

$$d_n = n\,D_n / n_n \text{ or } D_1 = n_1 d_1 / n$$
$$\text{and } \Sigma = D_n + d_n \text{ or } D_1 + d_1$$

With this value the other diameters can be found by eq. (*c*).

If one stepped pulley is designed first (for instance with equal increments of diameter at each step), let $d_1, d_2, d_3 \ldots$ be the diameters so determined. Then $D_1 = n_1\ d_1 / n$, where $n_1 / n$ is a velocity ratio suitable for that pair. Then $\Sigma = D_1 + d_1$, and the other diameters are given by the equations

$$D_2 = \Sigma - d_2\ ;\ D_3 = \Sigma - d_3 \quad . \quad . \quad . \quad . \quad (d)$$

*Case II. Open belt.*—Generally the question of designing speed cones for an open belt can be reduced to this: Given the diameters of one pair of pulleys, to find the diameters of another pair to work with the same belt, the speeds of driving shaft and driven shaft being also given. Thus let $D_0$ be the diameter of pulley on driving shaft at $n$ revolutions, which, working with a pulley of diameter $d_0$, gives a speed of $n_0$ revolutions of driven shaft. It is required to find the diameters $D_1$, $d_1$, of a pair of pulleys, for the same belt, to drive the driven shaft at $n_1$ revolutions. $\Sigma_0 = D_0 + d_0$, and $\Delta_0 = D_0 - d_0$, are determined directly from the data.

Now (*a*) treat the case as the same as that of a crossed belt. Then the following are first approximations :—

$$\Sigma'_1 = \Sigma_0 ; \ D'_1 = \frac{n_1}{n_1 + n} \Sigma'_1 ; \ d'_1 = \frac{n_1}{n_1 + n} \Sigma_1 ;$$

$$\Delta'_1 = \frac{n_1 - n}{n_1 + n} \Sigma'_1$$

For a second approximation, from formula (6) above :—

$$\Sigma''_1 = \Sigma_0 + \frac{\Delta_0{}^2 - \Delta'_1{}^2}{2 \pi c} ; \ D''_1 = \frac{n_1}{n_1 + n} \Sigma''_1 ;$$

$$d''_1 = \frac{n}{n_1 + n} \Sigma''_1 ; \ \Delta''_1 = \frac{n_1 - n}{n_1 + n} \Sigma''_1$$

Generally this is accurate enough for practical purposes. A third approximation is :—

$$\Sigma'''_1 = \Sigma_0 + \frac{\Delta_0{}^2 - \Delta''_1{}^2}{2 \pi c} ; \quad D'''_1 = \frac{n_1}{n_1 + n} \Sigma'''_1 ;$$

$$d''_1 = \frac{n}{n_1 + n} \Sigma'''_1$$

Thus suppose $D_0 = d_0 = 12$ in. ; the distance between the shafts 25 in. (taken small to exaggerate the errors of the method of calculation). Let the speed of the two shafts be 100 with these pulleys, and let it be required to find the diameters of a pair of pulleys to drive the second shaft at 200. Then for a first approximation, $n = 100$, $n_1 = 200$, $\Sigma'_1 = 24$ ; $D'_1 = 16$ ; $d'_1 = 8$ ; $\Delta'_1 = 8$. A second approximation gives : $\Sigma''_1 = 23{\cdot}593$ ; $D''_1 = 15{\cdot}728$ ; $d''_1 = 7{\cdot}864$ ; $\Delta''_1 = 7{\cdot}864$. A third approximation gives: $\Sigma'''_1 = 23{\cdot}606$ ; $D'''_1 = 15{\cdot}7374$ ; $d'''_1 = 7{\cdot}8687$. The results of the third are not sensibly different from those in the second approximation.

Now let it be required to find a third pair of pulleys to drive the second shaft at 400. Then $n = 100$ ; $n_1 = 400$ ; $\Sigma'_1 = 24$ ; $D'_1 = 19{\cdot}2$ ; $d'_1 = 4{\cdot}8$ ; $\Delta'_1 = 14{\cdot}4$. The second approximation gives : $\Sigma''_1 = 22{\cdot}680$ ; $D''_1 = 18{\cdot}144$ ; $d''_1 = 4{\cdot}536$ ; $\Delta''_1 = 13{\cdot}608$. The third approximation gives : $\Sigma'''_1 = 22{\cdot}821$ ; $D'''_1 = 18{\cdot}256$ ; $d'''_1 = 4{\cdot}564$. Even here the second approximation is not very inaccurate. With ordinary distances between the shafts the second approximation differs much less from the third.

*Graphic construction for an open belt when the angle between the straight portions of the belt does not exceed about* 18°.—Mr. C. A. Smith has given the following neat construction,[1] fig. 331.

[1] *Trans. Am. Soc. Mech. Eng.* x. 269.

Let the diameters $d_1$, $d_2$, $d_3$ of one stepped cone be given and the distance $l =$ A B between the shafts. Choose any one of the driven diameters, say $D_1$, and draw the tangent $a\ b$. Bisect A B in C, and draw C D at right angles to A B. Take C D $= \cdot 314\ l$. With centre D draw an auxiliary circle touching $a\ b$. In some cases this circle falls inside the belt. Draw tangents to the

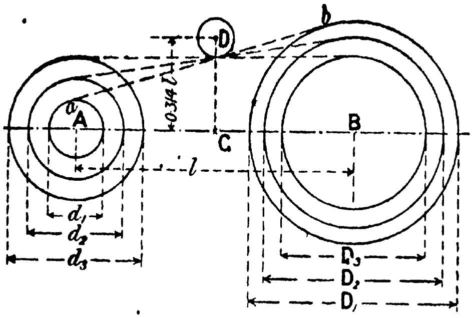



Fig. 331

circles $d_2$, $d_3$, touching the auxiliary circle. Circles with centre B touching these tangents will give the required diameters $D_2$, $D_3$.

283. *Resistance to slipping of a belt on a pulley.*—Let fig. 332 represent a belt strained over a pulley by tensions $T_1$, $T_2$, and on the point of slipping from $T_1$ towards $T_2$ in consequence of

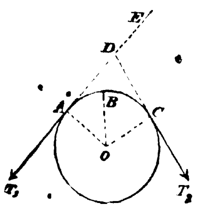



Fig. 332

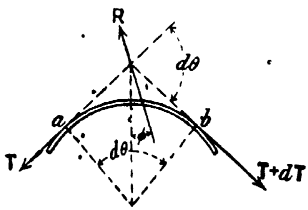



Fig. 333

the difference of tension $T_2 - T_1$. Then $T_2 - T_1$ must be equal to the frictional resistance of the belt on the pulley. Let B be any point of contact; T = the tension at B; $s =$ arc A B; $r =$ radius O A. The following theorem depends on the assumption that $\mu$ is constant for the arc of contact considered.

Suppose $a\ b$, fig. 333, represents a small arc $d\ s$ of the pulley,

in the neighbourhood of B, subtending an angle $d\,\theta$, the length $d\,s$ being greatly exaggerated for clearness. The tensions at $a$ and $b$ will be T and T + $d$ T, where $d$ T is the increment of tension in the arc $d\,s$. These tensions must be in equilibrium with the reaction R of the pulley on the part $a\,b$ of the belt. R acts through the intersection of the tensions, and at the angle of repose $\phi$ with the radius, where $\tan\phi = \mu$, the coefficient of friction. Resolving parallel to T,

$$T + R\sin\phi = (T + d\,T)\cos d\,\theta,$$

or since $d\,\theta$ is small, $R\sin\phi = d\,T$. Resolving at right angles to T,

$$R\cos\phi = (T + d\,T)\sin d\,\theta,$$

or since $d$ T is small compared with T and $d\,\theta$ is small, $R\cos\phi = T\,d\,\theta$. Dividing, $\tan\theta = d\,T/T\,d\,\theta = d\,T/d\,s$. Since $\tan\phi = \mu$,

$$\frac{d\,T}{T} = \mu\,d\,\theta$$

$$\int_{T_1}^{T_2}\frac{d\,T}{T} = \mu\int_{0}^{\theta} d\,\theta\,.$$

$$\text{Hyp. log. } (T_2/T_1) = \mu\,\theta$$

$$T_2/T_1 = \epsilon^{\mu\,\theta} \qquad . \quad . \quad (7)$$

where $\epsilon = 2{\cdot}718$ and $\theta$ is the whole angle of contact A O C in circular measure. If for simplicity $k$ is put for $\epsilon^{\mu\,\theta}$ then, for a belt driving a pulley, the limiting value of $T_2/T_1$ at which the friction balances the difference of tensions is $k$. But $T_2/T_1$ may have any less value.

The following forms of the equations are sometimes convenient :

Common log. $T_2/T_1 = k = 0{\cdot}434\,\mu\,\theta$ if $\theta$ is in circular measure.
$= 0{\cdot}007578\,\mu\,\theta$ if $\theta$ is in degrees.
$= 2{\cdot}729\,\mu\,n$ if $n$ is the fraction of the circumference embraced by the belt.

The antilogarithm of the quantity on the right of either of these equations is the value of $\epsilon^{\mu\,\theta}$ or $k$. The equations are the same, if for the total tensions $T_2$ and $T_1$, the tensions $t_2$ and $t_1$ per sq. in. of belt section are substituted.

284. *The coefficient of friction and values of k.*—Subject to

considerations given below, the coefficient of friction for leather belts on iron pulleys in normal working varies from $\mu = 0·3$ to $\mu = 0·4$. For wire rope running on the bottom of a grooved pulley $\mu = 0·15$, or, if the pulley is bottomed with leather or gutta percha, $\mu = 0·25$.

The arc of contact on the pulley of a pair on which that arc is smallest is, in a large number of cases, about $\theta = 165°$. Taking this value, the ratio of the tensions for different values of $\mu$ is as follows :—

| | | | | | | |
|---|---|---|---|---|---|---|
| $\mu =$ | 0·15 | 0·2 | 0·25 | 0·3 | 0·4 | 0·5 |
| $T_2/T_1 = k =$ | 1·54 | 1·78 | 2·06 | 2·37 | 3·16 | 4·22 |

It will be seen presently that, in normal driving, it is found that often $k$ for leather belts lies between 2 and 3. The following table gives the value of the ratio $k$ of the tensions for various arcs of contact, and for various values of the coefficient $\mu$.

*Greatest Value of the Ratio of Tensions on Tight and Slack Sides of Belting*

| Angle embraced by belt $\theta =$ | | | Ratio of tensions $k =$ | | | |
|---|---|---|---|---|---|---|
| In degrees | In circular measure | In fraction of circumference | $\mu = 0·2$ | $\mu = 0·3$ | $\mu = 0·4$ | $\mu = 0·5$ |
| 30 | ·524 | ·083 | 1·110 | 1·170 | 1·233 | 1·299 |
| 45 | ·785 | ·125 | 1·170 | 1·266 | 1·369 | 1·481 |
| 60 | 1·047 | ·167 | 1·233 | 1·369 | 1·521 | 1·689 |
| 75 | 1·309 | ·208 | 1·299 | 1·481 | 1·689 | 1·924 |
| 90 | 1·571 | ·250 | 1·369 | 1·602 | 1·874 | 2·193 |
| 105 | 1·833 | ·319 | 1·443 | 1·733 | 2·082 | 2·500 |
| 120 | 2·094 | ·334 | 1·521 | 1·875 | 2·312 | 2·851 |
| 135 | 2·356 | ·375 | 1·602 | 2·027 | 2·565 | 3·247 |
| 150 | 2·618 | ·417 | 1·689 | 2·194 | 2·849 | 3·702 |
| 165 | 2·880 | ·458 | 1·778 | 2·372 | 3·163 | 4·219 |
| 180 | 3·142 | ·500 | 1·875 | 2·566 | 3·514 | 4·808 |
| 195 | 3·403 | ·541 | 1·975 | 2·776 | 3·901 | 5·483 |
| 210 | 3·665 | ·583 | 2·082 | 3·003 | 4·333 | 6·252 |
| 240 | 4·188 | ·666 | 2·311 | 3·514 | 5·340 | 8·119 |
| 270 | 4·712 | ·750 | 2·566 | 4·112 | 6·589 | 10·55 |
| 300 | 5·236 | ·833 | 2·849 | 4·808 | 8·117 | 13·70 |

*Dependence of the coefficient of friction on the slipping of the belt on the pulley. Creep of belt.*—The earlier writers assumed that there was no slipping of the belt on the pulley, and that if slipping occurred the belt would cease to drive. So far is this from

being the case that there is always some slip of the belt, and belts will continue driving till the slip becomes considerable. Experiments by Professor Lanza show that $\mu$ increases with the amount of slip ('Trans. Am. Soc. Mech. Eng.' vii. 347). Thus for oak tanned leather belting run at a speed of about 1,500 ft. per min. with the hair side and flesh side next a cast-iron pulley the following results were obtained :

| V Ft. per min. | Speed of Slip Ft. per min. | Value of $\mu$ | |
|---|---|---|---|
| | | Hair Side | Flesh Side |
| 1,500 | 238 | ·78 | ·45 |
| 1,500 | 210 | ·82 | ·51 |
| 1,500 | 17 | ·34 | ·42 |
| 1,500 | 15 | ·33 | ·36 |

In conditions more like those of ordinary working—that is, with a less excessive amount of slip—the following were the results :

| Hair Side | | | Flesh Side | | |
|---|---|---|---|---|---|
| Ratio of Tensions $k$ | Speed of Slip, Ft. per min. | $\mu$ | Ratio of Tensions $k$ | Speed of Slip, Ft. per min. | $\mu$ |
| 2·12 | 2·09 | ·24 | 1·92 | 2·09 | ·21 |
| 2·40 | 2·61 | ·28 | 2·19 | 3·38 | ·25 |
| 2·64 | 6·92 | ·31 | 3·12 | 7·00 | ·36 |

Professor Lanza concludes from these tests that for permanent running and a belt speed of 1,500 ft. per min. the slip should not exceed three or four feet per minute and that then the average value of $\mu$ for leather belts on iron pulleys is 0·27.

There are a large number of similar results in a paper by Mr. Lewis ('Trans. Am. Soc. Mech. Eng.' vii. 549), and these show the important result that as the belt is more heavily loaded—that is, as the effective effort $T_2 - T_1$ is increased —the belt accommodates itself to the conditions by increasing the slip and consequently the co-efficient $\mu$. Thus for an open belt running at $V = 800$ feet per min. on 20-inch, cast-iron pulleys, the following is a sample of the results :—

| V feet per minute | $T_2 - T_1$ lbs. | $T_2/T_1$ | Velocity of slip, feet per minute | $\mu$ |
|---|---|---|---|---|
| 800 | 120 | 2·3 | 2 | ·27 |
| 800 | 180 | 3·7 | 4 | ·42 |
| 800 | 240 | 5·8 | 15 | ·57 |
| 800 | 300 | 9·0 | 61 | ·72 |

It is obvious, therefore, that the older view that a fixed value of $\mu$ could be found for belting is erroneous. The real question is what amount of slip is permissible in ordinary working. Now the general evidence is that the slip is not objectionable if it does not exceed about 1 per cent. of the belt velocity on each pulley. Hence the permissible slip increases with the belt speed, and higher values of $\mu$ may be taken as the belt speed is greater.

The following rule appears to give maximum values of $\mu$ within permissible working conditions ;

$$\mu = 0{\cdot}2 + 0{\cdot}004\sqrt{V} \qquad (8)$$

| | | | | | |
|---|---|---|---|---|---|
| V in ft. per min. | 500 | 1,000 | 2,000 | 4,000 | 6,000 |
| $\mu$ | ·29 | ·33 | ·38 | ·45 | ·51 |

285. *Ordinary theory of the tensions in an endless belt.*—In this treatment of the question the belt is assumed to be perfectly elastic—that is, the stretch is assumed to be proportional to the tensions. In some cases this is nearly true, but it has been shown that for leather belts it is far from true. For the case of leather belts an attempt at a more exact theory will be given later, but even for leather belts the ordinary theory has been useful, and in important respects gives true results.

Let an endless belt be strained over two pulleys with an initial tension $T_0$, which with the pulleys at rest is uniform throughout the belt. At the moment the driving pulley begins to move, the belt is stretched on the driving side and the tension increased, whilst the other side of the belt is shortened and the tension diminished. If the belt is perfectly elastic the lengthening of the tight and the shortening of the slack side must be equal in amount, and the average tension remains unaltered. That is,

$$\frac{T_2 + T_1}{2} = T_0 \qquad (9)$$

This process goes on till the force $T_2 - T_1$, tending to rotate

the driven pulley, is sufficient to overcome its resistance to motion, provided the limiting value of the ratio $T_2/T_1$ is not exceeded. Subject to this restriction the driven pulley rotates, and from that moment the condition of the belt is permanent.

*Effective tension or driving effort and power transmitted by a belt.*—Let P be the resistance at the circumference of the driven pulley. Then

$$P = T_2 - T_1 \qquad \qquad (10)$$

But if H is the number of horse-power transmitted, V the velocity of the circumference of the pulley or of the belt in feet per second,

$$P\,V = 550\,H \text{ and } P = \frac{550\,H}{V} \qquad \qquad (11)$$

If N = number of revolutions of pulley per minute and $d$ = diameter of pulley in inches, then $V = (\pi\, d\, N)/(12 \times 60)$, and consequently

$$P = 126{,}000\,\frac{H}{d\,N} \qquad \qquad (12)$$

*Tensions in a belt transmitting a given horse-power and on the point of slipping.*—Suppose that the value of P is obtained from the equations just given, and the value of $k$ from eq. 7, § 283, or the table corresponding to it. Then from equation (10)

$$\left.\begin{aligned} T_2 &= P\,\frac{k}{k-1} = x\,P \\ T_1 &= P\,\frac{1}{k-1} = y\,P \\ T_0 &= \tfrac{1}{2} P\,\frac{k+1}{k-1} = z\,P \end{aligned}\right\} \qquad \qquad (13)$$

The equations are the same if the tensions per sq. in. of belt section are substituted for the total tensions.

With an open belt the arc of contact on the smaller of two pulleys is less than 180°. With a crossed belt the arc of contact is greater than 180°. Hence for given pulleys and the same initial tension and coefficient of friction, a crossed belt will transmit more power than an open one. With an open belt the arc of contact on the smaller pulley is least and $\theta$ is to be taken for it. Even with a crossed belt, though $\theta$ is the same for the two pulleys, the belt slips on the smaller pulley.

286. *Theory of endless belt when the defective elasticity of leather is taken into account.*—It has already been stated (§ 279) that leather increases in stiffness as it is stretched. For such materials, if $f$ is the stress and $e$ the extension

$$f^n = c\, e$$

where $n$ and c are constants. Consider a belt on pulleys $l$ inches apart. The part of the belt on the pulleys may for the present purpose be neglected. Let $t_0$, $t_2$, $t_2$, be the initial tension and the tensions of the tight and slack sides, when running, per sq. in. of belt section, and $e_0$, $e_2$, $e_1$, the corresponding stretches per unit length. Then since the total initial stretch must be equal to the sum of the stretches when running,[1]

$$l\, e_2 + l\, e_1 = 2\, l\, e_0$$

$$e_2 + e_1 = 2\, e_0$$

$$t_2^{\,n} + t_1^{\,n} = 2\, t_0^{\,n}$$

Putting $n = 0{\cdot}5$, as in § 279,

$$\sqrt{t_2} + \sqrt{t_1} = 2\sqrt{t_0} \quad . \quad . \quad . \quad (14)$$

This equation is very approximately satisfied in the tests of Mr. Lewis. For example, the tensions being in pounds per sq. in.

| $t_0$ | $t_2$ | $t_1$ | $\sqrt{t_2} + \sqrt{t_1}$ | $2\sqrt{t_0}$ |
|---|---|---|---|---|
| 81·6 | 125 | 59 | 18·9 | 18·1 |
| ,, | 142 | 42 | 18·4 | ,, |
| ,, | 180 | 30 | 18·9 | ,, |
| 283 | 403 | 176 | 33·3 | 33·6 |
| ,, | 497 | 99 | 32·2 | ,, |
| 343 | 511 | 227 | 37·7 | 37·0 |
| ,, | 618 | 134 | 36·4 | ,, |

But $t_2 / t_1 = \epsilon^{\mu\theta} = k$, as to the use of which there is no practical doubt because $\mu$ is deduced from experimental observations of $t_2/t_1$ with actual belts. Inserting this value,

$$\sqrt{t_2} + \sqrt{(t_2/k)} = 2\sqrt{t_0}$$

$$t_2 = \frac{4\,k}{(\sqrt{k} + 1)^2}\, t_0 \quad . \quad . \quad . \quad . \quad (a)$$

Similarly,

$$t_1 = \frac{4}{(\sqrt{k} + 1)^2}\, t_0 \quad . \quad . \quad . \quad . \quad (b)$$

[1] This equation was given in *Machine Design* in 1882.

$$t_2 - t_1 = (k - 1)\, t_1 = \frac{4\,(k-1)}{(\sqrt{k}+1)^2}\, t_0$$

$$t_0 = \frac{(\sqrt{k}+1)^2}{4\,(k-1)}\,(t_2 - t_1) \quad . \quad . \quad . \quad (c)$$

Hence replacing $t_0$ in $(a)$ and $(b)$ by this value—

$$t_2 = \frac{k}{k-1}\,(t_2 - t_1) \quad . \quad . \quad . \quad . \quad (d)$$

$$t_1 = \frac{1}{k-1}\,(t_2 - t_1) \quad . \quad . \quad . \quad . \quad (e)$$

It can now be seen that equations $(d)$ and $(e)$ give identically the same values for $t_2$ and $t_1$ in terms of the effective driving effort $t_2 - t_1$ as equations (13), § 285, deduced from the ordinary theory. But the value of the initial tension $t_0$ for a given driving effort is different from that found by the ordinary theory.

Taking the values of $\mu$ and $k$ for an arc of contact of 165° as in § 284, the following is a comparison of the values of $t_0$ by the new and the ordinary theory.

| | | | | | | |
|---|---|---|---|---|---|---|
| $\mu =$ | ·15 | ·20 | ·25 | ·30 | ·40 | ·50 |
| $k =$ | 1·54 | 1·78 | 2·06 | 2·37 | 3·16 | 4·22 |
| | values of $t_0 / (t_2 - t_1)$ | | | | | |
| By eq. $(c)$ | 2·33 | 1·74 | 1·40 | 1·18 | 0·90 | 0·72 |
| By ordinary theory | 4·7 | 3·6 | 2·9 | 2·5 | 1·9 | 1·6 |

It is clear that for a given driving effort the initial tension necessary is less than has hitherto been assumed.

*Table to Facilitate the Calculation of the Belt Tensions*

| $\theta =$ | | | Values of $x$ for | | | | Values of $y$ for | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| In degrees | In circ. measure | In fractions of circumference | $\mu = 0·2$ | $\mu = 0·3$ | $\mu = 0·4$ | $\mu = 0·5$ | $\mu = 0·2$ | $\mu = 0·3$ | $\mu = 0·4$ | $\mu = 0·5$ |
| 30 | ·524 | ·083 | 10·09 | 6·89 | 5·29 | 4·35 | 9·09 | 5·88 | 4·29 | 3·34 |
| 45 | ·785 | ·125 | 6·89 | 4·76 | 3·71 | 3·08 | 5·88 | 3·76 | 2·71 | 2·08 |
| 60 | 1·047 | ·167 | 5·29 | 3·71 | 2·92 | 2·45 | 4·29 | 2·71 | 1·92 | 1·45 |
| 75 | 1·309 | ·208 | 4·35 | 3·08 | 2·45 | 2·08 | 3·34 | 2·08 | 1·45 | 1·08 |
| 90 | 1·571 | ·250 | 3·71 | 2·66 | 2·14 | 1·85 | 2·71 | 1·66 | 1·14 | ·840 |
| 105 | 1·833 | ·319 | 3·26 | 2·37 | 1·93 | 1·67 | 2·26 | 1·36 | ·924 | ·667 |
| 120 | 2·094 | ·334 | 2·92 | 2·14 | 1·77 | 1·54 | 1·92 | 1·14 | ·762 | ·541 |
| 135 | 2·356 | ·375 | 2·66 | 1·98 | 1·64 | 1·44 | 1·66 | ·984 | ·649 | ·444 |
| 150 | 2·618 | ·417 | 2·45 | 1·84 | 1·54 | 1·37 | 1·45 | ·840 | ·541 | ·370 |
| 165 | 2·880 | ·458 | 2·29 | 1·73 | 1·47 | 1·31 | 1·29 | ·730 | ·462 | ·311 |
| 180 | 3·142 | ·500 | 2·14 | 1·64 | 1·40 | 1·26 | 1·14 | ·638 | ·398 | ·262 |
| 195 | 3·403 | ·541 | 2·03 | 1·56 | 1·35 | 1·22 | 1·03 | ·563 | ·345 | ·223 |
| 210 | 3·665 | ·583 | 1·93 | 1·50 | 1·30 | 1·19 | ·926 | ·499 | ·300 | ·190 |
| 240 | 4·188 | ·666 | 1·76 | 1·40 | 1·23 | 1·14 | ·763 | ·398 | ·230 | ·140 |
| 270 | 4·718 | ·750 | 1·64 | 1·32 | 1·18 | 1·10 | ·639 | ·322 | ·179 | ·105 |
| 300 | 5·236 | ·833 | 1·54 | 1·26 | 1·14 | 1·08 | ·541 | ·262 | ·140 | ·079 |

## PRACTICAL CALCULATIONS OF BELTING

287. *Economical limits in belt transmission.* — Since the coefficient of friction $\mu$ varies with the creep of the belt on the pulley, there is no *definite* limit of $T_2 / T_1$ at which the belt ceases to drive, except for velocities of slip greater than could be tolerated in practice. Consider a belt driving against a practically constant resistance. It is initially tightened to a tension greater than is necessary to ensure driving. In running the belt gradually stretches, and the tensions $T_2$ and $T_1$ diminish while the driving effort $T_2 - T_1$ is constant. Hence the ratio $T_2 / T_1$ increases and at last reaches the limiting value at which an inconvenient amount of slipping occurs, and the belt must be tightened. This involves interruption of service more or less serious according to the kind of work being done. The initial tension and the driving effort should be so proportioned that frequent tightening is unnecessary. The design of belting depends very little on its breaking strength, although in any case the greatest tension $t_2$ should not exceed one-fourth of the strength at the joint; but it depends mainly on the conditions which secure durability and economy and freedom from trouble in working, and these are dependent chiefly on the tensions being moderate. Excessive belt tensions are likely to give rise to trouble with the shaft bearings.

*Mr. Lewis's tests of belt driving.*—Very valuable tests of belts were made by Mr. W. Lewis for Messrs. Sellers & Co. on a specially constructed machine ('Trans. Am. Soc. Mech. Eng.' vii. 549). These showed that the coefficient of friction $\mu$ calculated from the measured difference of tensions $T_2 - T_1$ by the formula above, increased with the velocity of slip from 0·25 to 1·0 and more. They showed that for a belt with a given initial tension the sum of the tensions $T_2 + T_1$ did not remain constant as assumed in the ordinary theory, but increased as the effective effort $T_2 - T_1$ increased. This point will be dealt with later. The efficiency of a single belt drive was about 97 per cent.

As a result of the tests Mr. Lewis judged that the coefficient $\mu$ might be taken at 0·4 in ordinary driving and the maximum tension $T_2$ at 275 lbs. for a laced, and at 400 lbs. for a riveted belt per square inch. On the ordinary theory this gives $T_2 / T_1 = 3$; $T_1 = 90$ or $133$; $T_2 - T_1 = 185$ or $267$ lbs. per sq. in.

*Mr. Taylor's investigation.*—Mr. F. W. Taylor made an important investigation of the durability of belts in a workshop extending over nine years ('Trans. Am. Soc. Mech. Eng.' xv. 204). The belts were all thin double belts and in two classes, shifting belts and cone belts, the latter more heavily loaded than the former. In tightening the belts, belt clamps were used, having spring balances between the clamps, so that the exact amount of initial tension could be weighed. The belts were all tightened to an initial tension $T_0 = 71$ lbs. per inch of width, or 192 lbs. per sq. in. of section. The final tension when they required tightening was again weighed. The belts were of four qualities, but all gave similar results. Those on oak-tanned belts may be taken as an example. After nine years the shifting belts were in good condition, but the cone belts were wholly or nearly worn out. The mean speed was about 1,140 ft. per min. for the shifting and 580 ft. per min. for the cone belts. The following figures are averages.

In the following table the tensions are given in pounds per sq. in., and $t_2$ and $t_1$ have been calculated from the observed $t_0$, and estimated value of the driving effort $t_2 - t_1$ by the ordinary theory :—

| | Shifting Belts | Cone Belts |
|---|---|---|
| Initial tension $t_0$ | 192 | 192 |
| Final tension before tightening $t'_0$ | 61 | 102 |
| Estimated effort $t_2 - t_1$ | 70 | 175 |
| Number of times tightened per year | 0·6 | 4·0 |
| $t_2$ after tightening | 227 | 280 |
| $t_1$ ,, ,, | 157 | 105 |
| $t_2$ before retightening | 96 | 190 |
| $t_1$ ,, ,, | 26 | 15 |
| $t_2/t_1$ after tightening | 1·45 | 2·67 |
| $t_2/t_1$ before retightening | 4·70 | 12·7 |

Mr. Taylor concludes that for the greatest economy in the use of belts the effective effort $t_2 - t_1$ should not exceed 35 lbs. per inch width of thin double belt, or, say, 109 lbs. per sq. in. of belt section. The initial tension to which belts should be tightened should not exceed 71 lbs. per inch width of double belting or, say, 240 lbs. per sq. in. of section. This corresponds on the ordinary theory to $t_2 = 290$ lbs. per sq. in.; $t_1 = 190$; or to $t_2 = 87$, $t_1 = 57$ lbs. per inch width of single belt. Also $t_2/t_1 = 1·5$.

288. *Table of average working tensions and driving efforts.*—The fundamental datum on which calculations of belts should depend is the driving effort which in practice it is found they will exert satisfactorily in given conditions. The following table gives calculations for driving efforts ranging from Mr. Taylor's value of 30 lbs. per inch width for double belting up to the higher values which are found in actual practice. All that can be said as to the choice to be made is that the lower the driving effort the greater will be the life of the belt and the less trouble it will give. Belts with cemented joints will in any case carry satisfactorily 25 per cent. more load than belts with laced joints.

The table assumes an arc of contact of 180°; and a coefficient of friction $\mu = 0{\cdot}3$, known to be a moderate value for leather on cast-iron pulleys. The tensions are calculated both for pounds per sq. in. and pounds per inch width of belt. If $P = t_2 - t_1$ is the driving effort, then from the table § 285, $t_2 = 1{\cdot}64\ P$; $t_1 = 0{\cdot}638\ P$; $t_2/t_1 = 2{\cdot}57$. More exact calculations may be made by using the values of $\mu$, and other data given above.

*Working Tensions in Belting in Pounds*

| | Tensions in lbs. per sq. in. of section | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Driving effort $P$ | 75 | 100 | 125 | 150 | 175 | 200 | 225 | 250 |
| Greatest tension $t_2$ | 123 | 164 | 205 | 246 | 287 | 328 | 369 | 410 |
| Slack tension $t_1$ | 48 | 64 | 80 | 96 | 112 | 128 | 144 | 160 |
| | **Tensions in lbs. per inch width of belt** | | | | | | | |
| *Single Belting* 0·18 *in. thick* | | | | | | | | |
| Driving effort $p$ | 14 | 18 | 23 | 27 | 32 | 36 | 41 | 45 |
| Greatest tension $t_2$ | 23 | 30 | 38 | 44 | 52 | 59 | 67 | 74 |
| Slack tension $t_1$ | 9 | 11 | 15 | 17 | 20 | 23 | 26 | 29 |
| *Double Belting* 0·36 *in. thick* | | | | | | | | |
| Driving effort $p$ | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| Greatest tension $t_2$ | 44 | 59 | 74 | 89 | 103 | 118 | 133 | 148 |
| Slack tension $t_1$ | 17 | 23 | 29 | 34 | 40 | 46 | 52 | 58 |
| *Treble Belting* 0·54 *in. thick* | | | | | | | | |
| Driving effort $p$ | 41 | 54 | 68 | 81 | 95 | 108 | 122 | 135 |
| Greatest tension $t_2$ | 67 | 89 | 112 | 133 | 156 | 177 | 200 | 221 |
| Slack tension $t_1$ | 26 | 34 | 43 | 52 | 61 | 69 | 78 | 86 |

In no case are the tensions in this table beyond the strength of good belting. In selecting the working stress it is desirable to take it smaller, if the diameter of the pulleys is small or

the driving effort is irregular. It may be taken somewhat greater as the speed of the belt is greater.

*Correction for arc of contact.*—In open belts the arc of contact on the smaller pulley will be rather less than 180°. With crossed belts it may be somewhat greater. If the same values of P are assumed, the values of $t_2$ and $t_1$ in the table must be multiplied by the following factors for the arcs of contact stated.

| | | | | | | |
|---|---|---|---|---|---|---|
| Arc of contact . . | 135° | 150° | 165° | 180° | 195° | 200° |
| Multiplier for $t_2$ . | 1·21 | 1·12 | 1·06 | 1·00 | ·95 | ·92 |
| ,, ,, $t_1$ . | 1·54 | 1·32 | 1·14 | 1·00 | ·88 | ·78 |

In a few cases, for the smaller arcs of contact and larger driving efforts the maximum stress $t_2$ may be larger than is advisable for belts with laced joints.

289. *Horse-power transmitted per inch width of belt.*—A convenient constant for use in designing belting is the horse-power per inch width of belt. Let $p$ be the driving effort reckoned per inch width of belt in lbs. and V the belt speed in feet per min. Then the horse-power transmitted per inch width of belt is

$$H = (p \text{ V})/33{,}000.$$

Taking the values of $p$ in the preceding table as examples:

*Single Belt*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $p$ = | 14 | 18 | 23 | 27 | 32 | 36 | 41 | 45 |
| H/V = | ·00042 | ·00055 | ·00070 | ·00082 | ·00097 | ·00110 | ·00125 | ·00137 |

*Double Belt*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $p$ = | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| H/V = | ·00082 | ·00110 | ·00137 | ·00164 | ·00191 | ·00219 | ·00246 | ·00273 |

*Lineal feet of belt V per min. one-inch wide per horse-power.*—For one horse-power, per inch width of belt,

$$V = 33{,}000/p \text{ ft. per min.}$$

Using the same values of $p$:—

*Single belt*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $p$ = | 14 | 18 | 23 | 27 | 32 | 36 | 41 | 45 |
| V = | 2360 | 1840 | 1430 | 1230 | 1040 | 917 | 805 | 734 |

*Double belt*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $p$ = | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| V = | 1230 | 917 | 734 | 612 | 524 | 459 | 408 | 368 |

*Square feet of belt per minute per horse-power transmitted.*—Another convenient constant in belt calculations is the number $n$ of square feet of belt which must pass over the pulleys per minute to transmit one horse-power. If $w$ is the width of belt in ft.,

$$n = w \text{ V} = 2750/p \text{ sq. ft. per min.}$$

*Single Belting*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ = 14 | 18 | 23 | 27 | 32 | 36 | 41 | 45 |
| $n$ = 196 | 153 | 120 | 102 | 86 | 76 | 67 | 61 |

*Double belting*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ = 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| $n$ = 102 | 76 | 61 | 51 | 44 | 38 | 34 | 31 |

290. *High-speed Belting.*—Part of the tension in a running belt is used in deviating it over the curved surface of the pulleys, and, apart from any stiffness in the belt, the normal pressure of the belt on the pulley, and consequently the frictional resistance to slipping, are less than they would be if there were no centrifugal action. The effect of centrifugal force may be neglected when the belt speed does not exceed 2,000 feet per minute; but it increases as the square of the velocity of the belt, and becomes important at higher speeds. In dynamo driving, belts are used at speeds of 6,000 to 7,500 feet per minute, and their driving effort for a given initial belt tension is much reduced by the centrifugal force. At some limiting speed, which may be 9,000 or 10,000 feet per minute for leather belts, the centrifugal force destroys the whole normal pressure of the belt on the pulley and the belt ceases to drive.

It is to some extent a practice to limit the belt speed on small pulleys. The greatest speed V is about $750 + 150\,d$ ft. per min., where $d$ is the pulley diameter in inches. The rule is purely empirical.

*Centrifugal tension.*—The weight of leather belting is about $w = 0{\cdot}43$ lbs. per sq. in. per foot length. The centrifugal force of one foot length of belt one inch square deviated over a pulley of $r$ feet radius at $v$ feet per second is $w\,v^2/g\,r$. The resultant centrifugal force of so much belt as covers half the pulley circumference is $2\,w\,v^2/g$. This produces in each of the straight segments of the belt a tension

$$t_c = w\,v^2/g = 0{\cdot}014\,v^2 \text{ lbs. per sq. in.} \qquad (15)$$

*Centrifugal tension, lbs. per sq. in.*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $v$ = 10 | 15 | 25 | 50 | 75 | 100 | 125 | ft. per sec. |
| V = 600 | 900 | 1500 | 3000 | 4500 | 6000 | 7500 | ft. per min. |
| $t_c$ = 1·4 | 3·1 | 8·8 | 35 | 79 | 140 | 218 | lbs. per sq. in. |

The centrifugal tension balances part of the tension producing friction on the pulleys. Hence the effective tensions preventing slip are $t_2 - t_c$ and $t_1 - t_c$ in lbs. per sq. in.

But if $k$ is the driving ratio of the tensions calculated from $\mu$

$$k = \frac{t_2 - t_c}{t_1 - t_c}$$

Let P be the driving effort $t_2 - t_1$ in lbs. per sq. in. of section,

$$P = t_2 - t_1 = t_2 - \frac{t_2}{k} - t_c\left(1 - \frac{1}{k}\right)$$

$$= (t_2 - t_c)\left(1 - \frac{1}{k}\right) \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$t_2 = \frac{k}{k-1}P + t_c \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Comparing this with the result in § 285, for a given value of $k$ $t_2$ is increased by the centrifugal tension. If, therefore, the maximum tension in the belt is limited by considerations of strength the centrifugal tension reduces the permissible driving effort. If the speed is such that $t_c = t_2$ no driving effort can be transmitted with the assigned limit of stress. For instance, if $t_2 = 350$ lbs. per sq. in. The driving effort is zero for a speed of 158 feet per second Taking the values of $t_2$ in the table in § 288 as limiting values of stress, the driving efforts P would be reduced as follows by the effect of centrifugal force

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Greatest tension $t_2$ | 123 | 164 | 205 | 246 | 287 | 328 | 369 |
| | Driving effort P in lbs. per sq. in. | | | | | | |
| At $v =$ 50 ft. per sec. | 54 | 79 | 103 | 129 | 154 | 179 | 204 |
| 100 " " | | 15 | 40 | 65 | 90 | 115 | 140 |

Comparing this with the table § 288, it will be seen how much the driving effort is reduced by centrifugal force at high speeds when the maximum stress in the belt is limited.

*Belt speed at which the greatest power is transmitted with a given maximum belt tension.*—The power transmitted by a belt is in lbs. per sq. in.

$$P\,v = (t_2 - 0{\cdot}014\,v^2)\left(1 - \frac{1}{k}\right) v \text{ ft. lbs per sec.}$$

Differentiating and equating to zero, the work is a maximum when

$$t_2 - 0{\cdot}042\,v^2 = 0$$

$$v = 4{\cdot}9\sqrt{t_2} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

| | | | | | | |
|---|---|---|---|---|---|---|
| $t_2 =$ | 400 | 300 | 200 | 100 | 50 | lbs. per sq. in. |
| $v =$ | 98 | 84 | 69 | 49 | 35 | ft. per sec. |
| V = | 5880 | 5040 | 4140 | 2940 | 2100 | ft. per min. |

291. *Single, double, and combined belting. Joints in belting.*—The leather used for belting is of ox-hide tanned with oak bark, and only the best part of the hide, termed the butt, is used. The butts are cut into strips of the width required, and joined together to form a belt of any required length. The joints are

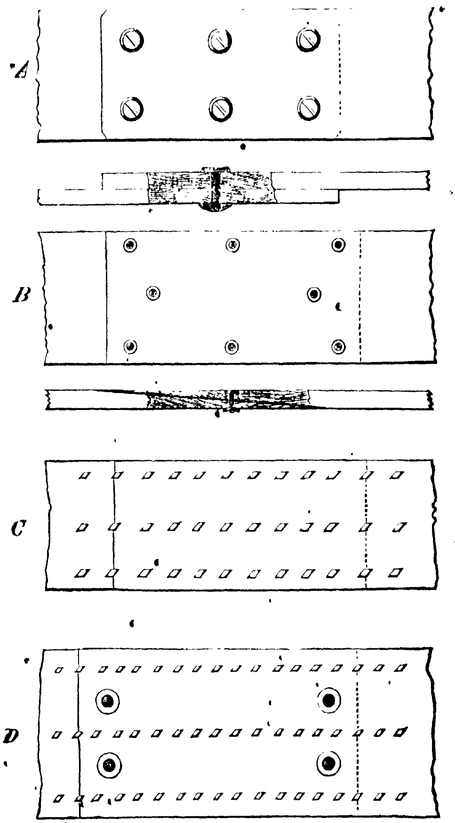


Fig. 334

made by paring down the ends of the strip, overlapping them, and cementing them with glue. They are then either sewn, laced, or riveted as an additional precaution. Fig. 334 *C* shows a cemented and laced joint; the overlap is about 7 inches long,

and the laces $1\frac{1}{2}$ inch apart, extending an inch beyond the overlap at each end. Sometimes a few rivets are used in addition to the lacing. Fig. 334 *B* shows a cemented and riveted joint, the overlap 6 to 7 inches long, and having about one rivet to $2\frac{1}{4}$ or $2\frac{1}{2}$ sq. ins. of overlap. Fig. 334 *D* shows a laced and riveted joint.

In an endless belt one joint must generally be uncemented, so that it can be easily broken when the belt requires to be tightened. This joint may be a laced joint, like that previously described, or it may be made with belt screws shown in fig. 334 *A*. These belt screws are of iron with a very flat nut. The length of overlap may be 6 ins., and there may be one screw to 6 or 8 sq. ins. of overlap. This joint is more clumsy than a laced joint, but is very easily broken or made. The laces commonly used are strips of white leather tanned with alum.

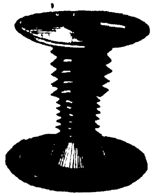


Fig. 335

A very convenient belt screw with a right- and left-hand screw thread has been introduced and is shown in fig. 335. The screws are made of steel or gun-metal, and are less likely to work loose than ordinary screws.

Another convenient belt fastening intended to replace laces

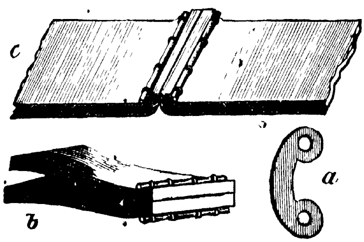



Fig. 336

is that shown in fig. 336. The fastener is shown at *a*, the belt in process of fastening at *b*, and the belt in running condition at *c*.

When a single belt would be of inconvenient width, a double belt is used. This is made by cementing two strips of leather together, and then sewing them or riveting them. There may be about one rivet to 3 to 4 sq. ins. of belt. The double belt is more rigid than a single belt, and does not work satisfactorily

*unless there is ample distance between the pulleys, and the pulleys are of sufficient diameter.*

When a very broad belt is required to connect two shafts which are not parallel (that is, when the belt has a half or quarter twist), it does not work well, because its rigidity prevents its lying down in contact with the pulleys. It comes in contact with the pulleys on one side only. Messrs. Tullis, of Glasgow, have in such cases employed several narrow belts instead of a single wide one. These run side by side on the same pulleys, and are kept parallel by cross strips of leather riveted to them. Thus, for instance, instead of a 12-inch belt, three four-inch belts may be used, connected by cross strips $1\frac{1}{2}$ inch wide, at intervals of about 12 inches. A combined belt of this kind runs parallel and fits the pulleys better than a very wide belt.

Leather has a smooth or grain side and a rough or flesh side. Usually belts are run with the flesh side next the pulley, and care is taken that in twisted belts the same side of the leather is kept next all the pulleys. In America it seems to have been found that the driving power of the belt is greater with the grain side next the pulley. Castor oil applied to the grain side of a belt makes it more supple. Dubbin and boiled linseed oil are sometimes applied to the flesh side.

292. *Cotton belting* can now be obtained, made of 4 to 10 thicknesses of American cotton duck stitched together. It is waterproof, and cheaper and stronger than leather. The ordinary widths are, for 4-ply, $1\frac{1}{2}$ to 6 ins.; for 6-ply, 3 to 12 ins.; for 8-ply, 6 to 30 ins.; and for 10-ply, 12 to 60 ins. According to a test made for the manufacturers, 8-ply cotton belting is twice as strong as double leather belting, the breaking stress being 1,135 lbs. per inch of width. The best way of making the joints is by butting the ends of the belts and using a special metal fastening. A test of this gave a breaking strength of 330 to 540 lbs. per inch of width (thickness of belt not stated). Ordinarily 4-ply is taken as equivalent to single-leather, and 8-ply to double-leather belting.

293. *Special forms of belting.—Chain belting or link belting* is made of a series of short links of leather strung together on wire pins (fig. 337). It is more flexible than ordinary belting, can be made of great width, and is very easily shortened by taking out a set of links and rejoining with a wire pin. Messrs. Tullis make this belt with an arched section to suit the curve

of the pulley. It then has an even bearing, and the wire pins are not bent. It is said to transmit 25 per cent. more power than an ordinary flat belt of the same width. Chain belting is often made of tapered section for half-twist driving. Chain-belts have been made up to 33 ins wide.

Fig. 338 shows a peculiar leather belt introduced by Messrs. Tullis, of Glasgow, and intended to work on pulleys having

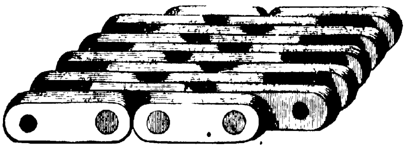


Fig. 337

V-shaped grooves round their circumference. When the grooves have sides inclined at 45°, the adhesion of the belt to the pulley is increased about 2·8 times, so that the grooved pulley is equivalent to a cylindrical pulley with a coefficient of friction, $\mu = 0{\cdot}8$ to $1{\cdot}0$. The V-shaped belt shown in fig. 338 has been used for some years in America. It is made of slices of leather riveted

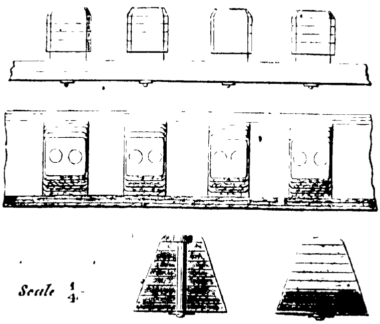



Fig. 338

together. The continuous part of the belt consists of three strips about ⅜ths of an inch in total thickness, and 2 ins. in average width Hence the belt section is about 1¼ sq. in. Several of these belts may be used side by side, precisely in the same way as the rope belts which are described in the next chapter. Messrs. Tullis state that the driving power of the belt is considerably greater than that of an ordinary rope belt.

294. *Belt-tightening pulley.*—When the distance between the pulleys is not great, and the belt is a short one, it loses its tension rapidly and requires frequent taking up. To obviate this a pulley pressed against the belt by a spring or weight carried on a lever may be used (fig. 339). This pulley should be on the slack side of the belt, and nearer the driving than the driven pulley.

295. *Belts connecting shafts which are not parallel.*—When two shafts are not parallel and do not intersect, they may still be connected by an endless belt, provided the pulleys are properly placed. The single and sufficient condition that the belt may run properly is this: The point at which the belt is delivered from each pulley must be in the plane of the other pulley. This condition can only be fulfilled for a belt which always runs in one direction.

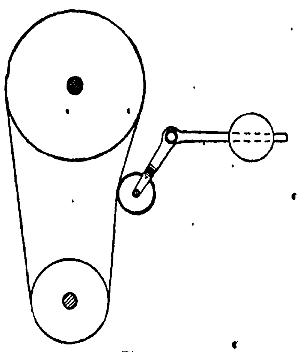


Fig. 339

Fig. 340 shows three views of this arrangement of belting applied to two shafts at right angles or quarter twist. The arrows show the direction of the motion of the belt. If this be followed, it will be found that the point at which the belt runs off each pulley is in the plane passing through the centre of the other pulley. The belt would in this case be said to have a quarter twist.

*Guide pulleys.*—When two shafts are not parallel, and whether their directions intersect or not, they may be connected by a single endless belt if intermediate guide pulleys are used. These guide pulleys alter the direction of the belt without modifying the velocity ratio of the shafts. Fig. 341 shows an elevation and plan of an arrangement of pulleys and guide pulleys: *a b* is the intersection of the middle planes of the principal pulleys. Select any two points *a* and *b* on this line and draw tangents, *a c*, *b d*, to the principal pulleys. Then *c a c* and *d b d* are suitable directions for the belt. The guide pulleys must be placed with their middle planes coinciding with the planes *c a c* and *d b d*. The belt will run in either direction.

Guide pulleys are sometimes used merely to lengthen the belt

between two shafts, which are too close together to be connected direct. Fig. 342 shows an arrangement of this kind. The middle planes of the guide pulleys are determined by the method just mentioned. It is, however, possible to place the guide pulleys with their axes parallel. Then the belt must be delivered from each pulley in the plane of the pulley on to which it is running. When this is provided for, it will be found that the belt will only run in one direction.

Figs. 343 and 344 show two arrangements of belting and

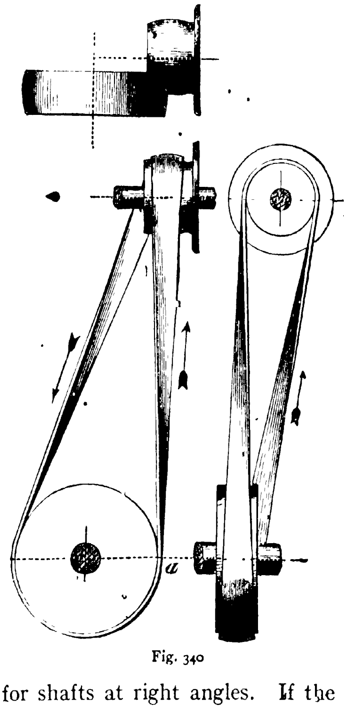



Fig. 340

guide pulleys for shafts at right angles. If the belts be traced round, it will be found that the rough side of the belt is always next the pulleys. It is to secure this that the belts have a quarter or half twist between the pulleys as shown.

296. *Rounding of pulley rim.*—When a flat belt is placed on a conical pulley, it tends to climb toward the larger end. If the pulley is made of a double conical form, or, still better, with

a rounded rim a little larger at the centre than at the sides, the flat belt keeps its place on the pulley and has no tendency to

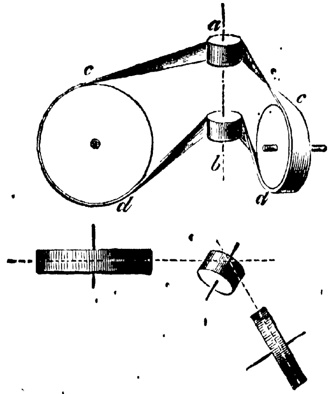



Fig. 341

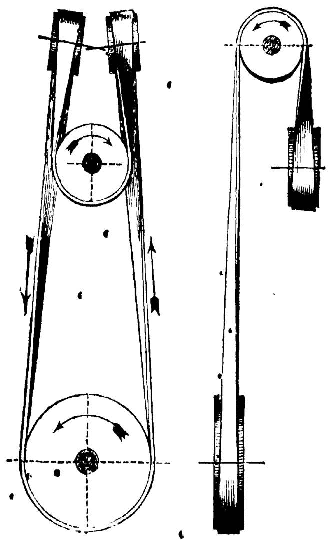


Fig. 342

slip off. The rounding of the rim may be $\frac{1}{8}$ in. to $\frac{1}{4}$ in. for pulleys up to 12 ins. broad, and $\frac{1}{8}$ in. to $\frac{1}{4}$ in. per foot of width for wider pulleys.

297. *Proportions of pulley. Rim of pulley.*—The pulley rim is a little wider than the belt it is intended to carry. Let $B$ = width of rim, $\beta$ = width of belt. Then,

$$B = \frac{9}{8}(\beta + 0 \cdot 4)$$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\beta =$ | 2 | 3 | 4 | 5 | 6 | 8 | 10 | 12 |
| $B =$ | 2·7 | 3·82 | 4·95 | 6·08 | 7·2 | 9·45 | 10·7 | 13·95 |
| $=$ | $2\frac{3}{4}$ | $3\frac{7}{8}$ | 5 | 6 | $7\frac{1}{4}$ | $9\frac{1}{2}$ | $11\frac{3}{4}$ | 14 |

The form of the rim in section is shown in fig. 345; at the edge the thickness may be

$$t = 0 \cdot 7\,\delta + \cdot 005\,D$$

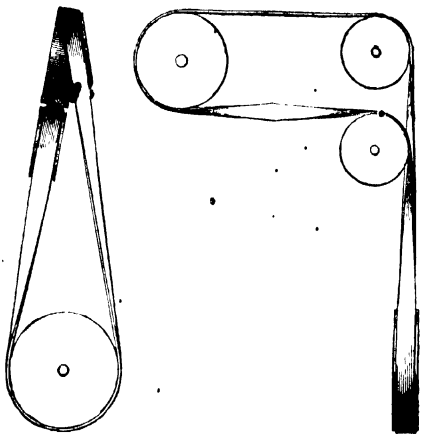


Fig. 343

where $D$ is the diameter of the pulley and $\delta$ the thickness of the belt.

The rim of the pulley is rounded to a radius 6 $B$ to 12 $B$. In very wide pulleys the middle part is left cylindrical. The diameter of the smaller of two pulleys should not be less than 24 times the belt thickness in any case, and generally not less than 35 times the belt thickness.

*Centrifugal tension in rim of pulley.*—Pulleys run at high speeds are liable to burst from the tension in the rim. Let $w$ = weight of a bar 1 sq. inch in section and 12 ins. long ($w$ = 3·36 lbs.). Then the weight of one foot length of a pulley

rim of section $\omega$ sq. ins. is $w\,\omega$ lbs. If $v$ = velocity in feet per second, and $r$ = radius in feet of pulley rim, then the centrifugal force of one foot length of rim $= \dfrac{w\,\omega\,v^2}{g\,r}$. Suppose the pulley divided by a diametral plane. Then the resultant centrifugal force of each half of the pulley rim, acting normally to the dividing plane, is

$$\frac{w\,\omega\,v^2}{g\,r} \times 2\,r = \frac{2\,w\,\omega\,v^2}{g}.$$

This force is balanced by the tensions on the two sections of

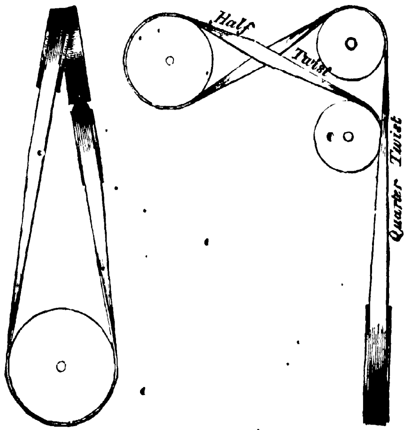



Fig. 344

the rim by the diametral plane. Consequently the whole tension in the rim is $\dfrac{w\,\omega\,v^2}{g}$, and the intensity of the stress is $\dfrac{w\,v^2}{g}$ lbs. per sq. in.

| | | | | | | |
|---|---|---|---|---|---|---|
| $v$ in ft. per sec. | = | 70 | 80 | 100 | 150 | 200 |
| Centrifugal tension in lbs. per sq. in. | = | 511 | 668 | 1043 | 2349 | 4175 |

The stresses due to the pull of the belt and those due to contraction in casting are additional to these stresses. Hence in practice the speed of pulley rims should not exceed 80 to 100 feet per second.

*Arms of the pulley.*—The arms of pulleys are of elliptical or segmental section, as shown in fig. 346. The latter form of section looks lighter than the elliptical section and is preferable. For a segmental arm the thickness $h_2 = \frac{1}{2}h_1$. For an elliptical arm the thickness $h_2 = 0{\cdot}4h_1$. The arms are either straight or curved. The curved arms are rather less liable to

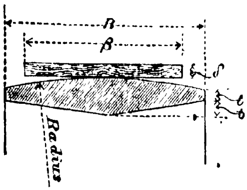



Fig. 345

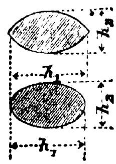


Fig. 346

fracture from contraction in cooling, but in all other respects the straight arms are preferable, being lighter and stronger. The section of the arms is diminished from the nave to the rim, so that if we put $h_1$ $h_2$ for the breadth and thickness of the arm, supposed produced to the centre of the shaft, the breadth and thickness at the rim will be $\frac{2}{3}\,h_1$ and $\frac{2}{3}\,h_2$, or $\frac{3}{4}\,h_1$ and $\frac{3}{4}\,h_2$.

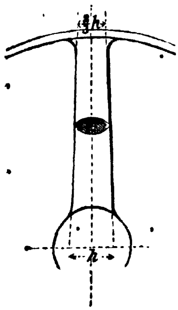


Fig. 347

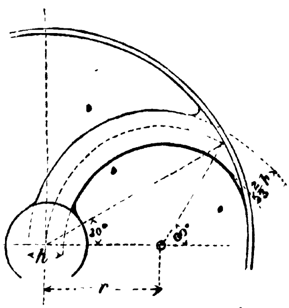


Fig. 348

Fig. 347 shows an ordinary straight arm, fig. 348 a curved arm, and fig. 349 an S-shaped or doubly curved arm. The figures indicate sufficiently the way in which the centre line of the arm is drawn. Let R be the radius of the pulley measured to the inside of the rim. Then in fig. 348, $r = 0{\cdot}577$ R; and in fig. 349, $r_1 = 0{\cdot}471$ R, and $r_2 = 0{\cdot}236$ R.

Let $\nu$ be the number of arms, B the breadth, and D the diameter of the rim. Then,

$$\nu = 3 + \frac{\text{B D}}{150}$$

the nearest whole number being taken.

| Width of Pulley B | Diameter of pulley in inches when the number of arms is | | | | |
|---|---|---|---|---|---|
| | 4 | 5 | 6 | 8 | 10 |
| 3 | 50 | 100 | 150 | .. | .. |
| 6 | 25 | 50 | 75 | 125 | 175 |
| 12 | 12 | 24 | 36 | 62 | 87 |
| 18 | 8 | 16 | 24 | 42 | 58 |
| 24 | 6 | 12 | 18 | 31 | 44 |

The number of arms is really arbitrary, and may be altered if necessary. In calculating the strength of the arms it will

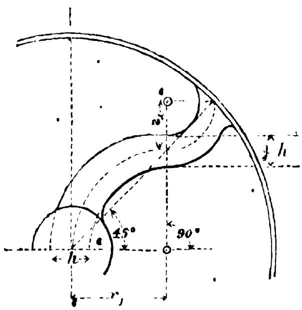



Fig. 349

be assumed that each arm is equally loaded, and also that each arm may be considered to be fixed at the nave and free at the rim. As these assumptions are only in a rough sense true, a large factor of safety must be allowed. Pulley-arms are also liable to be considerably strained by contraction in cooling. Hence a margin of strength must be allowed to meet this contingency. For these reasons the working stress on the cast-iron will be taken at $f = 2250$ lbs. per sq. in.

If P is the driving force transmitted by the belt, determined by eq. 12, § 285, and D is the diameter of the pulley, the greatest bending moment on each arm is—

$$M = \frac{1}{2}\frac{P\,D}{\nu}$$

For an elliptical section of width $h$ (measured at the centre of the pulley) and thickness $0{\cdot}4\ h$, the section modulus (Table IV, p. 70) is

$$\frac{\pi}{32} \times h^2 \times 0{\cdot}4\,h = 0{\cdot}0393\,h^3 \text{ nearly,}$$

and for a segmental section of width $h$ and thickness $0{\cdot}5\ h$, the modulus may be taken to be the same. Equating the bending moment and moment of resistance

$$\frac{1}{2}\frac{P\,D}{\nu} = 0{\cdot}0393\,f\,h^3$$

$$h = \sqrt[3]{\left(\frac{1}{{\cdot}0786\,f} \cdot \frac{P\,D}{\nu}\right)},$$

and putting $f = 2250$

$$h = 0{\cdot}1781\,\sqrt[3]{\frac{P\,D}{\nu}}.$$

Since in designing pulleys the driving force P will often be unknown, we may design the arms to resist the maximum driving force which is likely to be transmitted by a belt, the width of which is $\frac{4}{5}$ B. The driving force will be very often half the greatest tension in the belt, and will rarely exceed $\frac{4}{5}$th that tension, except when the belt embraces an unusually large arc. The greatest belt tension may be taken at 70 lbs. per inch width of the belt for single belting, and 140 lbs. for double belting. Hence, P will not exceed 56 and 112 lbs. per inch width of the belt, or 45 and 90 lbs. per inch width of the pulley.

$$P = 45\ B \text{ for single belts.}$$
$$= 90\ B \text{ for double belts.}$$

Inserting this value in the equations above,

$$h = 0{\cdot}6337\,\sqrt[3]{\frac{B\,D}{\nu}} \text{ for single belts.}$$

$$= 0{\cdot}798\,\sqrt[3]{\frac{B\,D}{\nu}} \text{ for double belts.}$$

These equations agree well with practice. If the arms are of wrought-iron $f$ may be taken equal to 9,000 lbs. per sq. in. If the section of the arms is different, the proper section modulus must be substituted for that assumed above.

*The nave of the pulley.*—The thickness of the nave may be

$$\delta = 0{\cdot}14 \sqrt[3]{\mathrm{B\,D}} + \tfrac{1}{4} \text{ (single belt)}$$
$$= 0{\cdot}18 \sqrt[3]{\mathrm{B\,D}} + \tfrac{1}{4} \text{ (double belt)}.$$

The length of the nave, $\lambda$, should not be less than $2\frac{1}{2}\,\delta$, and is often $\frac{2}{3}$ B. The key is to be proportioned by the rules in § 135. When the pulley is to run loose on the shaft the nave should be bushed with brass, and the length of the nave should be equal to B. Provision must also be made for lubrication. In large pulleys the nave may be strengthened by wrought-iron rings shrunk on.

298. *Split pulleys.*—When the pulleys are intended to be fixed on shafts which are bossed at the ends, they are often cast

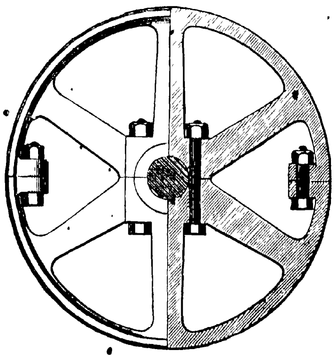


Fig. 350

in halves. The two halves can then be bolted together on the shaft without dismounting the shaft and without having recourse to cone keys. Fig. 350 shows a pulley of this kind. The net section of the bolt at the rim should be about a quarter the section of the rim, plus $\frac{1}{4}$ sq. in. and that of bolt at the nave about $\frac{1}{4}$ sq. in. plus a quarter the section of the nave calculated as above. The two half-pulleys may be made to grasp the shaft so tightly that relative motion is prevented by friction, and no keys are necessary.

If it is undesirable to cast the pulley in halves, the eye of the pulley must be bored out large enough to pass over the bosses at the ends of the shaft and slightly conical. Then three cone keys, described in § 133, are fitted in the space between

the pulley-eye and the shaft. Another plan is to use a conical sleeve, split on one side, like that shown in the drawing of the Sellers's coupling, fig. 191; this is drawn into the eye of the pulley by bolts. In either of these plans the pulley is fixed on the shaft by friction only.

Wrought-iron pulleys of ordinary size and of exceptionally large dimensions are made, and are preferable in many cases to cast iron. They are safer at high speeds because they are entirely free from strains due to contraction in cooling, and because, if they should break, their toughness would prevent them from flying to pieces.

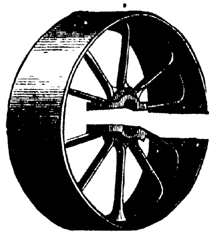


Fig. 351

Wrought-iron pulleys are generally made as shown in fig. 351. They are then virtually split pulleys. The split edges of the rim are joined by a lapping piece and screws.

*Weight of pulleys in lbs. per inch of width.*—The diameter being $d$ in ft., Mr. D. K. Clarke gives

$$\text{w} = 7{\cdot}6\,d - 1{\cdot}5 \text{ to } 12\,d - 9{\cdot}5 \text{ for rough castings}$$
$$= 7\,d - 1{\cdot}75 \text{ to } 11{\cdot}6\,d - 9{\cdot}25 \text{ for finished pulleys.}$$

299. *Management of belting.*—In fixing, repairing, or splicing a belt, it must be thrown off the pulley, and it then rests chiefly on the upper of the two connected shafts. If the shaft on which it rests is the driven shaft, no great danger is incurred; but if the belt rests on the driving shaft there is danger of the belt getting entwined round the shaft and so causing injury to the machinery and perhaps to the workman.[1] The danger is greater the more flexible the belt, and depends to some extent on the direction of motion of the shaft. The danger is greatest when the lower side is the tight side, and the slack side is liable to rest on it, as in fig. 352. The shaft may then grip the belt and roll up the two sides together. In fig. 353 the arrow is placed on the driving pulley. A and C are comparatively safe, B and D dangerous arrangements.

Fig. 352

*Belt perch.*—The simplest way of preventing the entwining

[1] Thwaite, *Factories and Workshops.*

of the belt is to fix a light belt perch over the shaft on which the belt rests when unshipped. Fig. 354 shows a simple perch

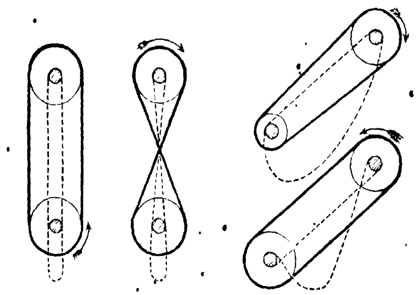


Fig. 353

of this kind. Where a pulley is placed close to a hanger it is desirable to fix a light guard to prevent the belt falling between

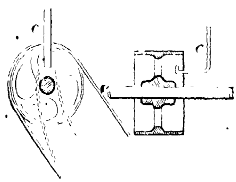


Fig. 354

the pulley and hanger, and it should be placed near the point where the belt advances towards the pulley in running.

# CHAPTER XV

## ROPE GEARING

300. Ordinary ropes of manila, Italian hemp or cotton may be used instead of leather belting when much power is transmitted. The pulleys for belts of this kind are made with circumferential V grooves, having an angle of 40° to 50°, so proportioned that the rope rests against the sides, not the bottom of the groove. The rope wedged in the groove has greater resistance to slipping than it would have resting on the bottom of the groove. The ropes most commonly used are patent ropes of three strands (fig. 355), white or untarred, and from 1 to 2 inches

Fig. 355

in diameter, or 3 to 6 inches in girth. They are placed on the pulleys with little initial tension. The joint in the rope is made by a splice which should be from 70 to 100 diameters of the rope in length. The pressure of the rope in the pulley groove is largely due to its weight. Hence to secure sufficient resistance to slipping the pulleys should be large and at a sufficient horizontal distance apart. If one pulley is nearly vertically over the other, the rope must be tightly strained and its durability is impaired. Usually the horizontal distance between the pulleys is 40 feet at least, and the rope works well on spans up to 90 feet. The ropes hang between the pulleys in catenary curves which approximate to parabolas. It is advisable to have the driving side of the rope on the lower side of the pulleys and the slack side above. Then in driving the two sides approach each other, and the arc of contact on the pulleys is increased. But in some cases this gives trouble from the swaying of the upper or slack segment of the rope. The slacker the ropes are consistently with obtaining sufficient frictional resistance to

slipping at the pulleys, the better, because the ropes are less squeezed in the grooves and wear longer.

The wear of ropes is partly on the surface, partly internal, and due to the friction of the fibres on each other. Cotton ropes are the best of all ropes for driving, from the flexibility of the fibres. Manila and hemp ropes are largely used, but they are less elastic and flexible, and have more internal friction and wear. The internal friction is diminished by occasionally opening the strands a little and lubricating them with castor oil or a mixture of tallow and graphite. In the United States a four-strand manila rope is used, into which a plumbago and tallow lubricant is introduced during manufacture.

301. *Strength of ropes.*—The breaking strength of white or untarred rope varies from 7,000 to 12,000 lbs. per square inch, and that of cotton rope is about 8,000 lbs. per square inch. For ordinary hauling purposes the working strength may be taken at $\frac{1}{8}$th of the breaking strength or at 880 to 1,500 lbs. per square inch. When, however, the rope is used for driving purposes durability is the consideration of chief importance, and the wear is excessive, unless a much smaller working stress is adopted. About 280 lbs. per square inch is as great a stress as is desirable in ropes used for driving.

Let $\delta$ be the diameter; $\gamma$, the girth; and G, the weight per lineal foot of the rope. The section of hawser-laid rope is about $\frac{9}{10}$ths of the area of the circumscribing circle. Hence,

$$\text{Area of section} = 0{\cdot}9 \times \frac{\pi}{4} \times \delta^2 = 0{\cdot}7\ \delta^2 = 0{\cdot}072\ \gamma^2$$

*Manila or hemp ropes.*—Breaking strength $= 6{,}000$ to $7{,}000\ \delta^2$ $= 617$ to $720\ \gamma^2$.

(Wet or tarred ropes are about one-fourth weaker.)

The working stress for ordinary hoisting purposes depends on the rope speed and the diameter of the sheaves or pulleys, and whether the use of the rope is continuous or intermittent. For slow crane work with rope speeds up to 100 ft. per minute, and on sheaves of a diameter not less than 8 times the rope diameter, the working stress may be $100\ \gamma^2$. For hoisting at speeds up to 300 ft. per minute on sheaves not less than 12 times the rope diameter, the working stress may be $40\ \gamma^2$.

For transmission ropes for continuous driving at speeds of 700 to 6,000 ft. per minute much smaller working stresses are permissible as will be stated below.

*Weight of ropes.*—The weight of ropes in lbs. per lineal foot is given by the following equations :—

| | |
|---|---|
| Hemp, dry . . . | $G = 0{\cdot}27$ to $0{\cdot}29\ \delta^2 = {\cdot}027$ to ${\cdot}029\ \gamma^2$ |
| Hemp, wet or tarred. . | $= 0{\cdot}32$ to $0{\cdot}34\ \delta^2 = {\cdot}032$ to ${\cdot}034\ \gamma^2$ |
| Cotton . . . . | $= 0{\cdot}28\ \delta^2 \qquad = 0{\cdot}028\ \gamma^2$ |

*Transmission Ropes*

302. *Resistance to slipping and driving effort of rope belts.*—The coefficient of friction of a rope on an iron pulley, when rolling is prevented, is ordinarily taken at 0·2 to 0·3,[1] but its value has not been satisfactorily determined. As driving ropes are wedged in grooved pulleys (fig. 356), the pressure between

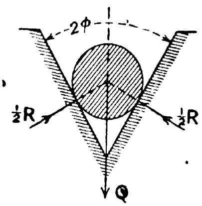



Fig. 356

the rope and the sides of the groove is greater than the radial force due to tension in the rope in the ratio cosec $\phi$ to 1. The angle $2\phi$ of the groove which has been found best is 40° to 50°, but for ropes of 1 in. diameter sometimes $2\phi = 30°$. If $Q$ is the radial force, the total normal pressure on the groove sides is $R = Q \operatorname{cosec} \phi$, and the frictional resistance to slipping is $\mu R = \mu Q \operatorname{cosec} \phi$. The equation for the ratio of the tensions in a flat belt (§ 283) is applicable if $\mu \operatorname{cosec} \phi$ is substituted for $\mu$. Then,

$$T_2/T_1 = \epsilon^{\mu\theta \operatorname{cosec} \phi}$$

$$\text{Com. log. } T_2/T_1 = 0{\cdot}434\,\theta \operatorname{cosec} \phi = k$$

where $T_2$ and $T_1$ are the tensions in the tight and slack sides in lbs., and $\theta$ is the arc of contact on the smaller pulley of a pair

[1] A reduction of some practical results communicated by Messrs. Pearce Brothers, of Dundee, gave $\mu \operatorname{cosec} \phi = 0{\cdot}57$ to $0{\cdot}88$ for ropes on ungreased grooved pulleys, and $\mu \operatorname{cosec} \phi = 0{\cdot}38$ to $0{\cdot}41$ when the pulleys were greased. These values correspond with average values $\mu = 0{\cdot}28$, pulleys dry, and $0{\cdot}15$ pulleys greased.

in radians. The following table gives calculated values of $T_2/T_1$ for an arc of contact of 165° and for various values of $\mu$ and $\phi$:—

| | When $2\phi$ is | | |
|---|---|---|---|
| $\mu =$ | 40° | 45° | 50° |
| | the value of $T_2/T_1$ or $k$ is | | |
| 0·2 | 5·37 | 4·49 | 3·91 |
| 0·25 | 8·19 | 6·55 | 5·51 |
| 0·3 | 12·45 | 9·51 | 7·73 |

In consequence of the wedging action the ratio $T_2/T_1$ is much greater than for flat belts. For an arc of contact of 165° and $2\phi = 40°$ to 45°, it is safe to take $T_2/T_1 = 7$.

*Centrifugal tension in the rope.*—As ropes are run at high speed the centrifugal tension is not negligible. Proceeding as in § 290, and taking the weight of rope to be $0·028\ \gamma^2$ lbs. per foot, the centrifugal force of one foot, travelling v feet per second on a pulley of radius $r$ feet, is $0·028\ \gamma^2\ v^2/g\ r$ lbs. Hence the centrifugal tension in the rope is $0·028\ \gamma^2\ v^2/g = 0·001\ \gamma^2\ v^2$ lbs. nearly.

If the working stress is not to exceed $20\ \gamma^2$ when running, the tension on the tight side due to the work transmitted must not exceed $T_2 = 20\ \gamma^2 - 0·001\ \gamma^2\ v^2$.

303. *Tensions in rope when driving.*—Taking $T_2/T_1 = 7$; $T_2 = 20\ \gamma^2 - 0·001\ \gamma^2\ v^2$; $= 20\ \gamma^2\ (1 - 0·00005\ v^2)$, we get for each rope,

$$T_1 = 2·9\ \gamma^2\ (1 - 0·00005\ v^2).$$

The driving effort is

$$P = T_2 - T_1 = 17·1\ \gamma^2\ (1 - 0·00005\ v^2).$$

The horse-power transmitted is

$$H = \frac{P\ v}{550} = ·031\ \gamma^2\ v\ (1 - 0·00005\ v^2).$$

The speed at which a rope will transmit the greatest amount of energy with the assumed limit of working stress is 82 feet per second or about 5,000 feet per minute. It appears that manila ropes work more satisfactorily at speeds of 3,500 to 4,000 feet per minute, but cotton ropes are run at speeds up to 6,000 feet per minute.

*Driving Effort and Horse-power transmitted by one Rope.*

| | | | | | |
|---|---|---|---|---|---|
| V = | 40 | 60 | 80 | 100 | 120 ft. per sec. |
| = | 2,400 | 3,600 | 4,800 | 6,000 | 7,200 ft. per min. |
| $P/\gamma^2$ = | 15·7 | 14·0 | 11·6 | 8·5 | 4·8 |
| $H/\gamma^2$ = | ·029 | ·025 | ·021 | ·016 | ·009 |

*Ordinary driving effort on rope belts.*—Experience shows that rope belts drive better and wear longer when lightly strained. As in the case of flat belts it is economical to reduce the tensions much below those they are capable of carrying. Hence in practice it will be found that many ropes are in use transmitting less power than the rules above permit. Let, for each rope—

$$P = C\gamma^2 (1 - 0{\cdot}00005\,V^2)$$
$$H = c\,\gamma^2 V (1 - 0{\cdot}00005\,V^2)$$

Then in ordinary practice $C = 8$ to $13$, and $c = 0{\cdot}015$ to $0{\cdot}024$. From data given by Mr. Combe who originally introduced rope driving in England, the working stress for manila ropes should diminish as the rope is larger. For manila rope Mr. Combe takes $C = 11{\cdot}6$ and $c = 0{\cdot}021$ for ropes one inch in diameter and $C = 7{\cdot}2$ and $c = 0{\cdot}013$ for ropes two inches in diameter.

Mr. Combe, however, states that when conditions are favourable, that is, when there is ample distance between the shafts, the ropes will carry 25 per cent. more power.

Cotton ropes are more costly than manila ropes, but they are more flexible and therefore wear less and last about three times as long. The ropes are made with three strands or with four strands and a central core. The former kind are preferred. For cotton ropes $c = 0{\cdot}02$ and $C = 11$.

*Weight, and Horse-power transmitted by Rope Belts.* $c = 0{\cdot}02$.

| Girth of rope ins. | Diameter of rope ins. | Weight of rope lbs. per ft. | Horse-power transmitted by each rope at speeds in ft. per sec. 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|
| $3\frac{1}{8}$ | 1 | ·28 | 5·6 | 7·2 | 8·6 | 9·6 | 10·3 | 10·6 |
| 4 | $1\frac{1}{4}$ | ·44 | 9·2 | 11·8 | 14·0 | 15·7 | 16·9 | 17·4 |
| $4\frac{3}{4}$ | $1\frac{1}{2}$ | ·64 | 13·0 | 16·6 | 19·8 | 22·2 | 23·9 | 24·6 |
| $5\frac{1}{2}$ | $1\frac{3}{4}$ | ·86 | 17·4 | 22·3 | 26·5 | 29·8 | 32·0 | 33·0 |
| $6\frac{1}{4}$ | 2 | 1·12 | 22·4 | 28·8 | 34·3 | 38·5 | 41·3 | 42·5 |

It is not convenient to use very large ropes, which would involve the use of very large pulleys, and, if much power has to

be transmitted, several ropes are used, running on multigroove pulleys. As many as thirty-six ropes have been used, transmitting power from one engine fly-wheel to several shafts. If there are $n$ ropes the power transmitted is $n$ H, where the value of H is given above.

304. *Pulleys for transmission ropes.*—The pulleys are usually of cast iron, and when motion is taken from a steam engine the fly-wheel is made the first driving pulley, grooves being turned in its rim. In rope transmission the durability of the ropes is of primary importance, and wear is chiefly due to the bending and unbending over the pulleys. Hence the pulleys must be of large diameter, usually at least 40 times the diameter of the rope for manila rope and 30 times for cotton rope. Under these conditions manila ropes may last three to five years and cotton ropes much longer. As the pulleys run at high speeds it is desirable that the sides should be cased to reduce the resistance due to the arms moving through the air.

It appears desirable that the ratio D/$\delta$ of the pulley diameter to the rope diameter should increase somewhat with the rope diameter.

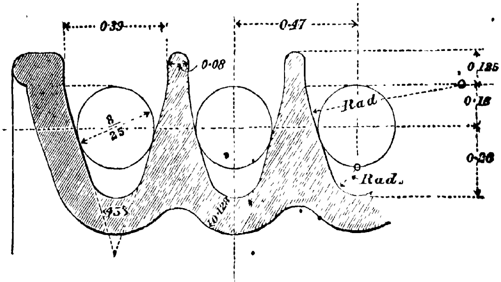
For manila rope . . . . $D = 18\,\delta + 10\,\delta^2$
For cotton rope . . . . $= 15\,\delta + 8\,\delta^2$

*Least Pulley Diameter.*

| | | | | | | |
|---|---|---|---|---|---|---|
| Diameter of rope D | $\frac{3}{4}$ | 1 | $1\frac{1}{4}$ | $1\frac{1}{2}$ | $1\frac{3}{4}$ | 2 |
| ,, ,, pulley | 19 | 28 | 38 | 50 | 62 | 76 Manila |
| ,, ,, ,, | 16 | 23 | 31 | 40 | 51 | 62 Cotton |

Fig. 357 shows the form of the grooves in the pulley-rim and proportions very commonly adopted. The unit for the proportional figures is $\gamma$, the girth of the rope. If the pulley is a guide-pulley merely, the rope should rest on the bottom of the groove, and the grooves are shallower. The sides of the groove are usually inclined at 40° to 50°. The pulleys are cast in one piece, when they are less than 8 feet diameter, unless they have to be fixed on shafts which have bosses at the ends, or require to be fixed after the shafts are in position. When this is the case they are cast in halves, and they are also usually cast in halves when they are from 8 to 12 feet diameter. Larger pulleys are cast in segments and bolted together. The grooves in each pulley must be accurately turned to the same gauge, and of the same diameter.

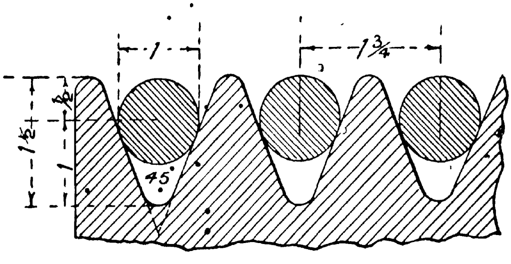
Very great care must be taken to have the pulley grooves of the same form and the ropes of the same diameter. If these conditions are not secured, different ropes will be virtually running on pulleys of different diameters, and some of the ropes will be severely strained. All the ropes intended to work on a




Unit = γ
Fig. 357

given pair of pulleys should be put on at the same time. They then stretch and decrease in diameter equally. The velocity of rope belting is generally 3,000 ft. to 5,000 ft. per minute.

Ordinarily the grooves are roughly shaped in casting the pulley, and the pulley rim is then turned. The finishing cut should be a very light cut by a tool of the exact shape of the




Unit = δ
Fig. 358

groove. In the United States the grooves are sometimes formed nearly accurate in the casting, and they are then smoothed by an emery block held against the wheel while revolving. Some pulleys are built up of arm sections and rim sections bolted together, so that by adding sections a rim of any width can be made.

Ropes have the peculiarity that they work well even if the shafts are not exactly parallel. For an inclination of not more than 3°, according to Mr. Combe, ordinary pulleys may be used. When the inclination is greater the form of groove is modified as shown in fig. 358. The grooves are triangular so that the ropes enter and leave without much rubbing on the edge.

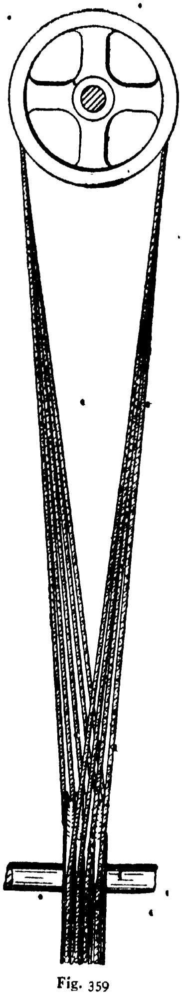


Fig. 359

Crossed rope drives are used, and rope drives for shafts at right angles. In crossed rope drives the ropes are put on so that they cross alternately on the right and left. Thus the driving side of the first rope comes next the driving side of the second where they cross, and the two ropes are running in the same direction. Similarly the trailing side of the second rope is adjacent to the trailing side of the third, and these are running in the same direction at the crossing. The grooves have a pitch greater than twice the diameter of the ropes, which also diminishes rubbing of the ropes. Fig. 359 shows a half crossed rope drive, with separate ropes to each groove, and fig. 360 the form of the grooves adopted when, as in this case, the ropes are inclined to the plane of the grooves. The pitch is one and a half times the rope diameter and the groove angle is 60°.

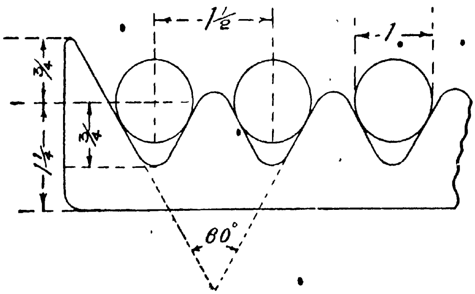
305. *Sub-division of power by ropes.*—Rope gearing affords great facilities for the distribution of power from a prime mover to various floors in a factory. Fig. 361 shows the distribution of power by ropes in a textile factory.

306. *Rope gearing with continuous rope.*—By introducing a guide pulley to return the rope from the last or $n$th groove of the driver to the first groove of the follower, a single continuous rope may be used instead of $n$ separate ropes, and there is then only one splice in the rope (fig. 362), and less distance is required between the shafts than

when there are separate ropes. The guide pulley is fixed in a frame which slides, and being loaded by a weight acts as a tightening pulley, keeping a constant tension in the ropes. This system, more largely used in the United States than in England, may be termed the American system.

If each bight of the rope exerts the same driving effort, $T_1 - t_1 = T_2 - t_2 = \ldots\ldots P$. Further, if each bight is on the point of slipping on the smaller pulley, $T_1/t_1 = T_2/t_2 = \ldots\ldots = k$. Then all the tight strands and all the slack strands are equally strained, and the total driving effort is the same as with $n$ separate ropes. It is doubtful if so uniform a distribution of the tensions can be realised, and the liability to breakage of the rope is greater than with separate ropes, and the breakage puts the whole transmission out of gear.

On the other hand, it is claimed for the American system




Unit = δ
Fig. 360

that the tension being automatically regulated by the tension pulley a less size of rope is required to transmit a given horse-power, that the ropes run with less swaying, and that the pulleys need not be at any great horizontal distance apart, and may even be vertically one above the other. The data on p. 485 have been given for transmissions of this type. The rope is Manila rope of three or four strands. The power stated is more than the power transmitted per rope in the English system and involves a greater rope tension.

307. *Relative merits of rope and toothed gearing in mills.*—It is difficult to give any conclusive opinion on the relative merits of toothed gearing and rope gearing when used in large mills. Unquestionably rope gearing has made great progress

since its introduction a few years ago. It is noiseless and free to a great extent from risk of serious breakdown. On the other

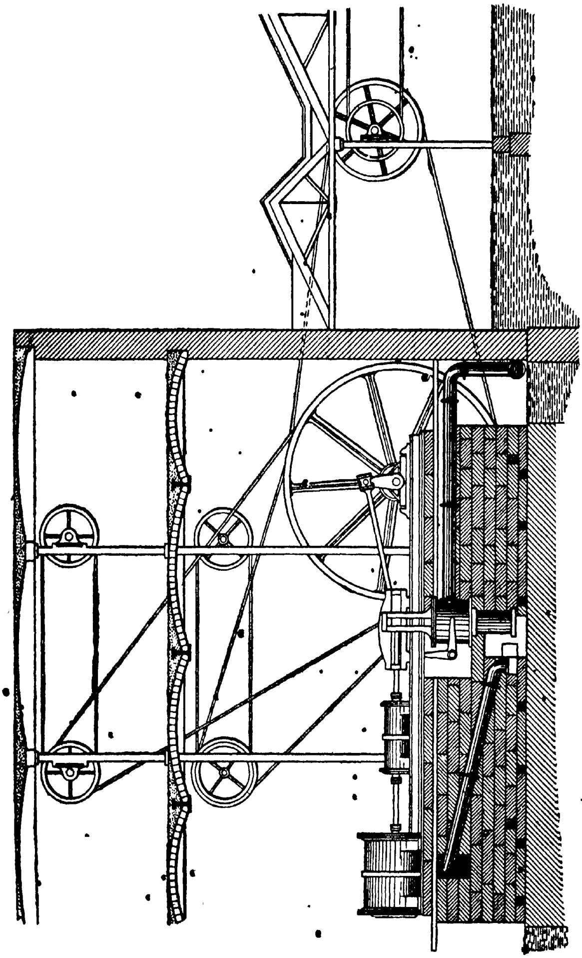


Fig. 361

hand, toothed gearing, especially if badly constructed or badly fixed, may give much trouble, and, if teeth break, serious delay

is occasioned. It appears that an engine flywheel fitted with ropes must be heavier than a spur flywheel, so much heavier, in fact, that in large engines the bearings of the flywheel shaft

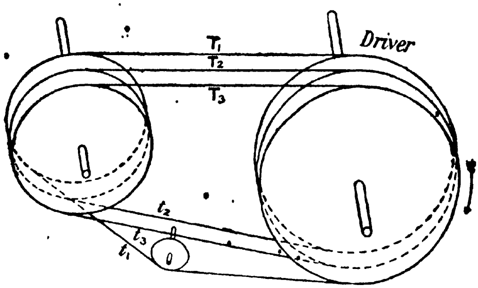



Fig 362

sometimes give trouble by heating. The ropes appear to take more space in the mill than gearing, and in the opinion of some engineers absorb ten per cent. more power than gearing.

*Rope Drives. American System.*

| Diameter of rope Ins. | Horse-power transmitted per groove at speeds in feet per minute. | | | | |
|---|---|---|---|---|---|
| | 1,000 | 1,500 | 3,000 | 4,500 | 6,000 |
| 1 | $5\frac{1}{2}$ | 8 | 16 | 24 | 33 |
| $1\frac{1}{4}$ | $8\frac{1}{2}$ | 13 | 25 | 38 | 51 |
| $1\frac{1}{2}$ | | 18 | 36 | 55 | 74 |
| $1\frac{3}{4}$ | | | 55 | 82 | 110 |
| 2 | | | 65 | 100 | 120 |

## CHAPTER XVI

### Wire Ropes and Telodynamic Transmission

308. *Different forms of wire rope.*—The manufacture of wire ropes was introduced, chiefly by Messrs. Smith & Newall, about 1838, and now wire ropes have largely superseded chains and hemp or manila ropes in many applications. A steel rope 6 in. in circumference and weighing 33 lbs. per fathom is as strong as a tarred hemp cable 19 in. in circumference and weighing 90 lbs per fathom. For equal strength a hemp rope must be about three times and a chain five times as heavy as a steel wire rope. Wire ropes are used for direct haulage, as in mines and steam ploughing; for wire ropeways supporting travelling carriers; for cable tramways; for the transmission of power to a distance, and for other purposes.

Wire ropes were at first made of charcoal iron wire, now they are almost always made of crucible steel. In ropes for suspension bridges, the wires are laid parallel and bound by an external serving. This is the strongest form of rope. For most purposes stranded ropes are used. A set of wires twisted round a core of wire or hemp forms a strand. A set of strands twisted round a core consisting of another wire strand or of a hemp rope forms a rope. The ropes with hemp cores are more flexible than those with wire cores. In the earlier construction the wires in the strands had a left-handed twist and the strands in the rope a right-handed twist. Now the Lang system is generally adopted, in which the wires in each strand and the strands in the rope twist in the same direction. The Lang ropes are more flexible than the older make, and the wires being less sharply bent wear better and are less liable to break. For crane ropes and some other purposes the older make is preferred.

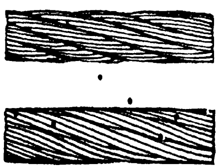


Fig. 363

Fig. 363, *a*, shows the older make, and *b* the Lang make of wire rope. Sometimes the wires are galvanised.

309. *Strength of wire.*—The strength of unannealed wire is greater than that of the bar from which it is drawn, and greater the smaller the diameter of the wire. The following results were obtained by Mr. H. Allen.[1] Broadly the ductility diminishes as the strength increases:

| | Diameter | | Area | Unannealed | | | Annealed | | |
|---|---|---|---|---|---|---|---|---|---|
| | B W G | Ins | Sq. in. | Elastic limit tons per sq in. | Breaking strength tons per sq in. | Elongation in 8 ins per cent. | Elastic limit tons per sq. in. | Breaking strength tons per sq. in. | Elongation in 8 ins. per cent. |
| Billet | — | | 4·0 | 17·0 | 28·1 | 28·1 | — | — | — |
| Rolled | No. 1 | 0·31 | 0·075 | 22·4 | 32·1 | 21·3 | 21·4 | 30·3 | 22·8 |
| Rolled | No. 2 | 0·27 | 0·057 | 20·0 | 29·8 | 21·1 | 20·4 | 28·8 | 19·8 |
| Rolled | No. 5 | 0·21 | 0·035 | 25·4 | 31·7 | 18·8 | 20·9 | 29·5 | 21·6 |
| Wire drawn from No. 1 | No. 3 | 0·25 | 0·047 | 27·5 | 48·8 | 1·9 | — | 27·1 | 15·2 |
| Wire drawn from No. 2 | No. 4 | 0·23 | 0·042 | 30·1 | 44·1 | 3·2 | 19·9 | 27·7 | 17·5 |
| Wire drawn from No. 5 | No. 8 | 0·16 | 0·020 | 47·3 | 56·4 | 1·0 | 18·8 | 26·6 | 21·2 |

The following table gives data of strength of various wires of small diameter.

| Material | Diameter ins | Bright or unannealed tensile strength tons per sq in. | Annealed tensile strength tons per sq. in. |
|---|---|---|---|
| Charcoal iron | 0·087 | 39·3 | 31·1 |
| Steel | 0·087 | 101·6 | 54·9 |
| Plough wire | 0·080 | 111·9 | — |
| Plough wire | 0·065 | 114·4 | — |
| Plough wire | 0·049 | 125·4 | — |

Messrs. Bullivant give the following average values of the strength of unannealed iron and steel wires used in the manufacture of wire ropes. Iron about 35 tons per square inch; Bessemer steel, 40 tons per square inch; mild Siemens-Martin steel, 60 tons per square inch; high carbon Siemens-Martin steel, 60 tons per square inch. The same 'improved' 80 tons per square inch; crucible cast steel, 'patented,' 100 tons per square inch; crucible cast steel, plough quality, 110 tons per square inch.

310. *Strength and weight of wire ropes.*—The breaking strength

[1] *Proc. Inst. C.E.* xciv. p. 235.

of ropes is proportional to the square of the diameter $\delta$ or to the square of the girth $\gamma$ of the rope. Similarly the weight in lbs. per foot length is proportional to the square of the diameter or girth. The following equations agree approximately with tables published by Messrs. Bullivant.

*Breaking Strength of Wire Ropes, in tons*

| | | |
|---|---|---|
| Charcoal iron wire | $11{\cdot}8\ \delta^2$ | $1{\cdot}2\ \gamma^2$ |
| Bessemer steel | $16{\cdot}8\ \delta^2$ | $1{\cdot}7\ \gamma^2$ |
| Crucible steel | $21{\cdot}7$ to $29{\cdot}6\ \delta^2$ | $2{\cdot}2$ to $3{\cdot}0\ \gamma^2$ |
| Plough steel | $35{\cdot}5$ to $39{\cdot}5\ \delta^2$ | $3{\cdot}6$ to $4{\cdot}0\ \gamma^2$ |

*Weight of Wire Ropes, in lbs. per foot*

| | | |
|---|---|---|
| All wire | $1{\cdot}68\ \delta^2$ | $0{\cdot}17\ \gamma^2$ |
| Hemp cores | $1{\cdot}48\ \delta^2$ | $0{\cdot}15\ \gamma^2$ |

311. *Wire ropes for hoisting.*—The causes of the gradual destruction of wire ropes in service are :—(*a*) Wear of the outer surface of the ropes. (*b*) Wear due to the rubbing and pressure of the wires against each other. (*c*) Fatigue of the steel due to the bending of the ropes over the pulleys or sheaves. Tramway cables suffer mainly from the wear of the outer surface due to the action of the gripper, and hence a stiff rope may be used composed of comparatively large wires permitting a good deal of wear before breaking. On the other hand, hoisting ropes which have to bend over small pulleys should be very flexible, and composed of small wires less strained by bending. If a wire rope is well oiled the internal friction and wear are much reduced.

Mr. Biggart has made some very interesting experiments on the life of wire ropes running on pulleys of different diameters.[1] The ropes were $\frac{7}{16}$ inch diameter or $1\frac{3}{8}$-inch circumference, and were subjected to a constant tension of $0{\cdot}7$ ton. The rope was composed of 72 wires 20 B.W.G. The following are the results :—

*Endurance of Steel Wire Ropes*

| | Number of Times the Rope passed over the Pulley before Breaking when the diameter of the pulley was | | | | | | |
|---|---|---|---|---|---|---|---|
| | $5\frac{1}{4}$ | $7\frac{7}{8}$ | $10\frac{1}{2}$ | $13\frac{1}{8}$ | $16\frac{7}{8}$ | $18\frac{3}{4}$ | 24 |
| Ordinary Rope A | 6075 | 10300 | 16000 | 23400 | 46800 | 72700 | 74100 |
| " " B | . | ... | 34800 | | ... | 100200 | 336600 |
| Lang's Lay C | | . | 53100 | 85200 | . | 392500 | . |
| Ordinary Rope A, oiled | | | 38700 | 37300 | .. | 163200 | 386100 |
| Lang's Lay C, oiled | .. | .. | 107600 | 142700 | .. | ... | ... |

[1] *Proc. Inst. C.E.* ci. 246.

*Working Stress in Wire Ropes*

The total stress in a rope consists of the direct stress due to the load and an additional stress due to bending over the pulleys. Let T be the total tension due to the load, $f$ the safe working stress, $f_t$ and $f_b$ the normal and bending stresses. Let $\delta$ be the diameter of the rope, D the diameter of the pulley, and $\nu$ the number of wires. Then

$$f = \text{or} > f_t + f_b$$
$$= \text{or} > \frac{4\,T}{\pi\delta^2\nu} + C\frac{\delta}{D} \text{ lbs. per sq. in.} \qquad (1)$$

The constant C may be taken at 8,000,000 to 11,000,000.

*Working Stress in Ropes*

| | Values of $f$ | |
|---|---|---|
| | Lbs. per sq. in. | Tons per sq. in. |
| Iron or Bessemer steel | 21,000 | 9·4 |
| Crucible steel | 35,000 to 50,000 | 15·6 to 22·3 |

These are maximum values for the greatest possible load.

*Bending stress in ropes passing over pullies.*—If a rope consisting of parallel wires of diameter $\delta$ is bent over a pulley of radius R, then in accordance with the principle in § 49, eq. 16, 17, the bending stress is

$$f_b = \frac{E\,\delta}{2\,R}$$

Till recently it has been assumed that this relation would also be true in the case of a rope of twisted strands, a somewhat modified value of E being taken, as in the statement above. But since ropes with twisted strands are much more flexible than ropes with parallel wires, it is reasonable to suppose that the bending stress in individual wires is less. The theory of a rope with twisted strands is very complicated. An attempt at a satisfactory theory has been made by Mr. R. W. Chapman (Proc. Australian Inst. of Mining Engineers, 1908), and the results are to some extent confirmed by some experiments made by him on the resistance of wire ropes to bending. Consider first a rope consisting of a single strand in which the wires make an angle $\alpha$ with the axis of the rope. Mr. Chapman finds the change of curvature of a wire due to bending over a pulley to be $\pm (\cos^2 \alpha)/R$. With an ordinary rope consisting of similar strands making an angle $\beta$ with the axis of the rope the extreme

change of curvature is $\pm (\cos^2 \alpha \cos^2 \beta)/R$, an expression true for ropes wound on the ordinary or the Lang lay. Hence, on this theory the bending stress is

$$f_b = \frac{E \delta}{2 R} \cos^2 \alpha \cos^2 \beta.$$

If $\alpha = \beta = 18°$, $\cos^2 \alpha \cos^2 \beta = 0{\cdot}81$, and the bending stress is only 0·81 of what it would be in a rope with parallel wires. In using this formula the value of E is the value for the steel—that is about 28,500,000 lbs. per square inch.

312. *Wire ropes for transmitting power.*—A method of wire-rope transmission has been in use, on the Continent chiefly, by which large amounts of motive power can be transferred to great distances, with an efficiency impossible with any other mode of mechanical transmission. This system, due to M. C. F. Hirn, is termed 'telodynamic transmission.' It has, however, not proved so important as it might have been, because the durability of the wire ropes has proved to be less than was hoped.

In belt transmission, we may increase the amount of power transmitted in three ways: by increasing the frictional bite of the pulleys, by increasing the strength of the belt, and by increasing the velocity of the belt. The first principle is applied in ordinary rope transmission by wedging the ropes in V-shaped grooves in the rim of the pulley. With wire ropes these wedge-grooves cannot safely be adopted, because of the injury done to the rope. On the other hand, wire ropes are stronger than hemp ropes, and if in addition they are run at the highest practicable velocity, a very great amount of power can be transmitted with comparatively light gearing. The principle of telodynamic transmission is, therefore, to use flexible wire rope belts of very great strength on ordinary pulleys, and to work them at very high velocities. This method of transmission promised at one time to be of very great importance. M. Hirn no doubt thought that by suitable arrangements the wear of the rope could be made very small. This has not proved to be the case. The ropes do not last in good condition more than about a year. The expense of replacing them is considerable, and has much restricted the adoption of what, in all other respects, is an admirable system of power transmission to great distances.

The pulleys are of large diameter, which tends to the preservation of the ropes by diminishing the bending action, and reduces

the influence of the stiffness of the ropes and the loss of work in journal friction. If the distance to which power is transmitted is very great, the transmissions are divided into relays with a separate rope for each. The relays are separated by stations. Each station is provided with a horizontal shaft upon which a double-grooved pulley is fixed, which is the driven pulley as regards the relay terminating there, and the driving pulley as regards the succeeding relay. The stations are usually arranged on masonry pillars more or less raised according to the configuration of the ground, for it is necessary that the rope should not touch the ground. Sometimes the power has to be partially distributed in its course; under these circumstances the shafts at the stations are made use of for the purpose. Frequently also intermediate pulleys are placed along a relay serving merely to support the rope. Occasionally a relay has been made 650 feet in length. Usually the length is 400 to 500 feet. The system is very efficient, and involves little loss in transmission. That loss is estimated at only about $2\frac{1}{2}$ per cent. + 1 per cent. in addition, for every 1,000 yards of distance.

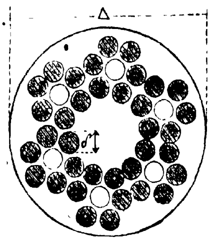



Fig. 364

313. *Form, strength, and weight of wire transmission ropes.*—The rope used consists of six or more strands wound upon a hemp core. Each strand consists of six or more wires also twisted round a hemp core. The strands are wound in the opposite direction to the wires in each strand. Fig. 364 shows the section of a rope, the shaded circles being sections of the wires, and the unshaded portions hemp. The angles of twist are usually 8° to 15° for the strands, and 10° to 25° for the rope. The wire diameter varies usually from $\frac{1}{50}$ to $\frac{1}{12}$ of an inch.

The ropes most commonly used have six strands, each containing six wires and a hemp strand at the centre. For these ropes with 36 wires the diameter of the rope is nearly $9\frac{1}{4}$ times the diameter of the single wires. Ropes of 42 wires are used with the middle hemp core replaced by a strand of six wires, and their diameter is about $10\frac{1}{2}$ times the diameter of a single wire. The number of strands and of wires in each strand is, however, arbitrary, and ropes of 8 strands, each of 10 wires, of 10 strands; each of 9 wires, and various other proportions, are adopted.

The relation between the diameter of the rope $\Delta$, the diameter of the wires $\delta$, and the number of wires $\nu$, is given very approximately by the formula

$$\frac{\Delta}{\delta} = \frac{\nu}{13} + 7 \quad . \quad . \quad . \quad . \quad (2)$$

The breaking strength of iron wire varies from 85,000 to 108,000 lbs. per sq. in., and the greatest working stress has been fixed at 25,600 lbs. per sq. in. Steel wire has a greater tenacity, and might be assumed to be capable of bearing a still higher working stress. At first steel wire ropes did not answer so well as ropes of iron wire. But according to M. Naville steel wire ropes are now preferred to those of iron. They are, however, worked only up to the same limiting stress, and in such conditions they last longer than iron. A rope running night and day lasts about 200 to 250 days, if of iron; and 250 to 300 days, if of steel. Moreover the steel ropes stretch less in working.[1] An iron rope requires tightening once in 60 days; a steel rope only once in 120 days.

The weight of wire rope per lineal foot is very nearly

$$= G = 3{\cdot}268\, \nu\, \delta^2 = 1{\cdot}341\, \Delta^2 \text{ lbs.} \quad . \quad . \quad (3)$$

*Splicing wire ropes.*—The following directions are abbreviated from those given by Messrs. Roebling.[2] Overlap the ropes for a distance of 20 feet. Unlay the strands for a length of 10 feet of each rope, and cut away for that distance the central hemp core. Now let $a$, $b$, $c$ . . . . be the strands of one rope taken in order, and $a'$, $b'$, $c'$ . . . . those of the other. Unlay $a$ for a further distance of 10 feet, and lay into the spiral groove so formed $a'$, and cut off $a$ and $a'$ so as to leave two short ends about 6 inches long. Next unlay about 10 feet of $d'$, and lay in the corresponding strand $d$, cutting off as before. Proceed by unlaying $b$ and laying in $b'$, and by unlaying $e'$ and laying in $e$, stopping about 4 feet of the previous cut ends. Lastly, unlay $c$ and lay in $c'$, and unlay $f'$ and lay in $f$, again stopping 4 feet short of the ends of the previously cut strands. To dispose of the cut ends, nip the rope about 6 inches on each side of the ends. Insert a stick and untwist the rope, cut out 6 inches of the hemp core, and force in the cut end into its place. Close the rope and hammer it even with a wooden mallet.

[1] Achard 'Transmission of Power,' *Proc. Inst. of Mech. Eng.* Jan. 1881.

[2] Stahl, *Transmission of Power by Wire Ropes.*

314. *Stresses in a wire rope belt.*—Used as a belt the wire rope is subjected to three different straining actions. (1) There is the longitudinal tension, due to the tightness with which the belt is strained over the pulleys, to the weight of the rope, and to the power transmitted. (2) There are stresses of tension and compression in the part of the belt which at any moment is bent to the curve of the pulley due to the bending. (3) There is a stress due to the centrifugal action of the part of the belt which is being bent. This last stress, though not insignificant, is sometimes left out of consideration.

Let $f_t$ be the greatest working stress due to the longitudinal tension of the belt, and $f_b$ the stress due to bending. For those wires which lie on the stretched side of the belt in passing over the pulley, the total stress is

$$f = f_t + f_b \quad . \quad . \quad . \quad . \quad (4)$$

When a cylinder of diameter $\delta$ is bent to a radius R, the bending moment at any point is § 49,

$$M = \frac{E\,J}{R} = \frac{E\,Z\,\delta}{2\,R} \quad . \quad . \quad . \quad . \quad (5)$$

where J is the moment of inertia, and Z the modulus of the section of the rope.

The moment of resistance to bending of a circular section of diameter $\delta$ is

$$f_b Z.$$

Equating these values,

$$f_b = \frac{E\,\delta}{2\,R} \quad . \quad . \quad . \quad . \quad (6)$$

If T is the total longitudinal tension in a rope having $\nu$ wires, each of $\delta$ inches diameter,

$$f_t = \frac{T}{\frac{\pi}{4}\delta^2\nu}$$

Hence the total stress in the most strained wire is

$$f = \frac{E\,\delta}{2\,R} + \frac{T}{\cdot 785\,\delta^2\nu};$$

hence

$$T = \cdot 785\left(f - \frac{\delta\,E}{2\,R}\right)\delta^2\nu \quad . \quad (7)$$

For a given value of the limiting stress $f$, T will be a maximum for pulleys of a given radius, when $\delta$ is so chosen that

$$\frac{d\,\text{T}}{d\delta} = 0,$$

or when

$$\frac{\text{R}}{\delta} = \frac{3\,\text{E}}{4f} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Putting $f = 25{,}600$, and $\text{E} = 29{,}000{,}000$,

$$\text{R}/\delta = 850.$$

That is, the longitudinal tension will be a maximum when the diameter of the wires is $\frac{1}{850}$th of the pulley radius.

When the ratio $\text{R}/\delta$ varies from these proportions, we have for the greatest safe working stress due to the longitudinal tension,

$$f_t = f - f_b = f - \frac{\delta\,\text{E}}{2\,\text{R}} \quad . \quad . \quad . \quad . \quad (9)$$

The deflection, when the rope is not working, should not be less than 18 inches. Sometimes when the pulleys are near together, the deflection of the rope will be too small with this tension. If this is the case, a lower value of the working tension should be adopted.

*Direct Stress and Stress due to bending, in Wire Rope Belts of Iron or Steel.*

| Ratio $\frac{\text{R}}{\delta}$ | Bending stress $f_b$ | Longitudinal stress $f_t$ | Total stress $f$ |
|---|---|---|---|
| 650 | 22,310 | 3,290 | 25,600 |
| 700 | 20,710 | 4,890 | |
| 750 | 19,330 | 6,270 | |
| 800 | 18,120 | 7,480 | |
| 850 | 17,060 | 8,540 | |
| 900 | 16,120 | 9,480 | |
| 950 | 15,270 | 10,330 | |
| 1,000 | 14,500 | 11,100 | |
| 1,100 | 13,180 | 12,420 | |
| 1,200 | 12,090 | 13,510 | |
| 1,350 | 10,740 | 14,860 | |
| 1,400 | 10,360 | 15,240 | |

The proportions most commonly adopted are those corresponding to $\text{R}/\delta = 1350$, so that the working longitudinal stress is only 14,860 lbs. per sq. in. The reason of this apparently low working stress will be obvious, if the bending action is considered.

315. *Total longitudinal tension of rope.*—Let T be the greatest tension in any part of the rope, exclusive of the bending stress.

Then since the section of metal in the rope is $\frac{1}{4}\pi\nu\delta^2$,

$$T = 0{\cdot}785\,\nu\,\delta^2 f_t.$$

Hence the size of wire for a given total tension is

$$\delta = \sqrt{\frac{4}{\pi f_t}}\,\sqrt{\frac{T}{\nu}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

| | | | | | | |
|---|---|---|---|---|---|---|
| $f_t =$ | 8000 | 9000 | 10000 | 12000 | 14000 | 16000 |
| $\sqrt{\frac{4}{\pi f_t}} =$ | ·01262 | ·01190 | ·01128 | ·01030 | ·00954 | ·00892 |

It has been stated already that $\delta$ is usually between $\frac{1}{80}$th and $\frac{1}{12}$th of an inch.

*Tension due to centrifugal force.*—The tension due to centrifugal force in a rope weighing G lbs. per foot, and travelling at $v$ feet per second, is

$$C = G\frac{v^2}{g}.$$

Inserting the value found in eq. (3) for G,

$$C = 3{\cdot}268\,\nu\,\delta^2\frac{v^2}{g} \quad . \quad . \quad . \quad . \qquad (11)$$

and dividing this by the section of the rope, the intensity of centrifugal tension is—

$$c = 0{\cdot}1293\,v^2 \text{ lbs. per sq. in.} \qquad (12)$$

| | | | | | | |
|---|---|---|---|---|---|---|
| $v =$ | 50 | 60 | 70 | 80 | 90 | 100 |
| $c =$ | 323 | 466 | 634 | 828 | 1047 | 1293 |

These stresses must be deducted from the stresses $f_t$ in the table, § 314, in order to find the safe tension due to the power transmitted.

316. *Driving force of belt, and power transmitted.*—The equations for the friction of a belt on a pulley given in Chapter XIV. are equally applicable for an iron wire rope, if proper values are taken for the coefficient of friction.

Taking $\mu = 0{\cdot}24$, and supposing that the belt embraces nearly a semicircle of the pulley, so that $\theta = 3$,

$$\epsilon^{\mu\theta} = 2 \text{ nearly.}$$

The ratio of the tensions on the tight and slack sides of the belt due to the resistance to slipping is,

$$\frac{T_2}{T_1} = e^{\mu\theta} = 2 \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

The driving effort of the belt is the difference of the tensions, that is,

$$\begin{aligned} P &= T_2 - T_1 \\ \therefore T_2 &= 2P \\ T_1 &= P \end{aligned} \quad \Bigg\} \qquad \ldots \qquad (14)$$

The work transmitted in foot-pounds per second is $P v$, and if $H$ is the horse-power transmitted,

$$P = \frac{550\, H}{v} \text{ or } H = \frac{P v}{550} \qquad \ldots \qquad (15)$$

We may put equation 10 in the form for calculating the size of rope from the driving force, instead of from the total tension. The tension, apart from the bending stress, must not exceed

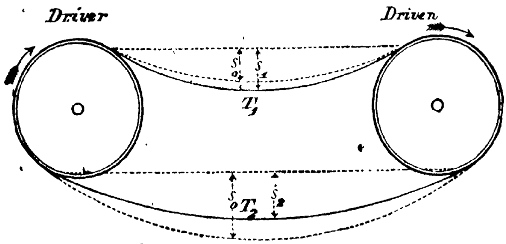



Fig. 365

$f_t - c$ lbs. per sq. in., and the total tension due to the work transmitted and the initial tension is $T_2$ or $2P$. Hence,

$$\delta = \sqrt{\frac{2 \times 4}{\pi(f_t - c)}} \sqrt{\frac{P}{v}} \qquad \ldots \qquad (16)$$

| $f_t - c$ | $\sqrt{\frac{8}{\pi(f_t - c)}}$ | $f_t - c$ | $\sqrt{\frac{8}{\pi(f_t - c)}}$ |
|---|---|---|---|
| 6000 | ·0206 | 10000 | ·0160 |
| 7000 | ·0191 | 12000 | ·0146 |
| 8000 | ·0178 | 14000 | ·0135 |
| 9000 | ·0168 | 16000 | ·0126 |

317. *Tightened belt.*—In some cases the diameter of the rope calculated by this rule will prove to be very small. Then it may be convenient to adopt a larger rope than is absolutely necessary. If this is done, either the size of the pulleys may be reduced, if desirable, the rope being capable of bearing a greater bending stress, or the tension in the rope may be increased beyond what is necessary to prevent slipping at the pulleys, with a view of reducing the deflection of the rope between the pulleys. In this

latter case, the tension $T_2$ may be calculated from the size of rope adopted; then $T_1$ is $T_2 - P$, and from these tensions the curves of the rope may be determined.

318. *Weight of ropes.*—The weight of wire ropes per lineal foot may be taken to be—

$$G = 3{\cdot}268\, \nu\, \delta^2 = 1{\cdot}341\ \Delta^2 \text{ lbs.} \qquad (17)$$

319. *Deflection of rope. Approximate equations.*—A parabola may be substituted for the catenary curve in calculating the tensions, without introducing any serious error. Suppose the two pulleys on the same level, then the lowest point of the curve is midway between them, and the tension at that point is

$$H = \frac{G\, l^2}{8\, s} \qquad (18)$$

G being the weight of the rope per unit of length; $l$ its horizontal projection, which is approximately equal to the distance between the pulleys; and $s$ the versed sine of the curve in which the rope hangs or the deflection of the rope at the centre of the span. The difference of tension at any two points of the curve is equal to the weight of a portion of rope of length equal to the difference of level of the two points. Consequently the tensions at the ends of the rope will be

$$T = H + G\, s = \frac{G\, l^2}{8 s} + G\, s \qquad (19)$$

Now let $T_2$ and $T_1$ be the tensions at the ends of the rope in the driving and slack sides of the belt, and $s_2$, $s_1$, the corresponding deflections. From equation 14 we have already $T_2 = 2\,P$ and $T_1 = P$. Putting these values in eq. 19 above, we get for the deflections,—

$$s_2 = \frac{P}{G} \pm \sqrt{\left\{\frac{P^2}{G^2} - \frac{l^2}{8}\right\}} \qquad (20)$$

$$s_1 = \frac{P}{2G} \pm \sqrt{\left\{\frac{P^2}{4\,G^2} - \frac{l^2}{8}\right\}} \qquad (21)$$

The deflection common to the two portions of the rope when not transmitting power is

$$s_0 = \sqrt{\left\{\tfrac{1}{2}(s_1{}^2 + s_2{}^2)\right\}} \qquad (22)$$

These equations determine the deflections of the span, when the driving force and the size of rope are given.

The greatest tension in the rope is

$$H_2 + G\, s_2 = \frac{G\, l^2}{8\, s_2} + G\, s_2 \qquad (23)$$

Let $f_t - c$ be the greatest permissible working stress, from the table § 314, and the values of the centrifugal stress in § 315, then

$$(f_t - c)\frac{\pi}{4}\nu\,\delta^2 = \frac{G\,l^2}{8\,s_2} + G s_2 \quad . \quad . \quad . \quad (24)$$

But $G = 3{\cdot}268\,\nu\,\delta^2$, and hence

$$f_t - c = 4{\cdot}16\left(\frac{l^2}{8\,s_2} + s_2\right) \quad . \quad . \quad (25)$$

This gives $f_t - c$, if $l$ and $v_2$ are assumed, or conversely determines $v_2$ in terms of the stress, if $f_t - c$ is assumed. Commonly $\delta$ is so chosen that $f_t - c = 14{,}000$ lbs. per sq. in. Then

$$\frac{l^2}{8\,s_2} + s_2 = 3364$$

$$s_2 = 1682 - \sqrt{\left(1682^2 - \frac{l^2}{8}\right)} \quad . \quad . \quad (26)$$

This gives for

| | | | |
|---|---|---|---|
| $l =$ | 420 | 500 | 600 feet |
| $s_2 =$ | 7 | 9 | 14 feet. |

320. *Efficiency of wire rope transmission.*—The experiments of M. Ziegler on the transmissive machinery erected at Oberursel give for the efficiency of a single relay

$$\eta_1 = 0{\cdot}962.$$

Hence if there are $m$ intermediate stations, the efficiency is approximately

$$\eta = \eta_1^{\frac{m+2}{2}} \quad . \quad . \quad . \quad . \quad (27)$$

| | | | | | | |
|---|---|---|---|---|---|---|
| No. of intermediate stations . . . | 0 | 1 | 2 | 3 | 4 | 5 |
| Efficiency . . . | 0·962 | 0·944 | 0·925 | 0·908 | 0·890 | 0·873 |
| Proportion of work wasted in per cent. . | 3·8 | 5·6 | 7·5 | 9·2 | 11·0 | 12·7 |

321. ***Pulleys for wire rope transmission.***—Wire ropes will not support without injury the lateral crushing which occurs when the rope rests against the sides of V-shaped grooves. Hence it is necessary to construct the pulleys with grooves so wide, that the rope rests on the rounded bottom of the pulley. It was found by Hirn that the wear of the rope was greatly diminished, and at the same time the frictional resistance to slipping was increased, by lining the bottom of the groove of the pulley with

gutta-percha or wood. The gutta-percha is softened and hammered into the groove, which is dovetailed in section. The wood may be inserted in short blocks, through a lateral opening, which is afterwards covered by a metal plate.

More recently, leather has been found to succeed better than either wood or gutta-percha. The leather is cut into pieces the shape of the notch, and placed in it edge upwards. When these pieces are filled in all round, the pulley is placed in the lathe, and the bottom of the groove is turned to the section required. This lagging of leather lasts on the average three years.

Fig. 366 shows the section of a pulley rim. The unit for the proportional figures is $w = \Delta + \cdot\frac{1}{2}$, where $\Delta$ is the diameter of the rope.

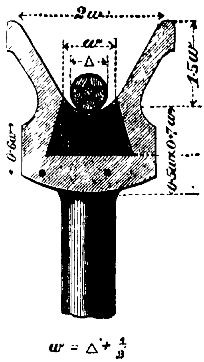



Fig. 366

The pulleys are often of cast iron, with cross-shaped arms, which may be calculated in the same way as the arms of toothed wheels. Sometimes they have oval curved arms like those of ordinary pulleys, and sometimes the arms are of round bar iron. These are cut to the right length and tinned at the ends. They are then placed in the sand mould, and the rim and nave cast round them. Such arms are usually placed sloping in the plane of the axis of the pulley, the slope being alternately in opposite directions. The pulley is thus rendered rigid enough to resist accidental lateral forces.

It has already been proved (eq. 9) that the radius of the pulley must not be less than

$$R = \frac{2(f-f_1)}{\delta E}.$$

Or, when $f = 25{,}000$, and the tension $f_1$ due to the work transmitted and the centrifugal force, does not exceed 8,000 lbs. per sq. in.,

$$R = 990\,\delta \text{ nearly.}$$

The pulleys commonly used are 12 to 15 ft. diameter.

When the distance to which the power is transmitted is great, intermediate guide or supporting pulleys are introduced to lessen the deflection of the rope. The supporting pulleys for

the tight side of the belt must be of the same size as the principal pulleys, those for the slack side may be smaller, in the ratio

$$\frac{R'}{R} = \frac{f - \frac{1}{2}f_t + \frac{3}{2}c}{f - f_t}$$

where $f$ is the total stress in the rope, $f_t$ the stress due to the longitudinal tension, including centrifugal force, $c$ the stress due to centrifugal force.

The pulleys are supported on shafts which rest in pedestals on masonry piers or timber trestles.

The weights of the most ordinary sizes of pulleys employed, including their shafts, are on the average as follows :—

| Diameter | Weight in lbs. | |
|---|---|---|
| | Single pulley | Double pulley |
| 18 ft. 0 in. | 6,232 | 8,267 |
| 14 „ 9 „ | 5,180 | 6,988 |
| 12 „ 4 „ | 2,425 | 4,078 |
| 7 „ 0 „ | 798 | 1,164 |

322. *Velocity of the rope.*—The rope is run at the highest safe velocity. That velocity is determined by the liability of the pulleys to burst, under the action of the centrifugal force, when the speed exceeds a certain limit. Let G be the weight of a cubic foot of cast iron, $v$ the velocity of the pulley rim in feet per second, $a$ its sectional area in square feet, R its radius in feet, and $f$ its tensile strength in lbs. per sq. ft. The tension in the rim due to centrifugal force is,

$$\frac{G\,a\,v^2}{g}$$

The resistance of the rim is $f\,a$. Equating these,

$$v = \sqrt{\frac{g f}{G}}$$

Thus, putting $G = 450$ lbs., $f = 4500 \times 144$,

$$v = 215 \text{ ft. per sec.}$$

The actual speed is never as high as this, a larger margin of safety being necessary. Usually the speed of the rope is from 60 to 100 feet per second.

Fig. 367 shows three arrangements of a wire-rope transmission. In A and B guide or supporting pulleys are used. The upper part of the rope is the driving side in A, and the lower part in B;

C is the arrangement adopted by Ziegler at Frankfort for transmitting 100 horse-power a distance of 984 metres.

323. *Duration of ropes.*—The ropes appear to last about a year. To preserve them from oxidation and improve their adhesion, they are coated with a mixture of grease and resin applied hot.

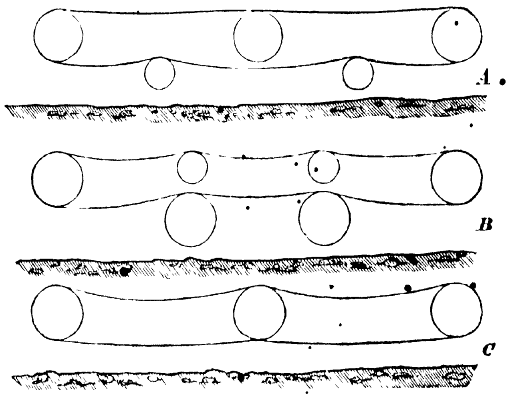



Fig. 367

The following table contains data taken from a paper by Achard[1]:—

*Wire Rope Transmission*

| Locality | Rope | | | Pulleys | | H P. transmitted | Total distance | | Velocity of belt ft. per sec. |
|---|---|---|---|---|---|---|---|---|---|
| | Diam. Δ ins. | Diam of wires δ in ins. | No of wires ν | Diam. 2R in ft | Distance apart in ft. | | Horiz. | Vert. | |
| Oberursel | 0·59 | ·060 | 36 | 12·30 | 394 | 94 | 3,153 | 146 | 73·8 |
| Schauff-hausen[2] | 0·95 | ·072 | 80 | 14·75 | 333 to 456 | 326 | 1,997 | — | 61·87 |
| Fribourg | 0·97 | ·070 | 90 | 14·75 | 502 | 300 | 2,510 | 269 | 65 |

[2] There are two cables. If one breaks, the other is capable of transmitting the power.

324. *Change of direction of the rope.*—When the direction of a rope requires to be changed either at a right angle or otherwise,

[1] *Annales des Mines*, viii. p. 229; *Proc. Inst. of Civil Engineers*, xlv. p. 267.

two plans may be adopted. A horizontal pulley may be used, in which case the pulley must have the same diameter as the other pulleys used in order that the bending stress may not be increased. More commonly, however, two vertical guide pulleys in the required directions in plan, are connected by bevil gearing. The splitting of the power transmitted to different points of application may be effected in the same way.

*Vertical rope.*—In the case of a vertical rope the initial tension on the lower pulley due to the weight of the rope would vanish, and that on the upper pulley would generally be insufficient unless special devices were used for producing the initial tension of the rope. Then tension or tightening pulleys may be used, like those for leather or rope belts.

*Stretching of ropes.*—In course of time the ropes stretch, and especially in summer sag so much that they become incapable of transmitting the required power. This may be remedied by resplicing the ropes, but this must be avoided as long as possible because it injures the ropes. Of late a mode of laterally compressing and stretching the ropes before use has been adopted which diminishes the stretching while working. According to Stahl the simplest way of neutralising the stretching which occurs in the working of the ropes is to increase a little the diameters of the pulleys by nailing wood strips to the material filling the bottom of the groove. Poplar or willow is used in pieces 1¼ inch thick and 45 to 70 inches in length. They are half cut through on one side with saw cuts, and steeped in water for two days to render them flexible. They are then nailed to the groove filling by wrought nails, long enough to pass through it and clinch themselves against the iron.

*Cost of Wire rope transmission.*—The cost is estimated in France at about £330 per mile, exclusive of the terminal stations. These cost about £1 per horse-power transmitted.

*Lateral swaying of the ropes.*—When a rope transmission is running well there should be little lateral swaying of the ropes, except the unavoidable motion produced by the wind. If swaying occurs it may be due to the pulleys being unbalanced or untrue, or to their not being in the plane of the rope; or it may be due to the pulley filling, or the rope being too much worn, or to bad splicing of the rope.

## CHAPTER XVII

### CHAINS AND GEARING CHAINS

CHAINS are used as structural tension members, for hoisting, and for transmission of energy as endless belts.

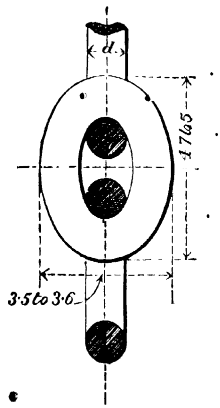
325. *Round iron link chains* are those most commonly used, and they are forged out of round iron bars of the best quality. When a tension is applied to such a chain, each link is subjected to a bending action additional to the tension, the bending being greatest at the extremities of the longer diameter of the link. Hence, on purely theoretical grounds the link should be stronger at the ends of the link. On the other hand, it would involve excessive expense to vary the section of the link, and the question of the best theoretical section is complicated by the uncertainty as to the strength at the weld. Welds in bars have been found to be from 10 to 40, or on the average 20, per cent. weaker than the bars themselves. Further, chains are constantly used on pulleys, and may then be subjected to bending action, perpendicular to the plane of the link. In designing a chain the links should be as small as possible, (1) because the greater the number of links in a given length the more flexible is the chain ; (2) because the less the transverse dimensions of the link the less is the bending action. The inside radius at the ends of the link must be a little greater than the radius of the iron of which the chain is made. Let $d$ be the diameter of the iron. Then $2{\cdot}6\,d$ and $1{\cdot}5\,d$ are about the smallest possible internal diameters of the chain link, and $4{\cdot}6\,d$ and $3{\cdot}5\,d$ the least outside diameters.

Very common proportions for ordinary chains are given in fig. 368. Such chain is termed close-link chain. Cheaper but weaker chain is made with longer links. Such chain may have the inside diameters $1{\cdot}5\,d$ and $4\,d$, and consequently the outside diameters $3{\cdot}5\,d$ and $6\,d$. In fig. 369, two forms of studded chain are shown. The stud resists the tendency of the link to collapse,

and renders the chain less liable to kink. The proportions of studded chain cables differ a good deal. The following are probably extreme proportions :—

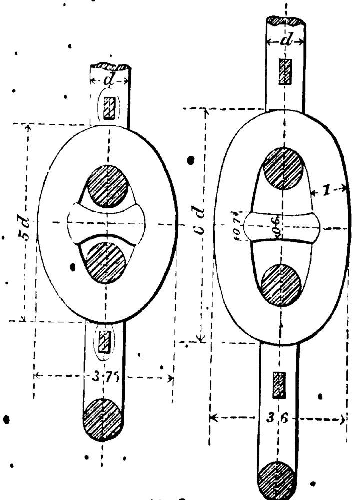
| | |
|---|---|
| Length of link, outside . . . . . . | $5\,d$ to $6\frac{1}{2}\,d$ |
| ,, ,, inside . . . . . . | $3\,d$ ,, $4\frac{1}{2}\,d$ |
| Width of link, outside . . . . . | $3\frac{1}{2}\,d$ ,, $3\frac{3}{4}\,d$ |
| ,, ,, inside . . . . . | $1\frac{1}{2}\,d$ ,, $1\frac{3}{4}\,d$ |
| Stud, diameter at ends . . . . . | $0{\cdot}7\,d$ ,, $d$ |
| ,, ,, centre . . . . | $0{\cdot}6\,d$ |

The end links of a length of cable are usually made of iron




Unit d.

Fig 368




Unit d

Fig. 369

of $1{\cdot}2\,d$ in diameter; they are a little larger than the other links, say $6\frac{1}{2}\,d$ in length and $4{\cdot}1\,d$ in width outside. Chain cable is often made in lengths of $12\frac{1}{2}$ or 25 fathoms. The lengths are joined by swivels and shackles.

A convenient method of drawing the elliptical form of chain links is shown in fig. 370. Set off $a\,c$, $c\,b$, the semi-diameters. Take the radius $d\,a$ a little greater than $1{\cdot}5\,d$, and draw the circular curve $a\,f\,e$ for the end of the link. Draw $d\,e$ parallel to $c\,b$. Join $b\,e$ and produce it to meet the arc in $f$. Join $f\,d$ and produce it to meet the smaller diameter of the link in $g$. Then

$g$ will be the centre for the arc $f\ b$, and $g\ b$ or $g\ f$ will be its radius.

326. *Strength of chains.*—The strength of the iron of which chains are made is about 21 to 25 tons per sq. in. Deducting one-fifth for the reduction of strength due to the weld, and to the presence of bending, the average tensile stress on the section of the chain at fracture might be expected to be $16\frac{1}{2}$ to 20 tons per sq. in. In actual tests the average breaking stress (neglecting bending) is about 15·5 tons for close-link crane chain and 17 tons for studded anchor chain.

The breaking strength of a link calculated from the tension and disregarding bending is

$$P_b = \pi d^2 f/2 = 1{\cdot}57\, d^2 f \text{ tons} \quad . \quad . \quad . \quad (1)$$

where $d$ is the diameter of the iron and $f$ the breaking stress in tons per sq. in. Using the test values above, $P_b = 24\, d^2$ for close link and $26{\cdot}7\, d^2$ for studded link chain.

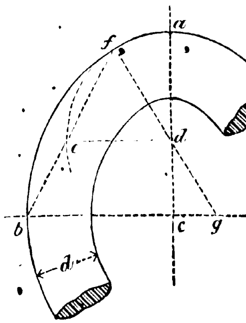


Fig. 370

The Admiralty rule for the proof stress of studded chain cable is—

Test load in tons $= 18\, d^2$,

corresponding to a stress of $11\frac{1}{2}$ tons per sq. in. of section. For close-link crane chains without studs,

Test load in tons $= 12\, d^2$,

corresponding to a stress of 7·7 tons per sq. in. of section.

327. *Working stress for chains.*—It is difficult to give definite rules for the working stress of chains because the circumstances in which they are used vary so greatly. Looking to the fact that they are always liable to be subjected to some bending action, the working stress is often stated at half of the proof load. This gives for the greatest safe load P in tons on a close-link chain of iron, $d$ inches in diameter,—

$$P = 6\, d^2 \text{ tons} \quad . \quad . \quad . \quad . \quad (2)$$

For crane chains subjected to shocks, the load should not exceed $P = 3{\cdot}25\, d^2$. This corresponds to a working stress irrespective of bending of 2 tons per sq. in.

## *Working Loads for Crane Chains*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Diameter of iron | $\frac{1}{4}$ | $\frac{5}{16}$ | $\frac{3}{8}$ | $\frac{7}{16}$ | $\frac{1}{2}$ | $\frac{9}{16}$ | $\frac{5}{8}$ | $\frac{3}{4}$ | $\frac{7}{8}$ | 1 |
| Load on chain in tons | ·20 | ·32 | ·45 | ·62 | ·81 | 1·03 | 1·27 | 1·83 | 2·49 | 3·25 |

## *Strength and Weight of Close-Link Crane Chains, and Size of Equivalent Hemp Cable*

| Diameter of iron $d$ in inches | Weight of chain in lbs per fathom | Breaking strength in tons | Testing load in tons | Girth of equivalent rope in inches | Weight of rope in lbs. per fathom |
|---|---|---|---|---|---|
| $\frac{1}{4}$ | 3·5 | 1·9 | 0·75 | 2 | $1\frac{3}{8}$ |
| $\frac{5}{16}$ | 6·0 | 3·0 | 1·10 | $2\frac{1}{2}$ | $1\frac{1}{2}$ |
| $\frac{3}{8}$ | 8·5 | 4·3 | 1·6 | $3\frac{1}{4}$ | $2\frac{1}{2}$ |
| $\frac{7}{16}$ | 11·0 | 5·9 | 2·3 | 4 | $3\frac{3}{4}$ |
| $\frac{1}{2}$ | 14·0 | 7·7 | 3·0 | $4\frac{3}{4}$ | 5 |
| $\frac{9}{16}$ | 18·0 | 9·7 | 3·8 | $5\frac{1}{2}$ | 7 |
| $\frac{5}{8}$ | 24·0 | 12·0 | 4·6 | $6\frac{1}{4}$ | $8\frac{1}{2}$ |
| $\frac{11}{16}$ | 28·0 | 14·6 | 5·6 | 7 | $10\frac{1}{2}$ |
| $\frac{3}{4}$ | 31·5 | 17·3 | 6·8 | $7\frac{1}{2}$ | 12 |
| $\frac{13}{16}$ | 37·0 | 20·4 | 7·9 | $8\frac{1}{4}$ | 15 |
| $\frac{7}{8}$ | 44·0 | 23·1 | 9·1 | 9 | $17\frac{1}{2}$ |
| $\frac{15}{16}$ | 50·0 | 26·1 | 10·5 | $9\frac{1}{2}$ | $19\frac{1}{2}$ |
| 1 | 56·0 | 29·3 | 12·0 | 10 | 22 |
| $1\frac{1}{8}$ | 71·0 | 36·3 | 15·3 | $11\frac{1}{4}$ | $27\frac{3}{4}$ |
| $1\frac{1}{4}$ | 87·5 | 44·1 | 18·8 | $12\frac{1}{2}$ | $34\frac{1}{2}$ |
| $1\frac{3}{8}$ | 105·8 | 52·8 | 22·6 | $13\frac{3}{4}$ | $41\frac{1}{2}$ |
| $1\frac{1}{2}$ | 126·0 | 62·3 | 27·0 | 15 | $49\frac{1}{2}$ |

## *Strength and Weight of Studded Link Cable*

| Diameter of iron $d$ in inches | Weight in lbs. per fathom | Breaking strength In tons | Test load in tons | Girth of equivalent rope in inches | Weight of rope in lbs. per fathom |
|---|---|---|---|---|---|
| $\frac{5}{8}$ | 24 | 9·5 | 7 | $6\frac{1}{2}$ | 9 |
| $\frac{11}{16}$ | 28 | 11·4 | $8\frac{1}{2}$ | $7\frac{1}{2}$ | 12 |
| $\frac{3}{4}$ | 32 | 13·5 | $10\frac{1}{2}$ | 8 | 14 |
| $\frac{7}{8}$ | 44 | 20·4 | $13\frac{3}{4}$ | $9\frac{1}{2}$ | $19\frac{1}{2}$ |
| 1 | 58 | 24·3 | 18 | $10\frac{1}{2}$ | $22\frac{1}{2}$ |
| $1\frac{1}{8}$ | 72 | 29·5 | $22\frac{1}{4}$ | 12 | $30\frac{3}{4}$ |
| $1\frac{1}{4}$ | 90 | 38·5 | $28\frac{1}{2}$ | $13\frac{1}{2}$ | $39\frac{1}{4}$ |
| $1\frac{3}{8}$ | 110 | 48·5 | 34 | 15 | $48\frac{1}{4}$ |
| $1\frac{1}{2}$ | 125 | 59·5 | $40\frac{1}{2}$ | 16 | 55 |
| $1\frac{5}{8}$ | 145 | 66·5 | $47\frac{1}{2}$ | 17 | 62 |
| $1\frac{3}{4}$ | 170 | 74·1 | $55\frac{1}{8}$ | 18 | $68\frac{3}{4}$ |
| $1\frac{7}{8}$ | 195 | 92·9 | $63\frac{1}{4}$ | 20 | 86 |
| 2 | 230 | 99·5 | 72 | 22 | 104 |
| $2\frac{1}{8}$ | 256 | 112 | $81\frac{1}{4}$ | 24 | 124 |
| $2\frac{1}{4}$ | 285 | 126 | $91\frac{1}{8}$ | 26 | 145 |

*Weight of Chains.*—The weight of chains in lbs. per fathom (of six feet) is:

$$w = 54\ d^2 \text{ to } 58\ d^2.$$

The stowage room required for chains is about $35\ d^2$ cubic feet for each 100 fathoms.

In all the above equations $d$ is to be taken in inches.

328. *Chain barrels.*—Sometimes a chain is coiled completely on to a chain barrel. Such barrels should have a spiral groove just wide enough to receive the edges of the links, and so deep that the alternate links lie flat on the cylindrical part of the barrel. The diameter of a chain barrel should be at least $24\ d$ to $30\ d$, and is usually determined with reference to the amount of chain to be coiled on it. In good machinery the barrel should

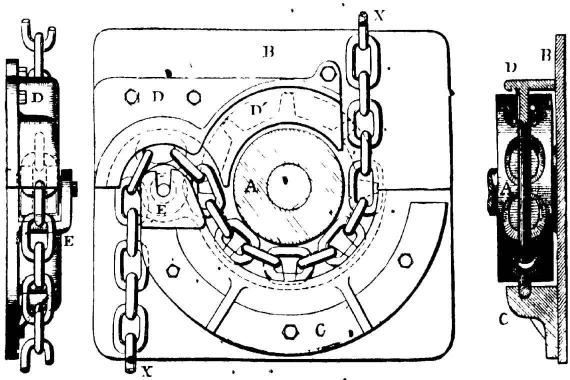



Fig. 371

take the chain in one layer. If one layer is coiled on another the chain is injured. The diameter of sheave for a chain should be at least $15\ d$.

*Pocketed chain Wheels.*—Some chain wheels are made with pockets which fit the chain. On such wheels the chain passing over half or three-quarters of a circumference, is so gripped that it will not slip, and the loose end may be run off on to the ground. Such wheels are narrow and moderate in diameter, and remain in the line of the pulling part of the chain. They occupy much less space, and injure the chain less than a chain barrel. The pitch of the pockets in the chain wheel must agree exactly with the pitch of the chain. The best material for the chain wheel is

soft cast-iron. To insure the engagement of the chain and wheel a chain guide is provided. Fig. 371 shows a pocketed chain wheel and guide. A is the chain wheel mounted on the frame B. C is the chain guide surrounding the lower half of the chain wheel, and bolted to B. Its inner side is grooved, and only a small space is left between it and the chain wheel. At E the chain guide carries a small roller, over which the chain passes down to a suitable receptacle. D is a chain stripper.

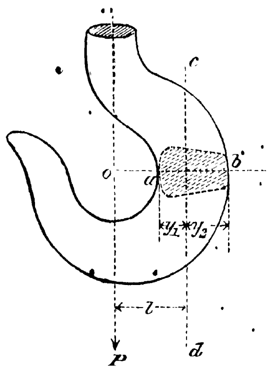


Fig. 372

329. *Straining action on a crane hook. Older theory.*—Consider a section of the crane hook through $o\,a\,b$ (fig. 372). Let P be the load supported by the hook, and let $l$ be the distance to a line $c\,d$ through the centre of figure of the section $a\,b$. Let $y_1$ $y_2$ be the distances from $c\,d$ to the extended and compressed edges of the section.

Then the section $a\,b$ is subject to the action of a bending moment P $l$, and a tension P (see Case I., § 76). Let A be the area of the section $a\,b$, and J its moment of inertia about the axis $c\,d$. Then the stress at $b$ is

$$f_2 = \mathrm{P}\left\{\frac{1}{\mathrm{A}} - \frac{l y_2}{\mathrm{J}}\right\}$$

and the stress at $a$ is

$$f_1 = \mathrm{P}\left\{\frac{1}{\mathrm{A}} + \frac{l y_1}{\mathrm{J}}\right\} \qquad (3)$$

*Straining action. Newer theory.*—The older theory dealt with the straining action as if the hook were a straight instead of a curved bar. It is now known that the stresses so determined are 40 or 50 per cent. too small. The following formulæ are due to an investigation by Professor Bach.

Let fig. 373 represent the ordinary sections of the hook, $n\,n$ being the neutral axis of the sections. Let O$a$ (fig. 372) $= a$. Then $l = a + y_1$, and $h = y_1 + y_2$. If P is the load, A the area of cross-section, and $f$ the working stress in tension, then

$$\mathrm{P} = f\frac{a}{y_1}\kappa\,\mathrm{A} \qquad (4)$$

where $\kappa$ is a function of the form of section. Let

$$b = (\text{P C})/(fa)$$

where $$c = (by_1)/(\kappa A).$$

Then the following are calculated values of C.

*Rectangular Section*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h/a =$ | 1 | 1·5 | 2·0 | 2·25 | 2·5 | 2·75 | 3 | 4 |
| C = | 12·6 | 7·3 | 5·1 | 4·4 | 3·9 | 3·6 | 3·2 | 2·4 |

*Trapezoidal Section*

$b/b_1$ = conveniently $(h/a)+1$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h/a =$ | 1 | 1·5 | 2·0 | 2·25 | 2·5 | 2·75 | 3·0 | 4·0 |
| C = | 15·0 | 9·0 | 6·4 | 5·6 | 5·1 | 4·6 | 4·2 | 3·3 |

*Elliptical Section*

| | | | | | |
|---|---|---|---|---|---|
| $h/a =$ | 1 | 1·5 | 2·0 | 2·5 | 3·0 |
| C = | 21·5 | 12·6 | 8·9 | 6·9 | 5·7 |

For crane hooks for hemp ropes $a = 0{\cdot}75\,d$ to $d$. For chain crane hooks $a=d$ to $1{\cdot}5\,d$, where $d$ is the diameter of the rope or of the chain iron. Generally $h/a = 2$ to 3. If a value is selected for this, the value of C can be found in the Table, and $b$ is obtained by the formula above.

For good wrought iron $f = 5$ to 7 tons per sq. in.

330. *Proportions of crane hook.*—A crane hook is most often required to receive a rope sling. The opening of the hook should therefore have a width $\delta =$ the diameter of the rope. The inside of the hook should have a broad rounded surface which will not injure the rope; and the section of the metal of the hook should be as deep as convenient in those parts where the bending is greatest. Fig. 374 gives conventional proportions for such a hook. The hook is forged of round bar flattened, as shown in the section on the left, to deepen the section where the bending is greatest. Taking $\delta$ as the opening of the hook, or diameter of the rope sling, the unit for the proportional numbers is $1{\cdot}3\,\delta$.

The outside diameter of the screwed shank of the hook may be $d = 0{\cdot}016\sqrt{\text{P}}$ when P is in pounds, and $d = 0{\cdot}75\sqrt{\text{P}}$ when P is in tons.

Fig. 375 gives also some conventional proportions for a crane hook. The unit for the proportional figures is

$$d = \sqrt{\text{P}},$$

where $d$ is in inches and P in tons.

Fig. 376 gives another scheme of conventional proportions for ordinary crane hooks, which agrees fairly closely with dimensions of different hooks stated by Mr. Towne. The unit of the figures is $d = \sqrt{P}$, as in the last case.

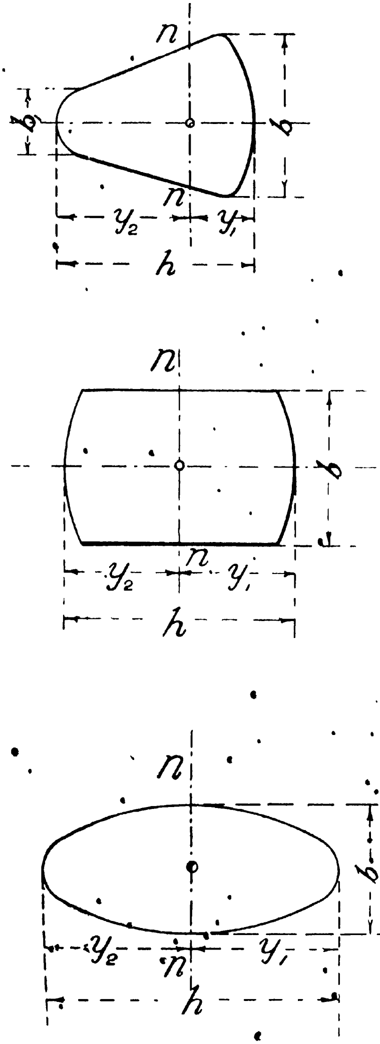


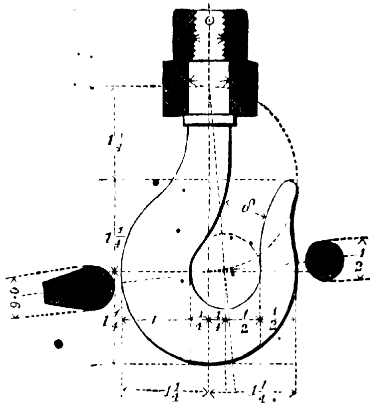

Fig. 373

331. *Plate-link chains.*—Chains of this kind are used when a great load is to be supported, as in the case of suspension bridges, patent slips, &c., or as gearing chains when the work transmitted is very heavy, as in the turning gear of cranes, dredger chains, &c. They are constructed with short or long links, according to the amount of flexibility required.

Let $d$ be the diameter of the pin in the eyes of the links. Then the shortest convenient length of link is about $2{\cdot}9\ d$. On the other hand, links are sometimes made 24 feet in length. Fig. 377 shows a simple flat-link chain with one and two links alternately, the double links being half the thickness of the single link.

Let $b$ be the width of the link, $\delta$ the thickness of the single link, $d$ the diameter of the pin, P the load on the chain. Then the stress on the link is given by the equation:

$$f_t\, b\, \delta = \text{P}$$



Unit = 1·3 $d$

Fig. 374

The shearing stress on the pin is given by the equation:

$$\frac{\pi}{4} d^2 f_s = \frac{1}{2}\text{P}$$

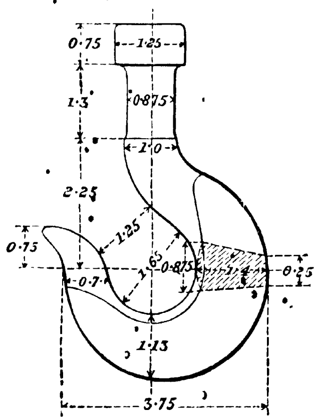



Fig. 375

And the bending stress on the pin is given by the equation:

$$\tfrac{3}{8} P \delta = f \frac{\pi}{32} d^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Suppose $f_t = 5$ tons per sq. in.; $f_s = 4$ tons per sq. in.; and $f =$ 5 tons per sq. in. Then the equations become:

$$b \delta = 0{\cdot}2 P \quad . \quad . \quad . \quad . \quad . \quad (a)$$

$$d^2 = 0{\cdot}159 P \quad . \quad . \quad . \quad . \quad . \quad (b)$$

$$d^3 = 0{\cdot}764 P \delta \quad . \quad . \quad . \quad . \quad . \quad (c)$$

Equation ($c$) will in general give a greater value of $d$ than ($b$). Equations ($a$) and ($c$) give

$$d^3 = 3{\cdot}82\, b\, \delta^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

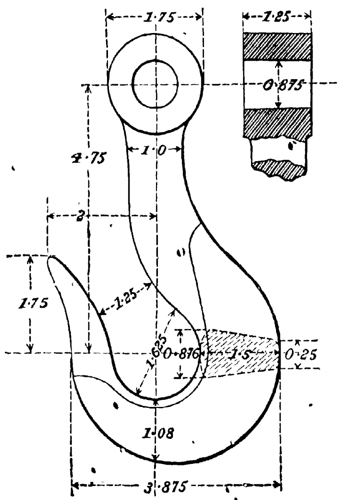



Fig 376

Experiment shows that if $d$ is less than $\tfrac{3}{4}\, b$, the link crushes in the eye and is weakened. Hence if

$$\delta < 0{\cdot}2785\, b,$$

$d$ must be taken not less than $\tfrac{3}{4}\, b$, and its bending resistance will be in excess.

Fig. 378 shows the forms of link eyes found by experiment to be strongest. A is the form arrived at by Mr. G. Berkley, and B that arrived at by Sir C. Fox. If links are short, they will not generally be made of the most economical form. They should

then have a form which includes the shape here indicated. Two proportional figures are given, one with the width of link, the other with diameter of pin as unit.

Three modes of fastening the pin in flat-link chains are shown in fig. 379. At *a*, the pin is simply riveted over the out-

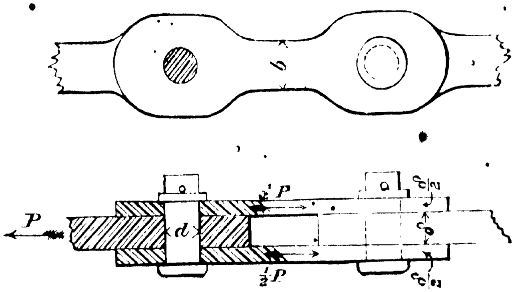



Fig. 377

side link. At *b*, a washer-plate is interposed, which secures greater freedom of motion in the links. At *c*, a washer and split pin are used; and this probably would be the best plan but for

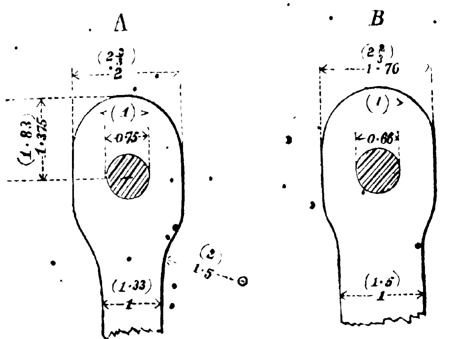



Fig. 378

the possibility of the split pin falling out when the chain is in use.

## Gearing Chains

332. *Gearing chains for great efforts at low speed.*—In cases where a considerable amount of work has to be transmitted between two shafts at a slow speed; the tension in a flexible

transmitter may easily be much greater than ordinary belts can sustain. In such cases iron or steel chains may be used, so formed that the links fit into the projections of toothed wheels on the shafts. There can then be no slipping of the belt on the toothed wheels; and as the chains may have almost any strength, an extremely great force can be exerted through the chain. Such chains are used on a large scale in cranes and lifting gear. The chains carrying the buckets of large dredgers act in the same way. Such chains forming a class of transmitting organs intermediate between belting and gearing, are termed gearing chains or pitch chains.* The chief objection to their use is, that however well they fit the toothed wheels at first, they are liable from stretching and wear to become of slightly greater pitch than the toothed wheel, and they then work badly. To obviate this as far as possible, the links should be short.

The commonest form of such a gearing chain (fig. 380) is a flat-linked chain, having two systems of links spaced some

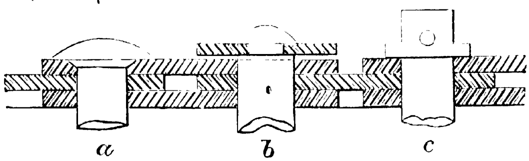



Fig 379

distance apart. The toothed wheel acts in the space between the two systems of links, and the teeth gear with the pins passing through the link ends. The pitch-line radius of the gear wheel for $n$ teeth is

$$r = l / \{2 \sin (180/n)\}.$$

A mode of construction in some respects better than this is to divide the toothed wheel into two parts, between which the chain is placed. The teeth of the wheel then gear with the alternate link ends on each side. With a chain of this kind, however, a larger toothed pulley is required than for the ordinary form. In a third form the pins project beyond the links and gear into the spaces of the gear wheel. Long-linked chains are sometimes used on a polygonal pulley without teeth. The polygonal pitch line of such a pulley has sides equal to the lengths of the links between the centres of the pins; and it has usually at least five or six sides. With four sides the twisting moment is too variable and the motion too irregular.

Fig. 380 shows the ordinary form of a flat-link gearing chain and its toothed pulley. The side views also show chains with one set of links in each system and with two sets.

Gearing chains of this type, under the name of stud chains, are largely used for hoisting purposes, and run on toothed wheels, termed sprocket wheels. The chain speed is generally not over 50 ft. per min. At higher speeds the wear of the studs is too great, the wearing surface being small and confined to a small arc on the pin.

333. *Strength of gearing chains.*—Let 2 T be the total tension on the loaded span of the chain, and 2 $i$ the whole number of links

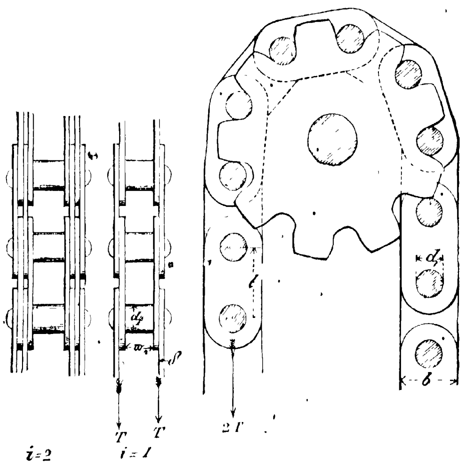



Fig 380

in the width of the chain. Then the tension in each half of the chain, having $i$ links, is T. Let $\delta$ be the thickness, $b$ the breadth of a link, and $d$ the diameter of the pin in the link eye. If $n$ T is the greatest tension in any one link, then the stress $f_t$ on a section through the link eye is given by the equation

$$n \mathrm{T} = (b - d)\,\delta f_t.$$

Usually $b=2{\cdot}5\,d$, and then

$$n \mathrm{T} = 1{\cdot}5\,d\,\delta f_t.$$

The average bearing pressure of the pin in the link eye, $f_c$, is given by the equation

$$n \mathrm{T} = d\,\delta f_c.$$

The pins are subjected to bending action which will increase with the total stress T and with the thickness $\delta$ of the links. Hence, whatever the distribution of the tension T amongst the links, the greatest bending moment on the pin will be

$$M = m\,T\,\delta,$$

and the stress due to bending will be given by the equation

$$f = \frac{32\,m\,T\,\delta}{\pi\,d^3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where $f$ is the greatest intensity of bending stress, and $m$ a coefficient depending on the distribution of the tension in the links.

Consider a part of the chain hanging freely between the toothed pulleys. For such a portion of chain the links may be assumed to carry equal portions of the load, and $n = 1/i$. Supposing the tension in each link to act at the centre of the link, we get for the greatest bending moment on the pin

$$M = \frac{T\,\delta}{i}\left(\tfrac{1}{2} - \tfrac{3}{2} + \tfrac{5}{2} - \ . \ . \ . \ . \ - \frac{4\,i - 1}{2}\right)$$
$$= T\,\delta\,.$$

Hence,

$$1{\cdot}5\,d\,\delta\,i\,f_t = \frac{\pi}{32}\,\frac{d^3}{\delta}\,f;$$

and supposing the stress in the link and pin equal, and taken at $f = 10{,}000$ lbs. per sq. in., we get

$$\frac{\delta}{d} = \frac{\cdot 256}{\sqrt{i}}$$

$$d = 0{\cdot}01614\sqrt{\frac{T}{\sqrt{i}}} \quad . \quad . \quad . \quad (8)$$

and if $f = 14{,}000$

$$d = 0{\cdot}01365\sqrt{\frac{T}{\sqrt{i}}} \quad . \quad . \quad . \quad (9)$$

334. *High speed gearing chains.*—The positive action with definite velocity ratio of gearing chains and their great strength are advantages, when compared with belts or ropes, and they are much less noisy than toothed gearing. Within a comparatively recent period their forms and manufacture have been so much improved that they have come into very extensive use even when power has to be transmitted at high speeds of revolution. The

development of this method of transmission is almost wholly due to Mr. Hans Renold, of Manchester, who has invented many improvements and has carried the manufacture of such chains to a remarkable degree of perfection. The chain drive is distinctly preferable to belts or ropes when the distance between the shafts is short, but it has also advantages in many cases where ordinarily a belt drive would be used. When chains are used for driving machine tools it is found that the positive action has the effect of reducing the chattering, and more work is done with less wear of the tool. A chain drive cannot be run at so high a speed as a belt or rope drive, and the connected shafts must be parallel and run in the same direction. Disengaging is effected by a friction clutch on one of the sprocket wheels. The efficiency of a chain drive is very high; the chain is fairly slack, and the strain on the journals being only that due to the power transmitted and not to initial tension, the pull on the bearings is less than with belts and the journal friction is less. In some tests of similar groups of machines belt and chain driven, the power expended was found to be 17 per cent. less with the latter. The endless chain has one coupling link, bolted instead of riveted. The chain is lengthened or shortened by adding or taking out links at this joint. It can be shortened, if it stretches or wears, by one link, or preferably by two links. Smaller adjustments of length are effected by moving the sprocket wheels.

335. *Forms of Renold transmission chains.*—There are three principal forms of transmission gearing chains. (1) The *block chain*, made up of straight or curved blocks connected by links and pins. The sprocket teeth act on the blocks. In the smaller sizes it is used for feed drives on machine tools, in the larger for conveyors and elevators. (2) The *roller chain* is similar to the stud chain, fig. 380, but has the rivets or pins surrounded by steel bushes, on which are rollers on which the sprocket teeth act. The wearing parts are hardened, and the wearing surface is greater than in stud chains. These chains are used for cycles, motors, and machine driving. (3) The *silent chain* is designed for high speeds, and runs even when worn with great smoothness and quietness. Segmental bushes are fixed in the links and bear on hardened pins. This allows a continuous film of oil between the rubbing surfaces. Tooth-formed projections of the links bear on the teeth of the sprocket wheels.

Fig. 381 shows a steel block chain. With this chain the transmission speed is usually 200 to 500 ft. per min., and as much as 100 H.P. can be transmitted. The chain is used for conveyor work, as the blocks are convenient for attaching buckets or special parts.

Fig. 382 shows an early modification of the simple stud chain, which is defective from the concentration of the wear on a small part of the pin. A tube is riveted between the pair of inside

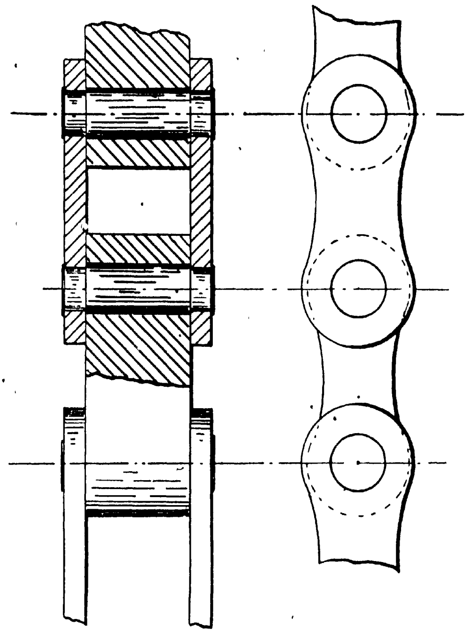


Fig. 381

links, and this is traversed by a pin riveted to the outside links. The pin gets a bearing surface on the whole length of the tube. In the case of a tramcar on the Bessbrook and Newry Tramway the motor is 20 H.P. and the power developed sometimes exceeds this. To transmit this when the car is running at 7 miles an hour the tension on the chain is 1,430 lbs., and the speed 460 ft. per min. At starting the tensions may reach 3,400 lbs., and the maximum speed may reach 1,300 ft. per min.

A steel chain is used of the pattern shown in fig. 382, with the tubes keyed and riveted to the inner links and the pins keyed and riveted to the outer links. The tenacity of the steel is 43½ tons per sq. in. Pitch of chain 2⅛ ins. Weight 8·5 lbs per foot. The breaking stresses are 13·8 tons (shear) on the pins; 10·2 tons on outside links, and 14·8 tons on inside links. The wearing surface is 16 sq. ins. per foot of chain.

Fig. 383 shows the roller bush chain, a further improvement (1880). It has the bush or tube riveted to the inside links and the pin traversing it and riveted to the outside links. But a loose roller is added, which diminishes friction and spreads the wear over a larger surface. The rivets, bushes, and rollers are

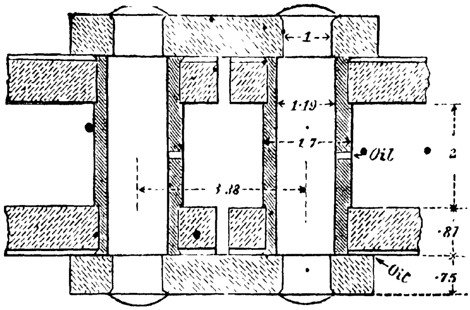



Fig. 382

case-hardened. Three series of these chains are made with short, medium, and long pitch.

The chains of short pitch are made of ½ to 3 in. pitch and of strength from 0·78 to 38 tons. The medium pitch chains from ¾ to 3¾ inch pitch and strength from 0·78 to 26 tons. The long pitch chains from 1 to 5 inch pitch and strength from 0·78 to 26 tons. For the short pitch chains let $p$ be the pitch, $b_1$ the width outside and $b_2$ the width inside the links, then the breaking load is very approximately

$$W = 11{,}000 \text{ to } 15{,}000\, p(b_1 - b_2) \text{ lbs.}$$

The short pitch chain is used at speeds up to 900 ft. per min., the medium up to 600 ft. per min., the long up to 400 ft. per min. But the greatest speed for quiet running depends on the pitch. Thus for the short pitch chain the speed may be

$$v = 1100 - 260\, p \text{ ft. per min.}$$

336. *Silent chains.*—Fig. 385 shows the form of the silent

block chain and sprocket teeth. With single blocks it is made of ½ to 1 in. pitch and of a strength of 1 to 3·7 tons. With multiple blocks it is made of ¾ to 1½ in. pitch and 1½ to 6 ins. wide. An improvement on this chain is the silent bearing chain, fig. 385. The links are fitted with segmental hardened bushes, which bear

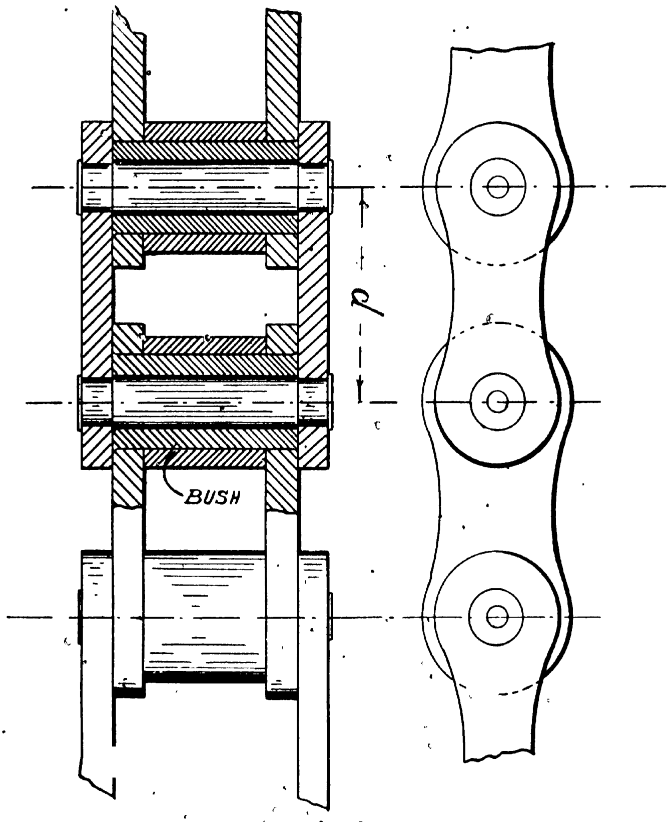



Fig. 383

on the pins. The hole in the alternate links is recessed to clear the bushes. As the chain stretches and wears it rises on the sprocket teeth so as to compensate for the increase of pitch. The chains are made with pitches from ¾ to 1¾ inch and with multiple links from ¾ to 10 ins. width. The strength ranges from 1 to

39 tons. If $p$ is the pitch and $b$ the width over the links, the breaking strength is approximately

$$W = 5200\ p\ b \text{ lbs.}$$

This chain can be used at speeds up to 1,250 ft. per min., or even more if special attention is paid to lubrication. Drives for as much as 500 H.P. have been constructed with these chains.

Mr. Renold allows the publication of the table of silent chains on p. 522.

337. *Working stress on the transmission chains.*—In chains for

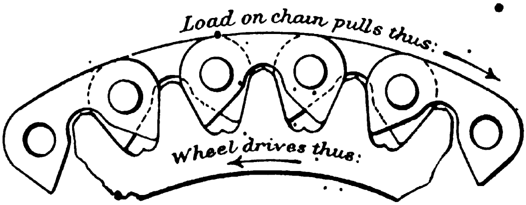



Fig. 384

hoisting a moderate factor of safety is sufficient, and the working load is one-fourth to one-fifth of the breaking load. Such chains are not continuously driven and the wear is not very serious. With transmission chains the case is different, and the load must be diminished to secure durability. The working load

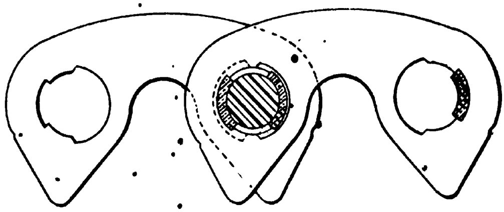


Fig 385

depends chiefly on the bearing area of the rubbing parts. For the block and roller chains, Mr. Renold allows bearing pressures of 600 to 1,000 lbs. per sq. in. For the bushed silent chains, which run faster, a bearing pressure of 650 lbs. per sq. in. This corresponds pretty nearly to taking the working load at one-thirtieth of the breaking load for silent bushed chains or one-thirty-fifth for silent block chains. The working load should be decreased if the conditions are severe—that is, if the load is

impulsive, the sprocket wheel very small, or the chain exposed to grit. The roller chain is used for many widely different

*Bearing Areas.*

*Bushed Silent Chain—2541 Series.*

| Chain No. | Pitch in. | Nominal width in. | Effective width in. | Bearing area sq. ins. | Breaking strength | Approx. HP on bearing surface with 1250 f.p.m. and 650 lbs. per sq. in. pressure | Approx. HP on breaking load with 1250 f.p.m. and 30 factor |
|---|---|---|---|---|---|---|---|
| 2,541 | ·75 | ·75 | ·79 | ·13 | 2,340 | 3·23 | 2·92 |
| | | 1·0 | ·96 | ·159 | 3,510 | 3·92 | 4·39 |
| | | 1·25 | 1·29 | ·214 | 4,680 | 5·26 | 5·86 |
| | | 1·5 | 1·62 | ·269 | 5,350 | 6·62 | 6·7 |
| | | 1·75 | 1·79 | ·297 | 5,850 | 7·3 | 7·32 |
| | | 2·0 | 2·12 | ·352 | 7,030 | 8·67 | 8·8 |
| | | 2·5 | 2·62 | ·434 | 9,380 | 10·7 | 11·7 |
| | | 3·0 | 3·11 | ·516 | 10,560 | 12·7 | 13·2 |
| 2,542 | 1.0 | 1·25 | 1·15 | ·287 | 5,010 | 7·06 | 6·25 |
| | | 1·5 | 1·48 | ·370 | 6,680 | 9·10 | 8·35 |
| | | 2·0 | 1·98 | ·495 | 10,020 | 12·2 | 12·5 |
| | | 2·5 | 2·48 | ·620 | 11,690 | 15·2 | 14·6 |
| | | 3·0 | 2·98 | ·745 | 15,030 | 18·3 | 18·8 |
| | | 4·0 | 3·97 | ·992 | 20,040 | 24·4 | 25·0 |
| | | 5·0 | 4·97 | 1·242 | 25,040 | 30·6 | 31·3 |
| | | 6·0 | 5·97 | 1·492 | 30,050 | 36·8 | 37·5 |
| 2,543 | 1.25 | 2·0 | 2·01 | ·637 | 12,550 | 15·7 | 15·7 |
| | | 2·5 | 2·42 | ·767 | 15,060 | 18·9 | 18·8 |
| | | 3·0 | 3·04 | ·963 | 17,570 | 23·7 | 22·0 |
| | | 4·0 | 4·07 | 1·290 | 25,100 | 31·8 | 31·4 |
| | | 5·0 | 5·10 | 1·62 | 30,100 | 40·0 | 37·6 |
| | | 6·0 | 6·13 | 1·942 | 37,600 | 47·8 | 47·0 |
| 2,544 | 1·5 | 2·0 | 2·01 | ·764 | 15,690 | 18·8 | 19·6 |
| | | 2·5 | 2·42 | ·92 | 18,830 | 22·6 | 23·5 |
| | | 3·0 | 3·04 | 1·155 | 21,970 | 28·4 | 27·75 |
| | | 4·0 | 4·07 | 1·547 | 31,390 | 38·2 | 39·2 |
| | | 5·0 | 5·10 | 1·938 | 37,670 | 47·8 | 47·2 |
| | | 6·0 | 6·13 | 2·33 | 47,090 | 57·5 | 58·8 |
| 2,545 | 1·75 | 3·0 | 2·86 | 1·27 | 26,270 | 31·3 | 32·9 |
| | | 4·0 | 3·85 | 1·71 | 35,030 | 42·10 | 43·8 |
| | | 5·0 | 5·08 | 2·25 | 43,790 | 55·4 | 54·7 |
| | | 6·0 | 6·06 | 2·69 | 52,550 | 66·2 | 65·6 |
| | | 8·0 | 7·94 | 3·52 | 70,060 | 86·5 | 87·5 |
| | | 10·0 | 9·916 | 4·4 | 87,580 | 108·2 | 109·5 |

purposes, and the working load may range from one-tenth to

one-thirtieth of the breaking strength, according to the use to which it is put. The largest factor of safety corresponds to the case where the chain runs continuously transmitting power.

338. *Chain or sprocket wheel.*—The sprocket wheels are usually of cast iron, machine cut, but in special cases of cast steel or bronze. The least and greatest number of teeth in either wheel should generally be as follows :—

| | Least number. | Greatest number |
|---|---|---|
| Block chain | 7 | — |
| Roller chain | 8 to 9 | 72 |
| Silent block chain | 19 | 114 |
| Silent bearing chain | 15 to 18 | 90 |

The greatest desirable velocity ratio is 6 to 8 for the roller chain and 5 to 6 for the silent chain.

*Length of chain.*—Let C be the distance between sprocket

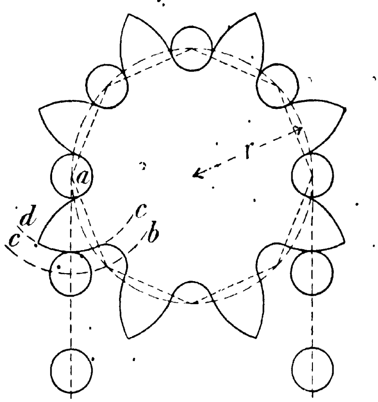



Fig 386

wheel centres in pitches, N and $n$ the numbers of teeth. Then the chain length, measured in pitches, is

$$L = 2C + \tfrac{1}{2}N + \tfrac{1}{2}n + \frac{1}{C}\left(\frac{N-n}{2\pi}\right)^2 \ldots\ldots\ldots\ldots (10)$$

When the chain is on the sprocket wheel the pin centres lie on a polygon, fig. 386, of as many sides $z$ as the wheel has teeth, which is strictly the pitch polygon. The length of one of the sides of the polygon is the pitch $p$ of the chain. This polygon is inscribable in a circle, the radius of which is given by the relation—

$$r = p / \left(2 \sin \frac{180}{z}\right)$$

*Form of teeth of wheel.*—If the chain were perfectly flexible, the path of a point in the chain, say the centre of a pin, when leaving the wheel, would be an involute of a circle, and the tooth curve would be a parallel to the involute distant from it by half the diameter of a pin. In fact, however, as the links are rigid,

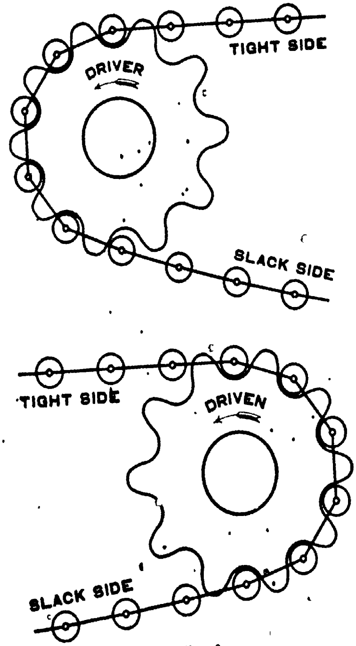



Fig. 387

ach link in unwinding describes a circle $b\ c$, fig. 386, struck from he centre $a$ of the pin which has not left the wheel, and the tooth orm is a parallel to this circle distant from it half the diameter of the pin. That is, the tooth form will be a circle $c\ d$ drawn from centre $a$ with radius $p - \frac{1}{2} d$, where $d$ is the pin diameter. To reduce the rubbing, and to allow for stretch and inaccuracy of the chain, it is desirable, however, to draw the tooth form with a rather shorter radius. The pin then touches the tooth curve

before beginning to unwind, but leaves contact with the tooth as the chain unwinds.

There is always a slight stretch of the chain in working which lengthens the pitch, and this, if excessive, may cause the chain to ride on the sprocket wheel teeth and break. Mr. Renold has found that practically any difficulty from this cause is obviated by a small modification of the driving sprocket wheel. He makes the pitch of this wheel slightly larger than the pitch of the chain (fig. 387). When new the part of the chain advancing on to the driving sprocket wheel first touches the teeth on the back or non-driving side, but in consequence of the difference of pitch touches the forward or driving side of the tooth near the point where it leaves the wheel. The chain can stretch a little without affecting the correct action or incurring a liability to ride the teeth.

339. *Chain drives for irregular loads. Spring sprocket wheels.*—In the case of drives subject to shocks or sudden variations of load, the positive action or inelasticity of the chain drives is sometimes disadvantageous. For such cases Mr. Renold has introduced a spring sprocket wheel—that is, a wheel in two parts, one keyed to the shaft, the other carrying the teeth and chain. The two parts are connected by springs which permit a small relative motion. Such spring wheels are used on pumps, compressors, rolling mills, power hammers, and machines such as rivet forging machines. The springs cushion the blow due to sudden changes of load.